CS 361: Theory of Computation

Assignment 3 (due 09/24/2025)

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LaTeX Source for Solutions: https://www.overleaf.com/read/wrpptzxwzfkx#835158

Problem 1. In this problem, we are going to prove the only-if direction of the Myhill-Nerode theorem. Let L be a language over the alphabet Σ. We will show that if the relation $\equiv_L$ over Σ^* has finite number of equivalence classes then L is regular. Let k be the number of equivalence classes defined by $\equiv_L$ over Σ. Let the set C contain exactly one representative string from each of the k equivalence classes, that is, $|C| = k$ and every pair of strings in C are mutually distinguishable wrt L. Let $e : \Sigma^* \to C$ be an onto function that maps any string in Σ^* to its representative element from C.

We now construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for L with exactly k states. Let $Q = C$, that is, create k states, labeled with strings from C.

(a) Identify the start and accept states of M in terms of the function e and language L.

Solution. $q_0 = \boxed{\text{STATE HERE}}$

$F = \boxed{\text{STATE HERE}}$

□

(b) State the transition function δ in terms of the function e.

Solution.

□

(c) Consider a string $w = w_1 w_2 \cdots w_n$, where each $w_i \in \Sigma$. Let $w[0] = \varepsilon$ and $w[i] = w_1 w_2 \cdots w_i$. What state is M in after reading i symbols from w, where $i = 0, \ldots, n$ (in terms of the function e)?

Solution. $\boxed{\text{STATE HERE}}$

□

(d) Using part (c) above, give a one-sentence justification of why M recognizes L.

Solution.

□

Problem 2. Use the Myhill Nerode theorem to answer the following questions about the language L_k defined for a fixed number $k \geq 1$ and $\Sigma = \{a\}$.

$$L_k = \{a^\ell \mid \ell \text{ is a multiple of } k\}$$

(a) State all the equivalence classes of the relation $\equiv_L$ over Σ^*. Prove that they form equivalence classes—that is—any pair of strings in the same class are indistinguishable wrt L, any pair of strings in different classes are mutually distinguishable wrt L and finally that the classes partition Σ^*.

Solution.

□

(b) Using (a) argue that L_k is regular. How many states does a minimal DFA for L require?

Solution.

□

Problem 3. Prove that the following languages are not regular using the **Myhill Nerode theorem**.

(a) $L = \{0^n 1^{m+n} 0^n \mid m, n \geq 1\}$.

Solution.

□

(b) $L = \{w \in \{0,1\}^* \mid w = w^R$ and $|w|$ is divisible by $3\}$. (That is, the set of binary strings that are palindromes and have length a multiple of 3.) *Hint. The divisibility by* 3 *is a bit of a distraction, think of ways of avoiding it.*

Solution.

□

Problem 4. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, identify whether it is regular or not regular. If it is regular, provide a DFA, NFA or regular expression for it. If it is not regular, provide a proof using the **pumping lemma** or **closure properties**.

(a) $L = \{w \mid \text{ substrings 01 and 10 appear the same number of times in } w\}$.

Solution.

□

(b) $L = \{w \mid \text{ substrings 00 and 11 appear the same number of times in } w\}$.

Solution.

□

(c) $L = \{xy \mid \text{where } x = 1^k \text{ and } y \text{ contains at least } k \text{ 1s, for some } k \geq 1\}$.

Solution.

□

(d) $L = \{xy \mid \text{where } x = 1^k \text{ and } y \text{ contains at most } k \text{ 1s, for some } k \geq 1\}$.

Solution.

□