CS 361: Theory of Computation

Assignment 1 (due 09/17/2024)

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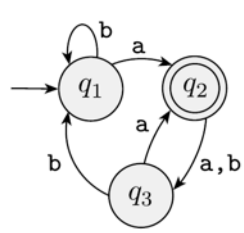
LaTeX Source for Solutions: https://www.overleaf.com/read/vqwcwffkbbcp#702e56

Problem 1. In class we proved that the power set of $\mathbb{N}$ is uncountable using proof by contradiction and a diagonalization argument. Using the same technique, prove that the set $\mathcal{B}$ of all infinite binary sequences is uncountable. An *infinite binary sequence* is an unending sequence of 0s and 1s.

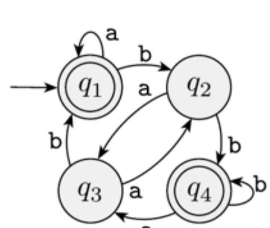
Hint: Suppose $\mathcal{B}$ is countable. Each $s_i \in \mathcal{B}$, it can be represented as $s_i = s_{i,1}, s_{i,2}, s_{i,3}, \ldots, s_{i,j}, \ldots$ where each $s_{i,j} \in \{0, 1\}$. The list of all s_i's can be represented in an infinite matrix where each cell (s_i, j) stores the bit $s_{i,j}$ as shown below.

s_i \ j	1	2	3	4	5	...
s_1	0	1	0	0	0	...
s_2	1	1	1	1	0	...
s_3	1	0	1	0	0	...
s_4	1	0	1	0	1	...
s_5	0	0	0	1	1	...
...	...	...	...	...	...	...
s_k	0	0	1	1	1	...
...	...	...	...	...	...	

Problem 2. The following are the state diagrams of two DFAs, M_1 and M_2.
Answer the following questions about these machines.



M_1



M_2

(a) What sequence of states does M_1 go through on input $aabb$?

(b) Does M_1 accept the string $aabb$?

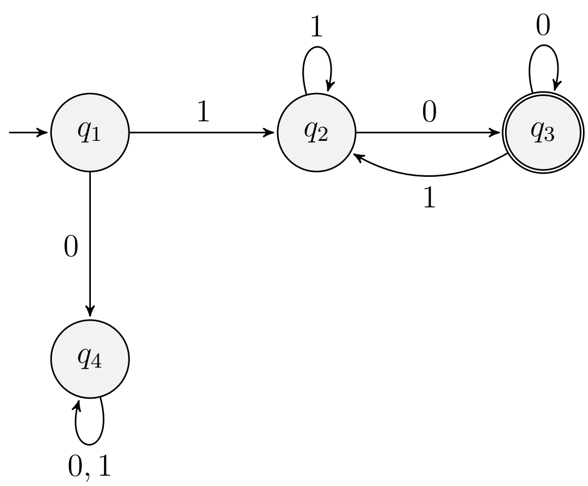
(c) Does M_2 accept the string ε?

Problem 3. Give state diagrams of DFAs recognizing the following languages. In all cases the alphabet is $\{0, 1\}$.

Remark: The first DFA is provided for reference of the LaTeXsource code using the `tikz` package. You can read more about how to typeset automata in tikz here: https://tikz.dev/library-automata. Alternatively, you may use any other software to draw the DFAs, or attach a clear picture of a hand-drawn figure.

(a) **Solved Example:** $\{w \mid w$ begins with a 1 and ends with a 0$\}$.

Solution.



□

(b) $\{w \mid$ starts with 0 and has odd length, or starts with 1 and has even length$\}$.

(c) $\{w \mid w$ every odd position of w is a 1$\}$.

(d) $\{w \mid w$ contains at least two 0s and at most one 1$\}$.

Problem 4. Let $L \subseteq \Sigma^*$ be a regular language. Show that the following two languages are also regular.

$$\text{SUFFIXES}(L) = \{x \in \Sigma^* \mid yx \in L \text{ for some } y \in \Sigma^*\}$$
$$\text{PREFIXES}(L) = \{y \in \Sigma^* \mid yx \in L \text{ for some } x \in \Sigma^*\}$$

Problem 5. Give state diagrams of NFAs with the specified number of states recognizing each of the following languages. Assume the alphabet is $\{0, 1\}$.

(a) The language $\{w \mid w$ contains an even number of 0s, or contains exactly two 1s$\}$. Give an NFA for this language with six states.

(b) The language that contains a pair of 1s separated by an odd number of symbols (0s or 1s). Give an NFA with 4 states for this language.