Lecture 11: Streaming (Updated)

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Introduction

Really Large Data

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- If is possible to store, can be very difficult to access particular pieces

Stream

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- Stream is incredibly long; you can't store all of the items (only log(items)!)
- Can't move forward or backward either; just come in one at a time

Streaming Model

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- What can we do in this situation?
- Note: very active area of research

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- Today: two data structures
    - Count-min sketch: More aggressive than a filter. Good guarantees for counting how many times a given element occurred in a stream.
    - HyperLogLog: Only uses a few bytes. Estimates how many unique items appeared in the stream.
- Note: no proofs today :(. The math behind these structures requires too much background

When to Use Streaming Algorithms?

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- Cache-efficiency! Streaming algorithms only require you to scan the data once.

Actual Applications

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- Google uses an improved HyperLogLog to speed up searches
- Reddit uses HyperLogLog to estimate views of a post

Count-Min Sketch

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Goal:

- Maintain a data structure on a stream of items

Count-Min Sketch

Goal:

- Maintain a data structure on a stream of items
- At any time, estimate how frequently a given item appeared

Example

You see the following items one by one:

adhesive

Example

You see the following items one by one:

flawless

Example

You see the following items one by one:

closed

Example

You see the following items one by one:

adhesive

Example

You see the following items one by one:

describe

Example

You see the following items one by one:

closed

Example

You see the following items one by one:

sea

Example

You see the following items one by one:

illustrious

Example

You see the following items one by one:

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You see the following items one by one:

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You see the following items one by one:

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Example

You see the following items one by one:

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Example

- Now, answer questions of the form: how many times did some item x_i occur in the stream?

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- Example: how many times did adhesive appear? How about closed?
    - (2 times and 3 times respectively)

Formally

- See a stream of elements $x_1, \ldots x_N$, each from a universe U[1]

[1] Like in the last lecture, this is just a requirement to make sure that we can hash them.

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- For some element $e \in U$, estimate how many i exist with $x_i = e$?

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- Today: pretty decent guess using $\lceil \frac{e}{\varepsilon} \rceil \ln(1/\delta) \log_2 N$ bits of space
  - ε and δ are parameters we can use to adjust the error
  - Don't depend on N, or $|U|$

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How would you do this with what you know right now?

- Keep a hash table with all elements
- Increment the counter each time you see an element
- $O(n)$ space, $O(1)$ time per query
- Pretty efficient! But we want way way less space.

Sketching: A first attempt

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  - Keep $n/100$ slots
  - For each item, with probability $1/100$, write the item down.

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Sketching: A first attempt

- If an item appears k times in the stream, we see it $k/100$ times in expectation.
- So, if we wrote an item down w times, we can estimate that it probably occurred $100w$ times in the stream.

Sketching: A first attempt

- But it's pretty loose. If our counter is just one off, that changes our guess by $+100$
- Could have a fairly frequent item that we never write down.
- Miss lots of information!

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Counters of length $\lceil \log N \rceil$ to have room

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- How can we query?

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How can we query q?

Since we always increase this counter when we see $x_i = q$

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But, also increase it when $h(x_i) = h(q)$, but $x_i \neq q$

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- Return $A[h(q)]$
- What guarantees does this give?
  - Always *overestimates* the number of occurrences
  - How much does it overestimate by?
  - Each of N items hashes to same slot with probability ε, so $N\varepsilon$ in expectation

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- Let's say we have two items; A appears 100 times and B appears 900

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Expectation is not that great!

- Let's say we have two items; A appears 100 times and B appears 900
- Query A: with probability $1 - \varepsilon$ we get 100; with probability ε we get 1000

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- How can you increase the reliability of a random process?
- For example, let's say we're rolling a die. We want to be sure we see a 6 at least once. How can we do that?

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- To guarantee a high-quality answer, we want to say that the solution is *likely* to be close to correct.
- How can you increase the reliability of a random process?
- For example, let's say we're rolling a die. We want to be sure we see a 6 at least once. How can we do that?
- Of course: roll the die many times!

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We'll use δ later.

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- Different hash function for each row

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- T has $\lceil \ln(1/\delta) \rceil$ rows
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The e is important for the analysis.

