CS 361: Theory of Computation

Assignment 6 (due 11/06/2024)

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LaTeX Source for Solutions: https://www.overleaf.com/read/xnthkrbmnjzr#eb8fca

Problem 1. Show that $\mathsf{EQ}_{\mathsf{CFG}} = \{\langle G_1, G_2\rangle \mid G_1, G_2 \text{ are CFGs and } L(G_1) = L(G_2)\}$ is undecidable. Is your reduction a mapping reduction? Justify your answer.

You may assume that $\mathsf{ALL}_{\mathsf{CFG}} = \{\langle G\rangle \mid G \text{ is a CFG and } L(G) = \Sigma^*\}$ is undecidable (Sipser, Theorem 5.13).

Problem 2. (a) Show that $\overline{\mathsf{EQ}_{\mathsf{CFG}}}$ is Turing-recognizable.

(b) Is $\mathsf{EQ}_{\mathsf{CFG}}$ Turing-recognizable? Justify your answer.

Problem 3. Let $\mathsf{REV}_{\mathsf{TM}} = \{\langle M\rangle \mid M \text{ is a TM such that } L(M) = (L(M))^R\}$. Recall that $L^R = \{w^R \mid w \in L\}$. In other words, the given TM accepts w if and only if it accepts w^R (the reverse of w). Show that $\mathsf{REV}_{\mathsf{TM}}$ is undecidable.

Problem 4. One of the following problems is Turing decidable and the other is not. Formulate each of them as a language and identify whether or not it is decidable. Justify your choice by either providing a decider or prove that one cannot exist. *Remark. To build intuition, it might be helpful to review two solved problems in Sipser: 5.10 and 5.11.*

(a) The problem of determining whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w.

(b) The problem of determining whether a Turing machine M on an input w ever attempts to move its head left three times in a row at any point during its computation on w.

Problem 5. A ***useless state*** in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable using a **mapping reduction** from a known undecidable problem.