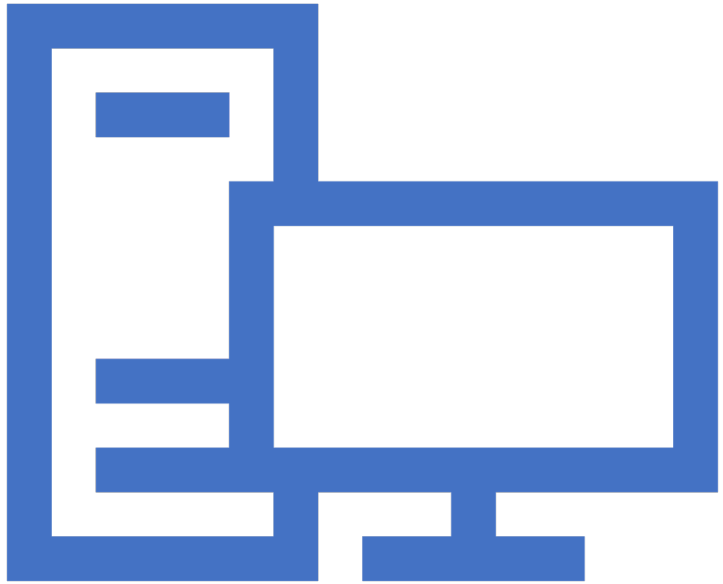
Applied Algorithms

Lecture 5: Cache efficiency



Admin

- Assignment 2 out
- Assignment 1 back soon
- Short(ish) code review Thursday

- Mini-midterm next week

Assignment 1

- Went pretty well
- Lots of great ideas and discussion!
- Everyone had very fast code
  - I thought I'd be near the middle ☹

Reminders about submissions

- Don’t need to beat my time (of course)
- Ties get good points
  - You are motivated to collaborate so long as you wind up in the ballpark of each other
- Times get much lower (~10% or so) during tests
  - One motivation to submit early

Changes

Feedback / late days

- The script should now tell you something about what went wrong
  - Wrong answer/seg fault/bad output/worse time/etc.
  - In errors.txt
- Assignments get 20% off per day late
  - Syllabus updated later today

Testing code

- Guess and check coding
  - Not reflective of the task
  - Debugging is a skill!
  - Loop invariants, etc: can you prove that your code always works?
- But: sometimes it happens
  - Time/results tradeoff
  - Black box issues: works on every input except the secret test

Testing moving forward

- I gave many more tests on Assignment 2
- Can “buy” five more tests for 5 points
  - Minor penalty
  - But, encourages you to exhaust other options

- This will all be updated on the assignment overview handout soon

In-class descriptions

- My goal was to give a “high-level” description in class, and a more detailed description on the assignment itself
- Interpreting a description into code is a part of the class
- Midterms don’t have a leaderboard, etc.

- But I think I’m going to back off this a bit and try to address more details
- I will edit slides over the next couple days

Edit distance

A few important points

- First: strings in C
- “Null-terminated”: just an array that ends in a special character
- ‘\0’
- This character has value 0 if (say) cast to an int

Storing solutions

- I mentioned last time that there are a few good ways
- Probably the easiest: build them bottom-up using arrays
  - Create a simple solution for the base case
  - After both recursive calls, find out the size of both solutions
  - Create a new array large enough to handle both together
  - Copy them over
  - Delete the solutions from the recursive calls
  - Pass the solution to the next level up

Subproblems

- Make sure your subproblems don't overlap!
- Think of it as partitioning both strings

- Hirshberg's works because:

  If ED(X,Y) = c, then if we divide X in half to get X[1,…,n/2] and X[n/2+1,…n], there must be a partition of Y into Y[1,…a] and Y[a+1,…n] such that

  ED(X[1,…,n/2],Y[1,…a]) + ED(X[n/2+1,…,n],Y[a+1,…,n]) = c

  (Proof on board)

Subproblems: example

- Edit distance between cde and aab
- String length/2 = 1

		a	a	b
	0	1	2	3
c	1	1	2	3

		b	a	a
e	1	1	2	3
d	2	2	2	3

Subproblems: example

- Edit distance between cde and aab
- String length/2 = 1

- Most efficient to match c to a (cost 1) and de to ab (cost 2)

- Recurse on Hirshberg's(c,a) and Hirshberg's(de, ab)

		a	a	b
	0	1	2	3
c	1	1	2	3

		b	a	a
e	1	1	2	3
d	2	2	2	3

Two towers

An optimization

- Almost all of the fastest solutions used an algorithmi
- Original idea: generate a table, binary search over it f
- $O\left(n2^{n/2}\right)$ time

0	0000
1.73	0010
3.87	0100
5.6	0110
8.88	1000
10.61	1010
12.75	1100
14.48	1110

31.76	0001
33.49	0011
35.63	0101
37.36	0111
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46.24	1111

Idea

- Generate two tables, sort bo
- Then do the binary search
- This improved performance

0	0000
1.63	0010
4.76	0100
5.9	0110
9.74	1000
12.15	1010
15.97	1100
18.72	1110

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Why?