- Different hash function for each row

Inserts

To insert x_i:

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We now have $\lceil \ln(1/\delta) \rceil$ counters for each item. How can we query?

Queries

Each entry is an *overestimate*.

Queries

Each entry is an *overestimate*.

- Find $\min_j T[j][h_j(x_i)]$.

Count-Min Sketch

- Table T with $\lceil \ln(1/\delta) \rceil$ rows, each with $\lceil e/\varepsilon \rceil$ columns. Cells of size $\lceil \log N \rceil$

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- To insert x: set $T[j][h_j(x)]$ for all $j = 0, \dots \lceil \ln(1/\delta) \rceil - 1$

Count-Min Sketch

- Table T with $\lceil \ln(1/\delta) \rceil$ rows, each with $\lceil e/\varepsilon \rceil$ columns. Cells of size $\lceil \log N \rceil$
- $\lceil \ln(1/\delta) \rceil$ hash functions; one for each row
- To insert x: set $T[j][h_j(x)]$ for all $j = 0, \ldots \lceil \ln(1/\delta) \rceil - 1$
- To query q: return $\min_{j \in \{0, \ldots, \lceil \ln(1/\delta) \rceil - 1\}} T[j][h_j(q)]$

Count-Min Sketch Guarantee

- On query q, let's say the filter returns that there were o_q occurrences

Count-Min Sketch Guarantee

So $o_q = \min_j T[j][h_j(q)]$

- On query q, let's say the filter returns that there were o_q occurrences

- In reality, the correct answer is $\widehat{o_q}$ occurrences

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Count-Min Sketch Guarantee

- On query q, let's say the filter returns that there were o_q occurrences
- In reality, the correct answer is $\widehat{o_q}$ occurrences
- First: always have $\widehat{o_q} \leq o_q$.
- Second: With probability $1 - \delta$, $o_q \leq \widehat{o_q} + \varepsilon N$

Count-Min Sketch Bounds

- $\left\lceil \frac{e}{\varepsilon} \right\rceil \left\lceil \ln \frac{1}{\delta} \right\rceil \lceil \log_2 N \rceil$ bits of space
- For any query q, if the filter returns o_q and the actual number of occurrences is $\widehat{o_q}$, then with probability $1 - \delta$:

$$\widehat{o_q} \leq o_q \leq \widehat{o_q} + \varepsilon N.$$

Example Insert

x

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7

Example Insert

x

$h_1(x)$

1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7

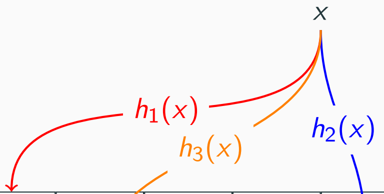
Example Insert

x

$h_1(x)$ $h_2(x)$

1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7

Example Insert



1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0
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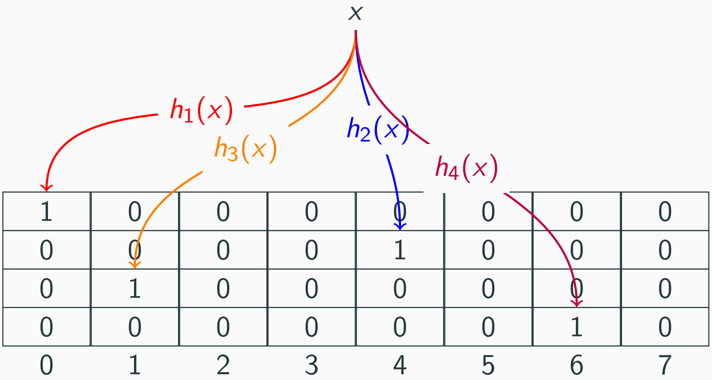
Example Insert



1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	1	2	3	4	5	6	7

Example Insert

y

1	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0
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Example Insert

y

$h_1(y)$

1	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	1	2	3	4	5	6	7

Example Insert

y

$h_2(y)$

$h_1(y)$

1	0	0	1	0	0	0	0
0	1	0	0	1	0	0	0
0	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0
0	1	2	3	4	5	6	7

