CS 361: Theory of Computation

Assignment 5 (due 10/15/2025)

Instructor: Shikha Singh

LaTeX Source for Solutions: https://www.overleaf.com/read/prfwnnchxqpz#6a5078

Problem 1. Give the state-diagram of the push-down-automata with four states recognizing the following language:

$$L = \{w \in \{a, b\}^* \mid w \text{ has an equal number of } a\text{s and } b\text{s}\}.$$

Solution.

□

Problem 2. A string y is said to be a *permutation* of the string x if the symbols of y can be reordered to make x. For example, the permutations of string $x = 011$ are $\{110, 101, 011\}$. For a language L, let $\text{perm}(L) = \{w \mid w \text{ is a permutation of some } x \in L\}$. For example, if $L = \{0^n1^n \mid n \geq 0\}$, then $\text{perm}(L)$ is the set of strings with equal number of 0s and 1s.

(a) Give an example of a regular language L over the alphabet $\{0, 1\}$ such that $\text{perm}(L)$ is not regular. Justify your answer.

Solution.

□

(b) Give an example of a regular language L over the alphabet $\{a, b, c\}$ such that $\text{perm}(L)$ is not context-free.

Solution.

□

Problem 3. Show that the set of context-free languages is closed under the reverse operation. That is, consider a context-free language L, then show that $L^R = \{w^R \mid w \in L\}$ is also context-free. *Remark. Construct a CFG G^R for L^R given a CFG G for L and argue that $L(G^R) = L^R$.*

Solution.

□

Problem 4. Are the following statements true or false? Provide a brief justification.

(a) Context-free languages are closed under intersection.

Solution.

□

(b) The intersection of a context-free language and a regular language is context-free.

Solution.

□

(c) For any context-free language there exists a deterministic push-down automaton that recognizes it.

Solution.

□

Problem 5. Prove that the following languages are not context-free:

(a) The language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s.

Solution.

□

(b) $\{t_1\#t_2\#\cdots\#t_k \mid k \geq 2,\ \text{each } t_i \in \{a, b\}^*,\ \text{and } t_i = t_j \text{ for some } i \neq j\}$

Solution.

□