CS 361: Theory of Computation

Assignment 4 (due 10/16/2024)

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LaTeX Source for Solutions: https://www.overleaf.com/read/ncgyjhymvfpw#bd3fba

Problem 1. In this problem, we will design a push-down-automaton that recognizes the language $L = \{w \in \{a, b\}^* \mid w \text{ has an equal number of } a\text{s and } b\text{s}\}$.

To simplify the construction (and the grading), your PDA must have four states and their purpose as described below:

- s: the start state
- q_{ab}: seen at least as many a's as b's so far
- q_{ba}: seen at least as many b's as a's so far
- f: accept state

Note that the stack of the PDA can be used to keep track of the difference in the number of a's and b's seen so far.

Complete the description of the PDA by providing the details of transitions between these states. For simplicity, assume that you can push more than one symbol on to the stack. For example, $(p, a, b) \to (q, cd)$ captures the transition that PDA moves from state p on reading input symbol a to state q and replaces b on the top of the stack with the symbols cd. You may draw the state-diagram or the PDA or list all of the transitions.

Problem 2. Show that the set of context-free languages is closed under the reverse operation. That is, consider a context-free language L, then show that $L^R = \{w^R \mid w \in L\}$ is also context-free. *Remark. It is sufficient to construct a CFG for L^R given a CFG for L. You do not need to prove correctness.*

Problem 3. Are the following statements true or false? Provide a brief justification for your choice (a counter-example for False statements is ideal when easy to state but not necessary).

(a) Context-free languages are closed under intersection.

(b) The intersection of a context-free language and a regular language is context-free.

(c) For any context-free language there exists a deterministic push-down automaton that recognizes it.

(d) If a language L satisfies the conditions of the pumping lemma for context-free languages then it must be context-free.

Problem 4. Consider the context-free grammar G:

$$\begin{aligned} S \to &\quad TT \mid U \\ T \to &\quad 0T \mid T0 \mid \# \\ U \to &\quad 0U00 \mid \# \end{aligned}$$

(a) Give a description of the strings generated by the grammar G in English.

(b) The pumping lemma for context-free languages states the existence of a pumping length p for $L(G)$. Identify the **minimum** value of p for $L(G)$ that satisfies the pumping lemma? Justify your answer.

Problem 5. Let B be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s. Show that B is not context free using the pumping lemma.