Example Insert

y

$h_2(y)$

$h_1(y)$

$h_3(y)$

1	0	0	1	0	0	0	0
0	1	0	0	1	0	0	0
0	2	0	0	0	0	0	0
0	0	0	0	0	0	1	0
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Example Insert

y

$h_2(y)$ $h_3(y)$ $h_1(y)$ $h_4(y)$

1	0	0	1	0	0	0	0
0	1	0	0	1	0	0	0
0	2	0	0	0	0	0	0
0	0	0	0	0	0	1	1
0	1	2	3	4	5	6	7

Example Query

q

28	10	78	9	26	69	39	28
85	40	52	70	11	84	65	99
56	82	34	75	99	35	14	55
10	20	17	80	92	89	71	13
0	1	2	3	4	5	6	7

Example Query

q

$h_1(q)$

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q

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56	82	34	75	99	35	14	55
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The estimated number of occurrences for q is 28.

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- Small sketch (size based on error rate)
- Always overestimates count
- Bound on overestimation is based on stream length

Parameters in Assignment CMS

- 300 entries in each row, 4 rows

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Parameters in Assignment CMS

- 300 entries in each row, 4 rows
- 32-bit counters (wasteful!)
- 7.3MB of data summarized in 4.8KB
- Really accurate still: in 1.2 million word stream, can estimate num occurrences of each word within $+1500$
- Often more accurate! Also: feel free to try 1000 or 10000 entries per row; it gets quite accurate

Hyper Log Log Counting

Setting up

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- OK not really. But, it stores a lot of information about the stream
- Common question: how many unique elements are there in the stream?
- (Compare to CMS: stores approximately how many there are of *each* element)

Cool way to solve this

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- Idea: let's look at a rare event in these hashes. The more often it happens, the more distinct hashes we must be seeing!

Cool way to solve this

- Let's hash each item as it comes in
- Then instead of a list of items, we get a list of random hashes
- Idea: let's look at a rare event in these hashes. The more often it happens, the more distinct hashes we must be seeing!
- In particular: how many 0s does each hash end with?

Hashes ending in 0s

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- So if we only see two different hashes, it's very unlikely that either will end in 10 0's.
- If we see $2^{10} = 1024$ distinct hashes, it's pretty likely that one will end with 10 0's.

Hashes ending in 0s

- What is the probability that a hash ends in 10 0's? Answer: 1/1024
- So if we only see two different hashes, it's very unlikely that either will end in 10 0's.
- If we see $2^{10} = 1024$ distinct hashes, it's pretty likely that one will end with 10 0's.
- Note "distinct!" All of this comes back to estimating how many *unique* elements there are. Unique elements give a new hash, and a new opportunity for many zeroes. Non-unique elements don't give a new hash.

Example

You see the following hashes one by one:

0010001010101001

Example

You see the following hashes one by one:

0010110010111101

Example

You see the following hashes one by one:

0001000111101111

Example

You see the following hashes one by one:

0000001011000011

Example

You see the following hashes one by one:

0110010010011100

Example

You see the following hashes one by one:

1000101011100001

Example

You see the following hashes one by one:

0110100100111101

Example

You see the following hashes one by one:

0011101001100010

Example

You see the following hashes one by one:

011000000001110

Example

You see the following hashes one by one:

0011001110001111

Example

You see the following hashes one by one:

1111100010110000

Example

You see the following hashes one by one:

1111110101011100

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You see the following hashes one by one:

1100010011010011

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How many unique items were there?

Example 2

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How many unique items were there? Was it more or less than the last one?

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- Answer: 1st had 14 items, 2nd had 3
- Notice that only one hash in the second example ended with 0
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- One of the items in the first example ended with 4 0's
  - Unlikely if there were 2 elements!

Intuitive loglog counting

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- Any given hash has k 0s with probability $1/2^k$
- So it seems that, there are probably something like 2^k items
- But if we're just off by 1 or 2 zeroes, that affects our answer by a lot!

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- Answer: It's complicated. (And outside the scope of the course.)

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- **Make sure to add 1 to your count of the number of zeroes**

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- Let b be a bias constant.[2] For $m = 32$, $b = .697$.
- Return bm^2/Z.

