CS 361: Theory of Computation

Assignment 4 (due 10/01/2025)

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LaTeX Source for Solutions: https://www.overleaf.com/read/nsjzrrkvkhrs#bec5c3

Problem 1. Are the following statements true or false? Justify your answers with an explanation or a counterexample.

(a) Every nonregular language is infinite.

Solution.

□

(b) The class of regular languages are closed under set difference.

Solution.

□

(c) Suppose $L_0, L_1, \ldots, L_i, \ldots$ is a countably infinite sequence of regular languages. Then, their union $\cup_{i \geq 0} L_i$ is also regular.

Solution.

□

(d) If $|A| = n$ and $|B| = m$, then $|A \circ B| = m \cdot n$.

Solution.

□

(e) $L_1 = L_2$ if and only if $L_1^* = L_2^*$.

Solution.

□

Problem 2. Non-regular languages may satisfy the pumping lemma. Consider the language

$$F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}.$$

(a) Prove that F is not regular by first arguing that the related language $\{ab^n c^n \mid n \geq 0\}$ is not regular.

Solution.

□

(b) Show that F satisfies the conditions of the pumping lemma. In particular, show that there exists a pumping length p (specifically consider $p = 3$), such that all $w = a^i b^j c^k \in F$ with length $|w| \geq p$ there exists a breakdown of w into x, y, z satisfying the conditions of the pumping lemma.

Solution.

□

Problem 3. Give a **one-sentence explanation of the the bug** in the following (incorrect) attempts to use the pumping lemma.

Attempt 1. We show that $L = \{ww^R \mid w \in \{0,1\}^*\}$ is not regular using pumping lemma. Suppose L is regular. Let p be the pumping length given by the pumping lemma. Consider the string $s = 0^p110^p$. Then $s \in L$ and $|s| \geq p$. Then it is possible to divide s into three pieces $s = xyz$ such that $xy = 0^p$, $|y| > 0$, and $z = 110^p$. This satisfies the conditions (1) and (2) of the pumping lemma. However, the string xz will have less than p leading zeroes and exactly p trailing zeroes which is no longer a palindrome. Thus, L is not regular.

Solution.

□

Attempt 2. We show that $L = \{ww^R \mid w \in \{0,1\}^*\}$ is not regular using pumping lemma. Suppose L is regular. Let p be the pumping length given by the pumping lemma. Consider all strings $w \in L$ of the form $w = \{\sigma 11 \sigma^R \mid \sigma \in \Sigma^* \text{ and } |\sigma| = p\}$. Then $w \in L$ and $|w| \geq p$. Among all ways to divide w into three pieces $w = xyz$ that satisfy the conditions (1) and (2) of the pumping lemma, we have that y must contain a non-empty substring of σ. However, removing y means that xz is now of the form $\sigma' 11 \sigma$, where $|\sigma'| < |\sigma|$ and $xz \notin L$. This is a contradiction, and thus L is not regular.

Solution.

□

Problem 4. Give context-free grammars for the following languages.

(a) $\{w \in \{0,1\}^* \mid w \text{ contains more 1s than 0s}\}$.

Solution.

□

(b) $\{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$.

Solution.

□

(c) $\{w \in \{0,1\}^* \mid w \text{ is not a palindrome, i.e., } w \neq w^R\}$.

Solution.

□