- Are the asymptotics better?
  - Nope
- Constants?
  - No, we're doing the same work
  - (We'll discuss Thursday: can avoid the searches entirely. But even implementations that kept binary searches were sped up)

The answer

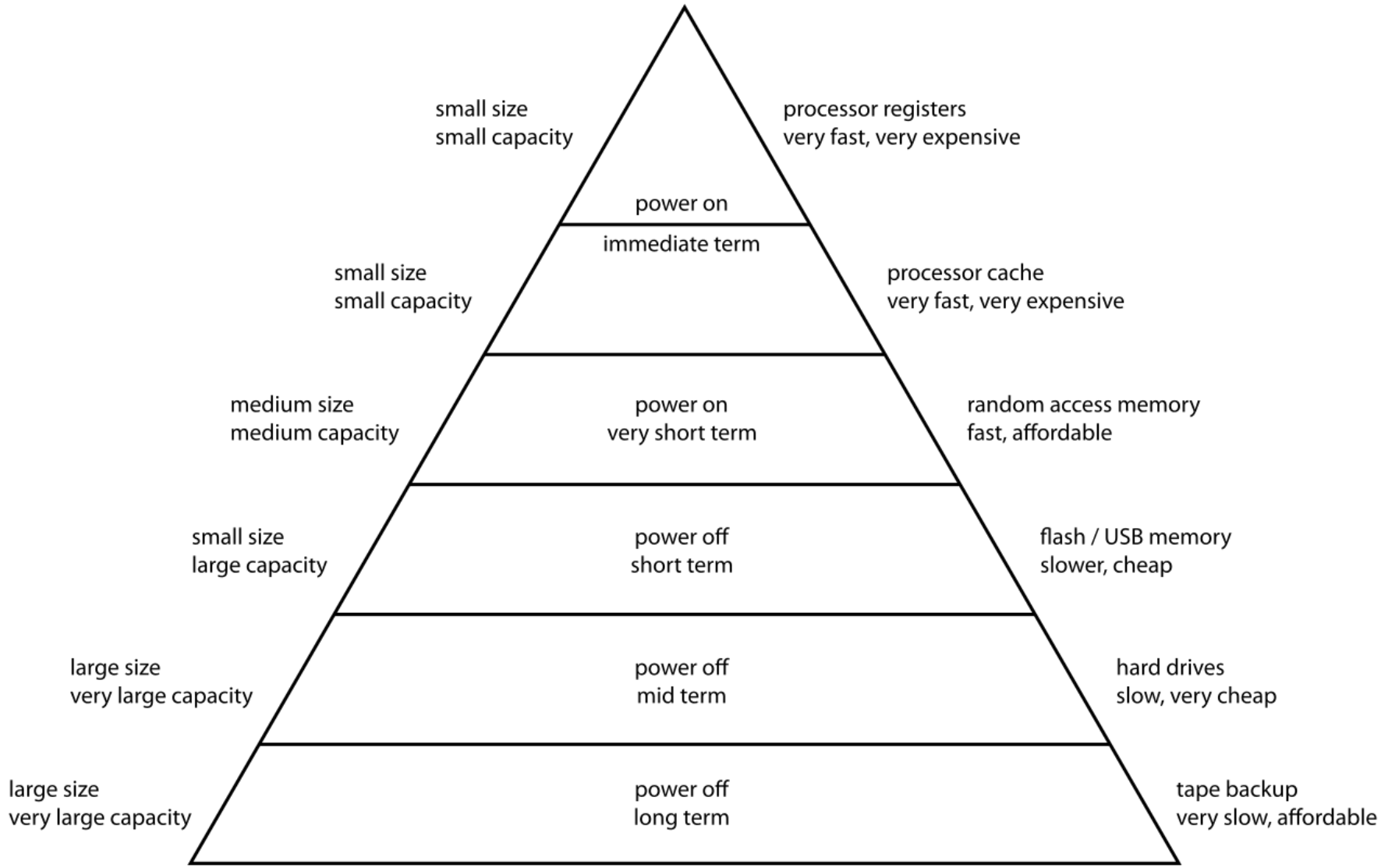
- Cache efficiency!

Cache

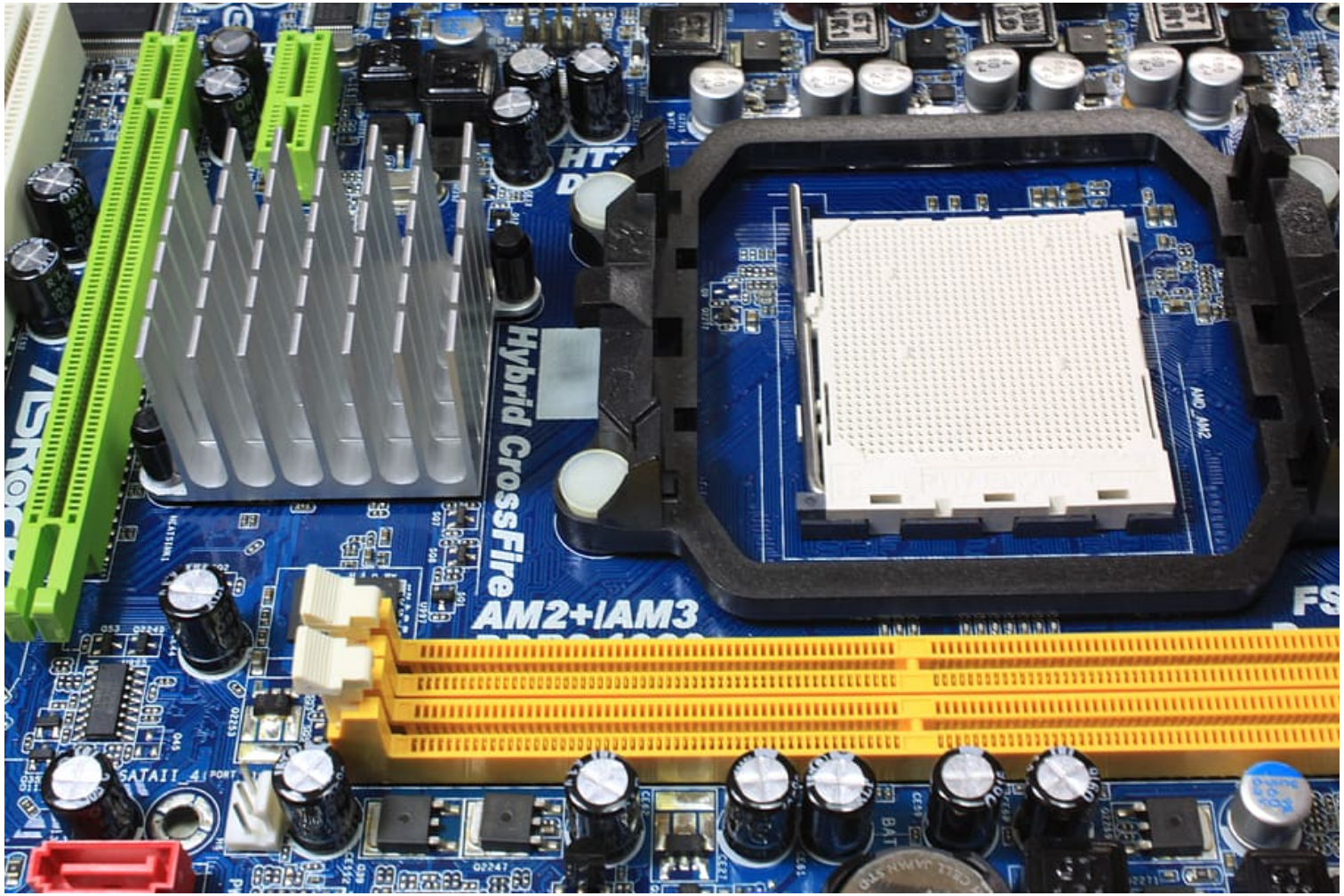
- Your CPU needs to fetch data before it can process it
- Your computer would never run if it ran off the hard drive
- Caching to the rescue!