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x_1

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index $= 110$ Remaining: $0100010001111101111111101010$

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index $= 110$ Remaining: $011110001100100001111010010110$

0	0	0	0	0	0	2	0
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The remaining hash ends with 1 zero, so we want to store 2. The counter stores 2, so we keep it as-is.

Example (with $m = 8$; in practice m is higher)

x_3

Example (with $m = 8$; in practice m is higher)

$$x_3$$

$$h(x_3) = 110011011101100000011010000001$$

Example (with $m = 8$; in practice m is higher)

$$x_3$$

$$h(x_3) = 11001101110110000001101000000 1$$

index $= 001$ Remaining: 11001101110110000001101000000

0	0	0	0	0	0	2	0
000	001	010	011	100	101	110	111

Example (with $m = 8$; in practice m is higher)

x_3

$h(x_3) = 1100110111011000000011010000001$

index $= 001$ Remaining: $1100110111011000000011010000$

0	0	0	0	0	0	2	0
000	001	010	011	100	101	110	111

Example (with $m = 8$; in practice m is higher)

$$x_3$$

$$h(x_3) = 110011011101100000011010000001$$

index $= 001$ Remaining: $110011011101100000011010000$

0	0	0	0	0	0	2	0
000	001	010	011	100	101	110	111

The remaining hash ends with 4 zeroes, so we want to store 5.

The counter stores 0, so we store 5 in the slot.

Example (with $m = 8$; in practice m is higher)

$$x_3$$

$$h(x_3) = 110011011101100000011010000001$$

index $= 001$ Remaining: $110011011101100000011010000$

0	5	0	0	0	0	2	0
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x_4

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$$x_4$$

$$h(x_4) = 10001001110110111011011011 1001$$

index $= 001$ Remaining: $10001001110110111011011011 1$

0	5	0	0	0	0	2	0
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The remaining hash ends with 0 zeroes, so we want to store 1.

The counter stores 5, so we keep the slot as-is.

Example (with $m = 8$; in practice m is higher)

$$x_4$$

$$h(x_4) = 100010011101101110110110111001$$

index $= 001$ Remaining: $100010011101101110110110111$

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The remaining hash ends with 1 zero, so we want to store 2. The counter stores 2, so we keep it as-is.

At the end of the day

Have an array of counters:

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- Sum up $(1/2)^{M[j]}$ across all $j = 0$ to $m - 1$; store in Z
- Return bm^2/Z. Here $m = 8$. We would have to look up the value of b for 8. (No one does HyperLogLog with 8)

Discussion

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- How big do our counters need to be?
- Need to be long enough to count the longest string of 0s in any hash
- Size $> \log \log(\text{number of distinct elements})$ (hence the *loglog* in the name)
- 8-bit counters are good enough, so long as the number of elements in your stream is less than the number of particles in the universe
- Note: one thing to be careful of is hash length. But 64 bit hashes should be good enough for any reasonable application (and 32 bits is usually fine)

HLL in the Assignment

- We'll use $m = 32$ counters
- Bias constant is .697

HLL Beyond the Assignment

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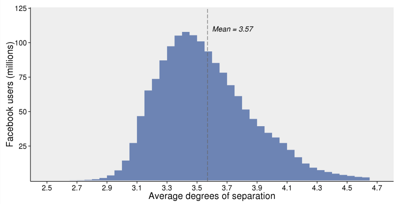
- Google developed HyperLogLog++ to help deal with these problems
- Other known improvements as well

One More Cool Thing



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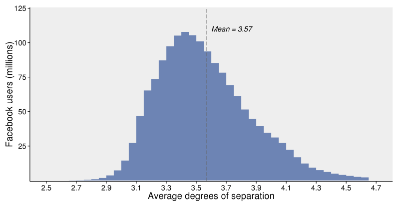

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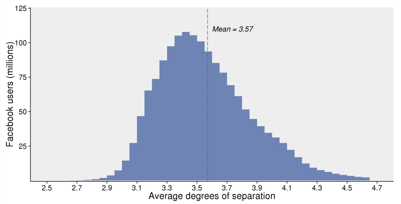

- Facebook developed an HLL-based algorithm to calculate the *diameter* of a graph
- Usually takes $O(n^2)$ time!
- Theirs is essentially linear, gives extremely accurate results