CS 361: Theory of Computation

Assignment 10 (due 12/03/2025)

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LaTeX Source for Solutions: https://www.overleaf.com/read/jxxtnsrgctcp#bba409

Note. This assignment is **optional** and it will replace the lowest grade on your HWs so far. (If it is the lowest grade, then it won't be included). If you choose not to turn it in, please do study the questions for the final exam.

Problem 1. Consider the following scheduling problem. You are given a list of final exams $F_1, \ldots, F_k$, a list of students $S_1, \ldots, S_l$, for each students a subset of exams that need to take, and a number h. The goal is to determine if it is possible to schedule the exams using only h slots, such that no student is required to take two exams in the same slot. Show that this problem is NP hard by reducing from either 3-Color or HamiltonianCycle.

Solution. □

Problem 2. State whether the following statements are True or False and provide brief justifications for each.

(a) $\mathsf{PRIMES} \in \mathsf{coNP}$, where PRIMES is the problem of determining whether a given number (in binary representation) is prime or not.

Solution. □

(b) If a language $L \in \mathsf{NSPACE}(\mathsf{f}(\mathsf{n}))$, then $L \in \mathsf{SPACE}(\mathsf{f}^3(\mathsf{n}))$.

Solution. □

(c) If a deterministic Turing machine uses $O(n)$ space, it must run in at most $2^{O(n)}$ time.

Solution. □

Problem 3. Show that $\mathsf{NP} \subseteq \mathsf{PSPACE}$ and that any PSPACE-hard language is also NP hard.

Solution. □

Problem 4. In this problem we will show that if every NP-hard language is also PSPACE-hard, then $\mathsf{PSPACE} = \mathsf{NP}$.

(a) Consider an NP-complete language L. Show that if every NP hard problem is PSPACE hard, then L is PSPACE-complete.

Solution. □

(b) Let L be PSPACE-complete and $L \in \mathsf{NP}$. Use this to show that $\mathsf{PSPACE} \subseteq \mathsf{NP}$.

Solution. □