- Fast memory close to the processor, to minimize time spent reading data

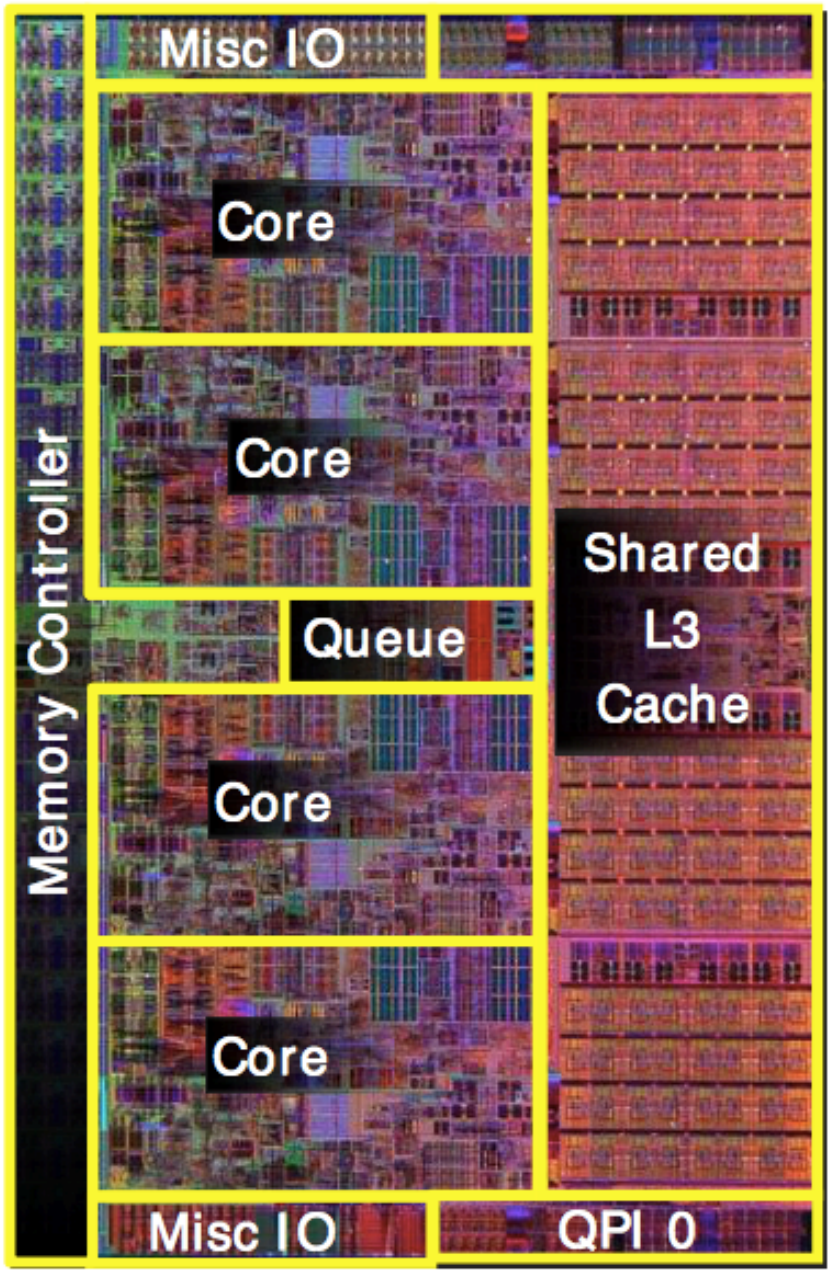
Computer Memory Hierarchy



RAM: pretty big and slow

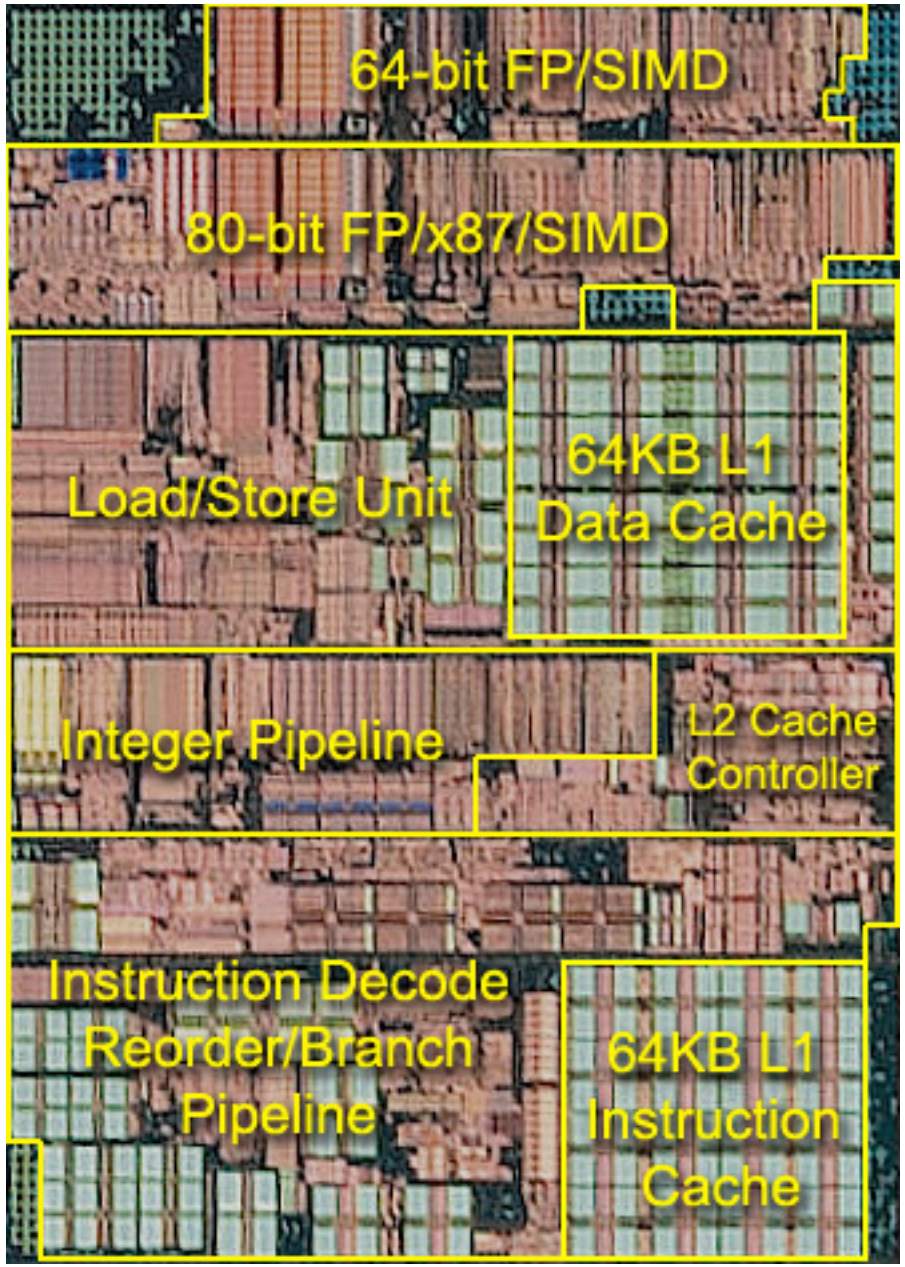


L3/L2 Cache



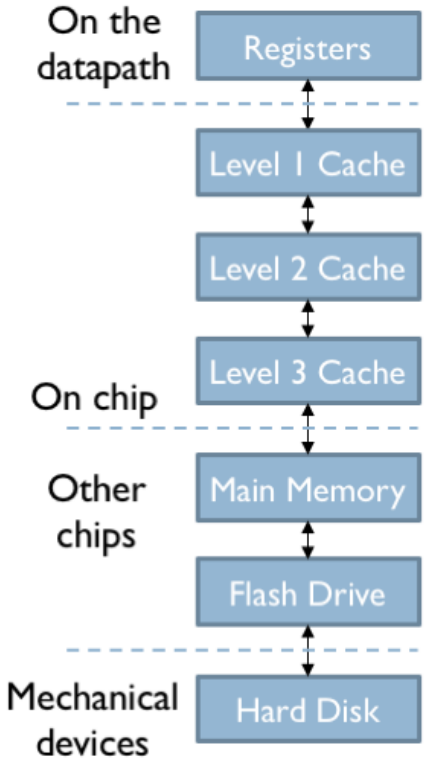
QPI: Intel® QuickPath Interconnect (Intel® QPI)

L1 Cache


64-bit FP/SIMD
80-bit FP/x87/SIMD
Load/Store Unit
64KB L1
Data Cache
Integer Pipeline
L2 Cache
Controller
Instruction Decode
Reorder/Branch
Pipeline
64KB L1
Instruction
Cache

A Typical Memory Hierarchy

- Everything is a cache for something else…



Access time	Capacity	Managed By
1 cycle	1 KB	Software/Compiler
2-4 cycles	32 KB	Hardware
10 cycles	256 KB	Hardware
40 cycles	10 MB	Hardware
200 cycles	10 GB	Software/OS
10-100us	100 GB	Software/OS
10ms	1 TB	Software/OS

What goes in the fast memory?

- Can’t fit much
- Who controls it?

How do these work?

- Your computer decides what is stored where
  - It's very good at it
- Also maintains consistency, etc.

- How to decide what data should be stored in the valuable L1/L2 caches?
  - Computer knows nothing about your program or your data
  - Most recently used data! It's likely to be accessed again

One note:

- This is a massive simplification on modern machines
- TLBs, shared memories, new hardware
- Modern machines have many levels of cache
- Not always even clear that “level” is meaningful

Getting back to two towers

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Getting back to two towers

- Your table probably did not fit in cache---it was stored in RAM
- Before: had to access RAM almost every step (~21 steps) during the binary search

- After sorting: only have to access RAM ~once!

