CS 361: Theory of Computation

Assignment 8 (due 11/12/2025)

Instructor: Shikha Singh

LaTeX Source for Solutions: https://www.overleaf.com/read/jynjyzhgxqbx#6332ba

Reminder of Course Academic Integrity Policies. As stated in the syllabus, looking up complete or partial solutions on the internet or AI tools (such as ChatGPT) is not permitted. Assignment problems are a way to practice concepts studied in class and do not require sophisticated techniques beyond what we have seen in class. If you have questions or need help getting started, please reach out to the instructor or the TAs.

Problem 1. Let $\mathsf{REV_{TM}} = \{\langle M \rangle \mid M \text{ is a TM such that } L(M) = (L(M))^R\}$. Recall that $L^R = \{w^R \mid w \in L\}$. In other words, the given TM accepts w if and only if it accepts w^R (the reverse of w). Show that $\mathsf{REV_{TM}}$ is undecidable.

Solution.

□

Problem 2. A ***useless state*** in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable using a **mapping reduction** from a known undecidable problem.

Hint. Reduce from E_{TM}. Make sure there are no extra useless states in a TM than the one you want to check for.

Solution. □

Problem 3. One of the following problems is Turing decidable and the other is not. Formulate each of them as a language and identify whether or not it is decidable. Justify your choice by either providing a decider or prove that one cannot exist. *Remark. To build intuition, it might be helpful to review two solved problems in Sipser: 5.10 and 5.11.*

(a) The problem of determining whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w.

Solution. □

(b) The problem of determining whether a Turing machine M on an input w ever attempts to move its head left three times in a row at any point during its computation on w.

Solution. □