CS 361: Theory of Computation

Assignment 3 (due 10/09/2024)

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LaTeX Source for Solutions: https://www.overleaf.com/read/jdggcphmmkdc#938624

Note. This assignment is a slightly longer two-week assignment. I recommend you finish Problems 1-3 by Oct 2 as an intermediate checkpoint. Students can turn in their partial work by Oct 2 for preliminary feedback and can revise their final submission based on it.

Problem 1. In this problem, we are going to prove the only-if direction of the Myhill-Nerode theorem. Let L be a language over the alphabet Σ. We want to show that if the relation $\equiv_L$ over Σ^* has finite number of equivalence classes then L is regular. Let k be the number of equivalence classes defined by $\equiv_L$ over Σ. Let $[x]$ denote the equivalence class of any string $x \in \Sigma^*$. Our goal is to show that there exists a DFA $M = (Q, \Sigma, \delta, q_0, F)$ for L that has exactly k states.

(a) Describe the states of the DFA M as well as the start state.

(b) Identify the accept states F of M.

(c) Describe the transition function δ in terms of [] notation. Is this a well-defined function?

(d) State a one-line explanation of why this DFA recognizes L.

Problem 2. Are the following statements true or false? Justify your answers with an explanation or a counterexample.

(a) All subsets of a regular language are regular.

(b) The class of regular languages are closed under set difference.

(c) Suppose $L_0, L_1, \ldots, L_i, \ldots$ is a countably infinite sequence of regular languages. Then, their union $\cup_{i \geq 0} L_i$ is also regular.

(d) $L_1 = L_2$ if and only if $L_1^* = L_2^*$.

Problem 3. Prove that the following languages are not regular.

(a) $L = \{0^n 1^{m+n} 0^n \mid m, n \geq 1\}$

(b) $L = \{w \in \{0,1\}^* \mid w = w^R \text{ and } |w| \text{ is divisible by } 3\}$. (That is, the set of binary strings that are palindromes and have length a multiple of 3.)

Problem 4. For each of the following languages over the alphabet $\Sigma = \{0, 1\}$, identify whether it is regular or not regular. If it is regular, provide a DFA, NFA or regular expression for it. If it is not regular, provide a proof using any of the methods discussed in class.

(a) $L = \{w \mid$ substrings 01 and 10 appear the same number of times in $w\}$.

(b) $L = \{w \mid$ substrings 00 and 11 appear the same number of times in $w\}$.

(c) $L = \{1^k w \mid w$ contains at least k 1s, for $k \geq 1\}$.

(d) $L = \{1^k w \mid w$ contains at most k 1s, for $k \geq 1\}$.

Problem 5. We would like to show that if non-regular languages may satisfy the pumping lemma. Consider the language

$$F = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}.$$

(a) Show that F is not regular by using closure property of regular languages.

(b) Show that F satisfies the conditions of the pumping lemma.

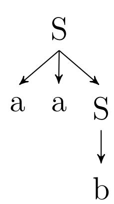
(c) Explain (in a sentence) why parts (a) and (b) do not contradict each other.

Problem 6. Give context-free grammars for the following language and draw the parse tree for the indicated strings. The first part is solved so you can refer to the tikz code for the parse tree.

(a) $\{w \in \{a, b\}^* \mid$ length of w is odd$\}$. Draw a parse tree for the string aab in this language using your grammar.

Solution. **Solved Example**

$S \to a \mid b \mid aaS \mid abS \mid baS \mid bbS$.



□

(b) $\{w \in \{0, 1\}^* \mid w$ contains more 1s than 0s$\}$. Draw a parse tree for the string 1110 in this language using your grammar.

(c) $\{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$. Draw a parse tree for the string $aabbbccc$ in this language using your grammar. Is your grammar for this language ambiguous? Explain your answer.