- If you sort both tables, the binary search has an almost-negligible cost

How cache works

- Data is moved around in large(ish) “chunks”
  - Idea: nearby data is also, likely, useful
- Called a “cache line”
- Aligned (this is important for fine tuning, but we will mostly ignore)
- L2/L1 cache line: ~64 bytes

What can we do with this?

- Can we model it?

- Can we predict how well an algorithm will perform, or is it purely experimental?

Simplifying assumptions

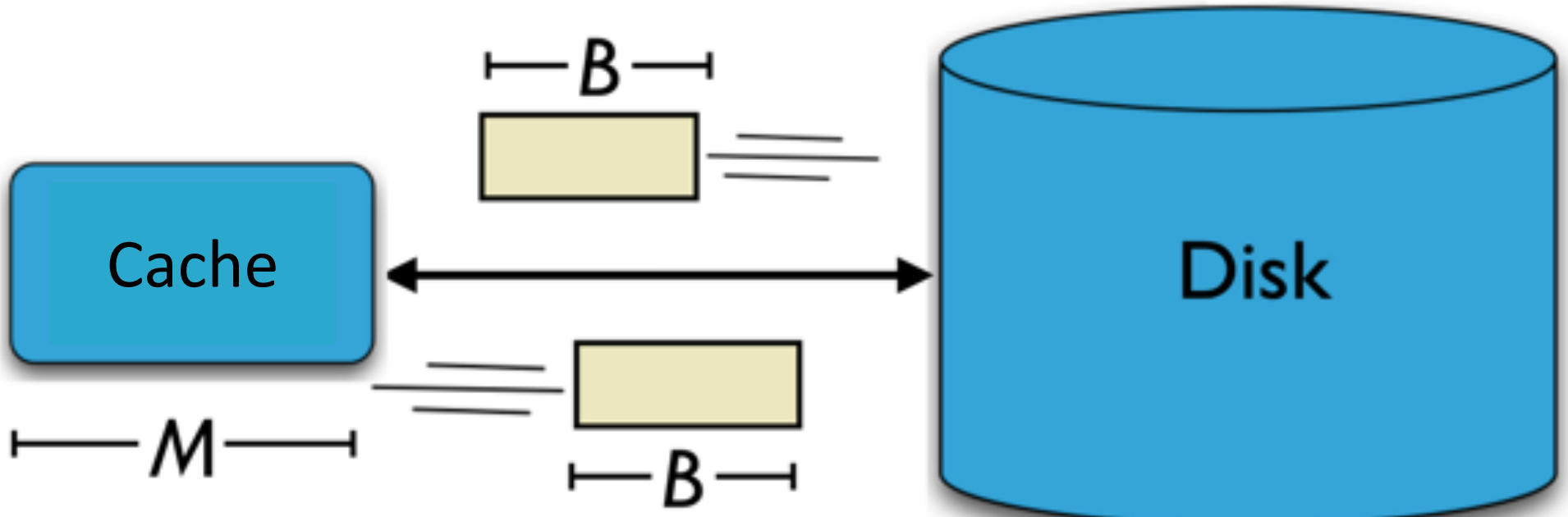
- Just look at *one* level of the memory hierarchy
  - It's very common that one is the bottleneck
- Assume that the computer always keeps the right things in memory
  - Pretty close to practice
- Just focus on moving items around
  - Ignore computation time (can analyze separately)

External Memory Model

Also called “DAM” or “Disk Access Model”

Components

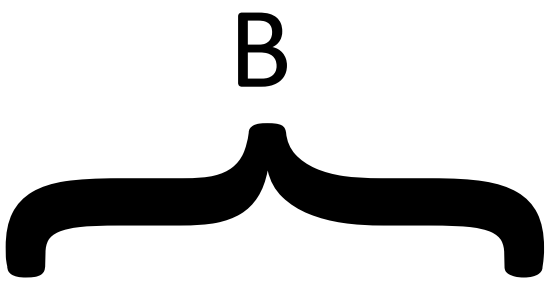
- We have a small cache (of size M)
- Rest of memory is unbounded size. Let's call it the "disk."
  - It's not always a hard disk (oftentimes it's the RAM), but "disk" emphasizes its size and slowness
- Can only compute on data that is in cache
  - But computation is free
- Move data around in blocks of size B
- Each movement is called an "I/O"
  - Some people call it a "disk access" or a "cache miss"
- Analyze using big-O notation, parameterized by M and B


B
Cache
Disk
M
B

Components

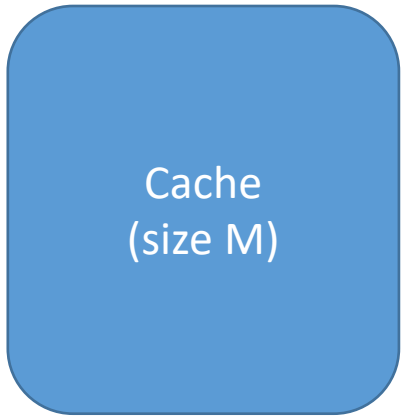
How do we read in an item from disk? (say 78)

Cache
(size M)



1	3	4	3	78	2	99	47	53	61

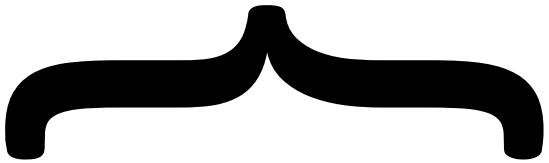
Components



How do we read in an item from disk? (say 78)

Costs 1 to bring it into memory

Now we can compute using 78! And we get 3 and 2 if they're useful.



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Simple example

- How many I/Os does it take to find the minimum element in a list?
- Well, how can we move memory around to do it?

