## CS 361: Theory of Computation

Assignment 7 (due 11/20/2024)

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**LATEX** Source for Solutions: https://www.overleaf.com/read/nyjsmzbbzqph#af253b

Problem 1. Consider the language

 $\bigcap_{CFG} = \{ \langle G_1, G_2 \rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) \cap L(G_2) = \emptyset \}.$ 

Prove that  $\cap_{CFG}$  is undecidable using the reduction from PCP discussed in Lecture 17. Hint. Use the CFLs  $L_A$  and  $L_B$  to prove that if  $\cap_{CFG}$  is decidable, then PCP is decidable.

**Problem 2.** Say that a variable A in a CFG G is **useless** if there exists a derivation for every  $w \in L(G)$  that does not use A. Consider the following language:

USELESS<sub>CFG</sub> = { $\langle G, A \rangle \mid A$  is a useless variable in G}.

Show that USELESS<sub>CFG</sub> is undecidable. *Hint*. Reduce from ALL<sub>CFG</sub>.

**Problem 3.** Show that a language A is Turing recognizable iff  $A \leq_m A_{\text{TM}}$ .

**Problem 4.** Show that the class P and NP are closed under the operations of union and intersection.