

CS 361: Theory of Computation

Assignment 7 (due 10/29/2025)

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L^AT_EX Source for Solutions: <https://www.overleaf.com/read/mpktnhcxvfyh#6210a2>

Problem 1. Consider the problem of determining whether the language of a given DFA M is finite. Express this problem as a language and show that it is decidable.

Solution.

□

Problem 2. A language is prefix-free if no member is a proper prefix of another member. Let $\text{PrefixFree}_{\text{REX}} = \{R \mid R \text{ is a regular expression and } L(R) \text{ is prefix-free}\}$. Show that $\text{PrefixFree}_{\text{REX}}$ is decidable.

Solution.

□

Problem 3. In class we proved that the power set of $\mathbb{N}$ is uncountable using proof by contradiction and a diagonalization argument. Using the same technique, prove that the set $\mathcal{B}$ of all infinite binary sequences is uncountable. An *infinite binary sequence* is an unending sequence of 0s and 1s.

Solution.

□

Problem 4. (a) Show that $\text{EQ}_{\text{CFG}} = \{\langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) = L(G_2)\}$ is undecidable. You may assume that $\text{ALL}_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is undecidable (Sipser, Theorem 5.13). Reduce from ALL_{CFG} .

Solution.

□

(b) Show that $\overline{\text{EQ}_{\text{CFG}}}$ is Turing-recognizable.

Solution.

□

(c) Is EQ_{CFG} Turing-recognizable? Justify your answer.

Solution.

□

Problem 5. Say that a variable A in a CFG G is **useless** if there exists a derivation for every $w \in L(G)$ that does not use A . Consider the following language:

$$\text{USELESS}_{\text{CFG}} = \{\langle G, A \rangle \mid A \text{ is a useless variable in } G\}.$$

Show that $\text{USELESS}_{\text{CFG}}$ is undecidable.

You may assume that $\text{ALL}_{\text{CFG}} = \{\langle G \rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is undecidable (Sipser, Theorem 5.13). Reduce from ALL_{CFG} .

Solution.

□