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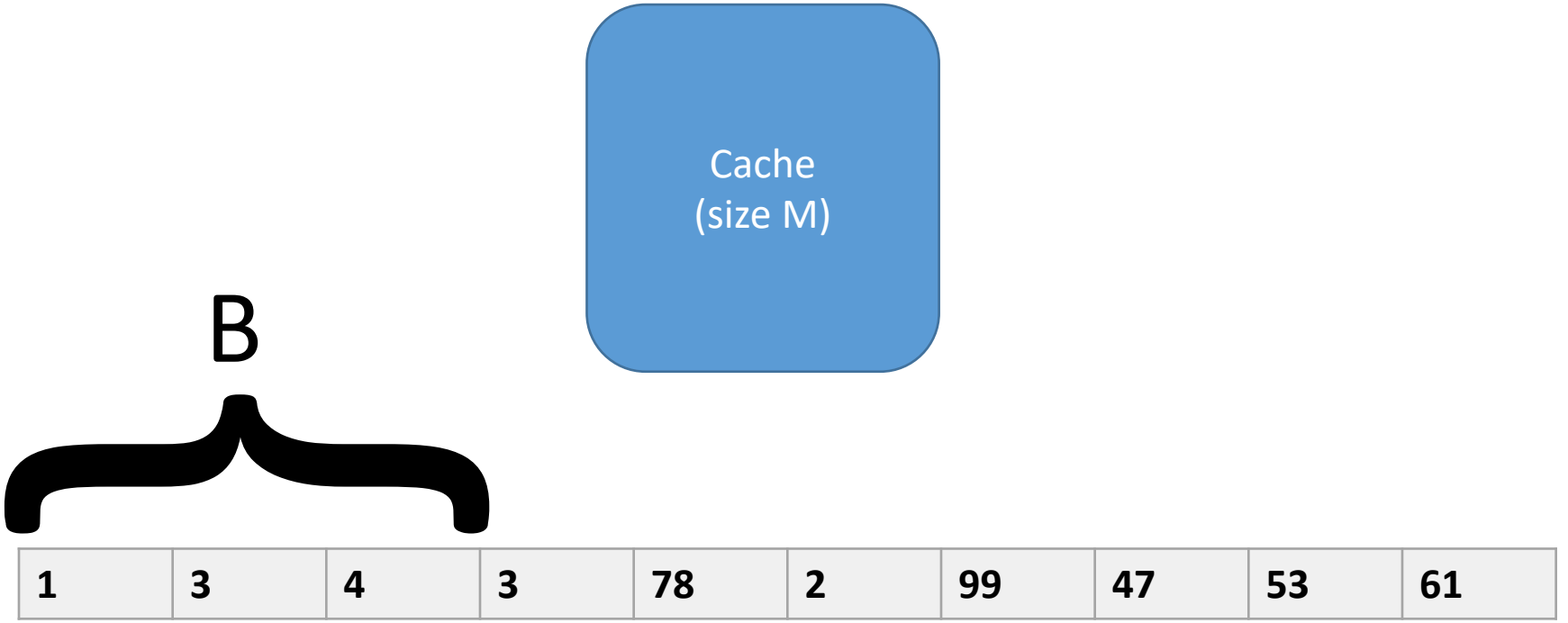

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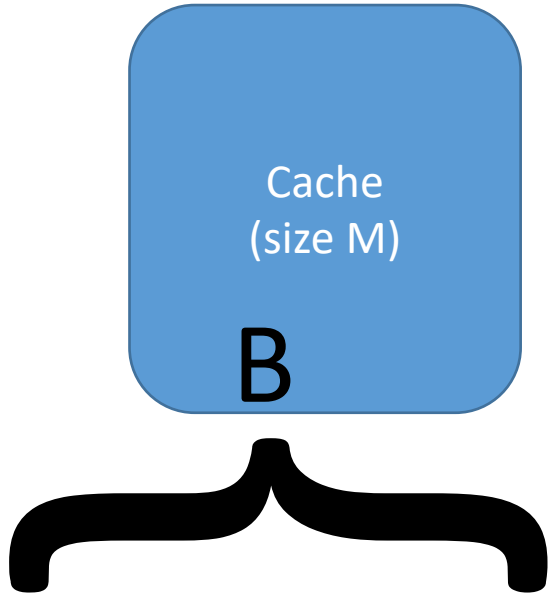


1	3	4	3	78	2	99	47	53	61

Finding the minimum

- How many I/Os does it take?

- Why doesn’t M come into the picture? Does any value of M work?

Binary search

- Let’s assume you do just one binary search. How many I/Os does it take?

- How do we find out how long binary search takes when counting operations normally?
  - Each operation divides the size of the input in two
  - We stop, at the latest, when we get to a range of size 1

Binary search

- When do we stop?
  - When we get to a range of size B
- So if h is the height of our recursion, we have $n/2^h = B$
- Solving, $h = \log_2 n/B$
- Not very good! (Although better than 1 I/O for EVERY operation)

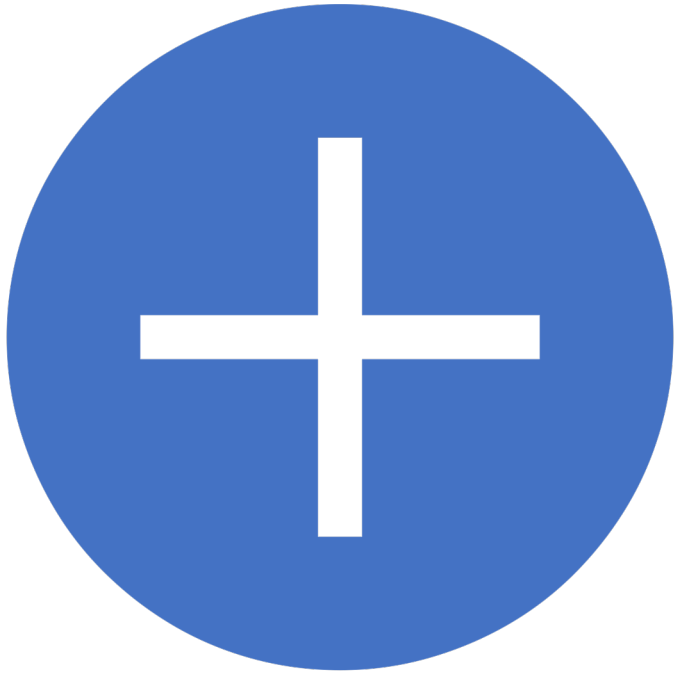
Quicksort

- How many I/Os does it take to partition?
- Basically a linear scan! $O(n/B)$

- At each "level" of quicksort we need to partition all array items
- How many levels?
  - Divide by 2 until size M
  - $O(\log_2 n/M)$

