CS 361: Theory of Computation

Assignment 5 (due 10/30/2024)

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LaTeX Source for Solutions: https://www.overleaf.com/read/nwttqjgvfhpk#71b116

Problem 1. Show that the class of decidable languages are closed under complement and intersection.

Problem 2. A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form $\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$. At each point, the machine can move its head right or stay in the same position. A transition that is of the form $(p, a) \to (q, b, S)$ means that the TM has changed its state from p to q and replaced the symbol on its current head position from a to b but the position of the head has not changed. How powerful is such a TM? We approach this question in two parts.

(a) Show that for each TM, M, with stay put instead of left, there exists a TM, M', that always moves right on each transition such that $L(M) = L(M')$. *Hint. Show that a maximal contiguous sequence of stay-put transitions can be replaced by a single right transition without affecting the language of the machine.*

(b) Now show that the Turing machines with stay put instead of left recognize exactly the class of regular languages by constructing an equivalent DFA. You do not need to prove correctness formally, just give a construction of an equivalent DFA.

Decidable Properties of DFAs and CFGs. The following problems use reductions to problems discussed in class.

Below, is a solved example for your reference. The goal is to convey the format and level of detail you should include in these types of questions.

Solved Example. Let $A = \{\langle R, S\rangle \mid R$ and S are regular expressions and $L(R) \subseteq L(S)\}$. Show that A is decidable.

Solution. We know that $\mathsf{E_{DFA}}$ is decidable, so let S be a TM that decides $\mathsf{E_{DFA}}$. The following TM decides A.

Y = "On input $\langle R, S\rangle$:

1. Since regular languages are closed under intersection and complementation, construct a DFA M that recognizes the language $L(R) \cap \overline{L(S)}$.
2. Run S on input $\langle M\rangle$.
3. If S accepts, then *accept*. Otherwise, *reject*."

□

Problem 3. Let $\Sigma = \{0, 1\}$. Consider the problem of determining whether a given CFG G generates *some* string in $L(1^*)$ is decidable. Express this problem as a language and show that it is decidable.

Problem 4. Consider the problem of determining whether or not a given DFA accepts a string with more 1s than 0s. Express this problem as a language and show that it is decidable.

Problem 5. (Bonus Problem: *Optional*) Show that the problem of determining whether a CFG generates all strings in $L(1^*)$ is decidable.