- Overall I/Os: $O((n/B)\log(n/M))$

Matrix multiplication



The problem

- Given two $n \times n$ matrices A, B
- Want to compute their product C:
- $c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$

- Example:

1	2
8	-1

$\times$

2	3
-2	7

$=$

-2	17
18	17

How do we do this?

```
for i = 1 to n
        for j = 1 to n
                for k = 1 to n
                        C[i][j] = A[i][k] + B[k][j]
```

How many I/Os does it take?

Every addition requires an I/O for B: $O(n^3)$

Can we improve this?

```
for i = 1 to n
        for k = 1 to n
                for j = 1 to n
                        C[i][j] = A[i][k] + B[k][j]
```

How many I/Os does it take?

Inner loop gets B additions per I/O: $O(n^3)/B$

I am given two functions for finding the product of two matrices:

```
void MultiplyMatrices_1(int **a, int **b, int **c, int n){
     for (int i = 0; i < n; i++)
         for (int j = 0; j < n; j++)
             for (int k = 0; k < n; k++)
                 c[i][j] = c[i][j] + a[i][k]*b[k][j];
 }

void MultiplyMatrices_2(int **a, int **b, int **c, int n){
     for (int i = 0; i < n; i++)
         for (int k = 0; k < n; k++)
             for (int j = 0; j < n; j++)
                 c[i][j] = c[i][j] + a[i][k]*b[k][j];
}
```

I ran and profiled two executables using `gprof`, each with identical code except for this function. The second of these is significantly (about 5 times) faster for matrices of size 2048 x 2048. Any ideas as to why?

c algorithm matrix matrix-multiplication gprof

share improve this question

edited Sep 13 '11 at 2:47
templatetypedef
295k ● 80 ● 725 ● 933

asked Sep 13 '11 at 0:29
kevlar1818
2,639 ● 4 ● 19 ● 39

add a comment

nswers

active oldest votes

I believe that what you're looking at is the effects of locality of reference in the computer's memory

Can we improve further??

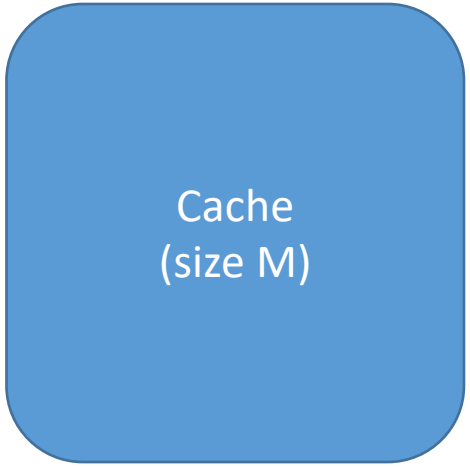
- Our goal is to perform all $O(n^3)$ multiplications with the fewest possible I/Os.

- Restated: our goal is to have each I/O result in the maximum possible number of multiplications.

- What is the most efficient way we can use our cache???

Can we improve further??

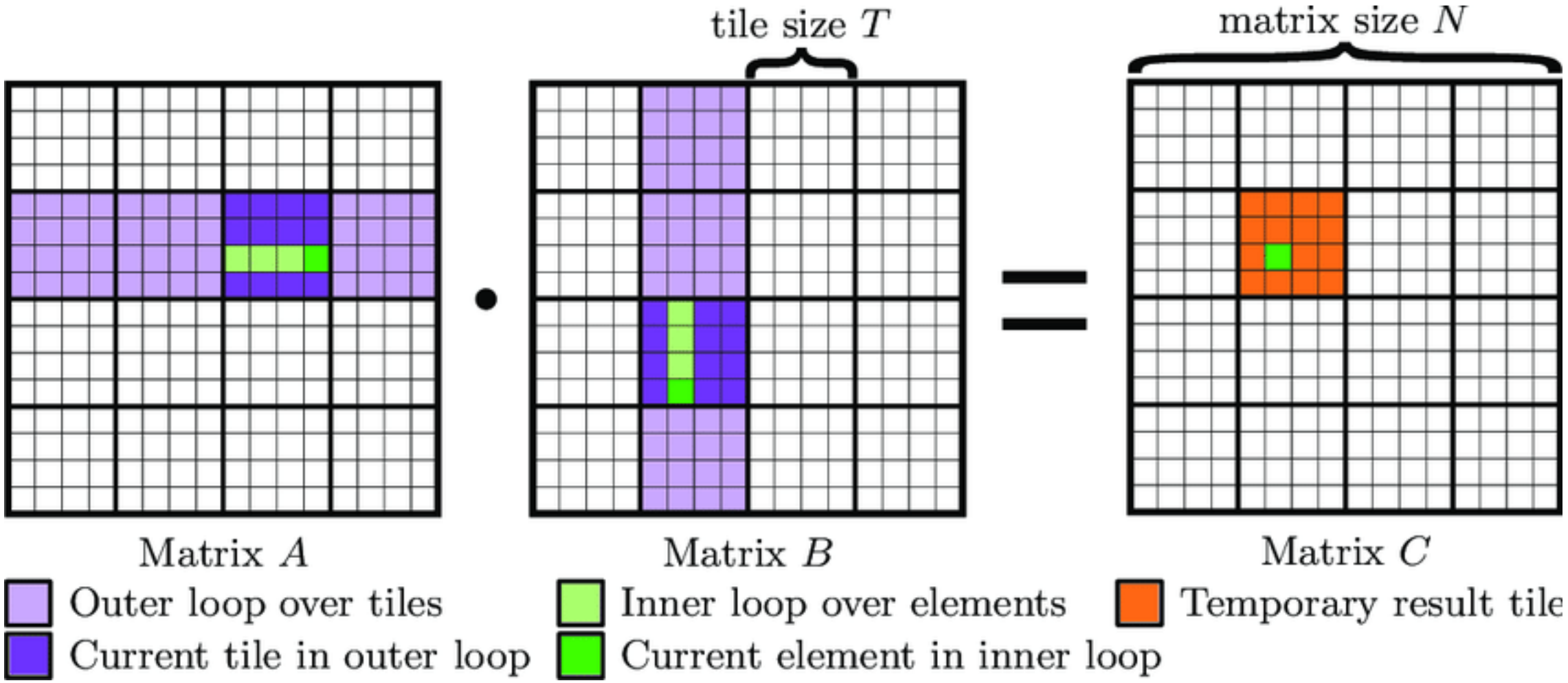
- If we have three matrices, each of total size < M/3, we can fit them all in cache and multiply them
- How many I/Os does this take?
- How many multiplications do we get out of it?



How can we take advantage of this?

- Can we partition matrix multiplication into a series of multiplications of matrices of size at most M/3?

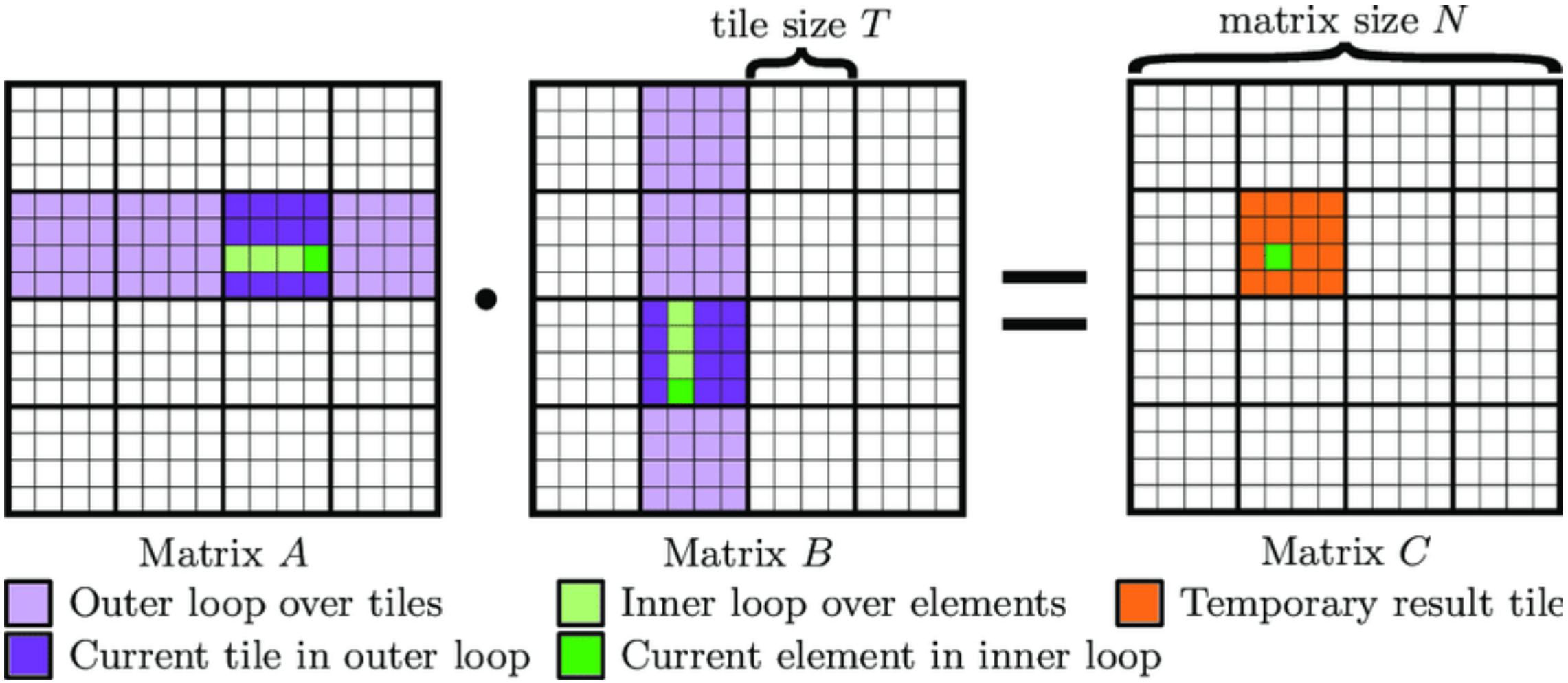
Blocking (tiling)



Blocking (tiling)

- Partition each matrix into ”tiles” (ideally, should fit in memory)
- Outer loop: perform a normal matrix multiplication of two $n/\sqrt{M} \times n/\sqrt{M}$ matrices
- Inner loop: for each tile, multiply the matrices as usual

Blocking (tiling)



Analysis

- How many tiles do we need to multiply?

- How many I/Os does it take to multiply two tiles and store the result?

What about sorting?

- Quicksort: $O((n/B)\log(n/M))$
- Can we do better?
- What does the cache of size M get us?

What about trees?

- What is binary search wasting?
- How can a tree structure resolve this?

Carrying back to practice

- How do we implement this? Do we plug in our best guess for M? What level of the hierarchy do we use?
- Cache improvement: predicted by model
- Constants: experimentation