# Applied Algorithms Lec 6: External Memory; Optimization

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Williams College

# Admin

- Next week I will not be here:
  - We will still have lecture (Shikha will teach you about filters, probability and hashing)
  - We will *not* have lab
  - Start Assignment 2 early even though it is due next week!
- Any questions about Assignment 2?
- Today: discuss Assignment 1, external memory practice, discuss other considerations for optimization

Let’s go over Hirshberg’s, and do one (abbreviated) run on the board

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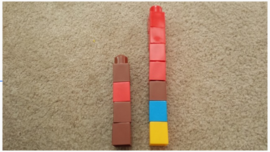
- Use `includegraphics` to put the plot into latex

# Looking back at Assignment 1



- Quick reminder of Implementation 1 vs Implementation 4
- First extra credit: use external memory model to explain why Implementation 4 was faster
- Second extra credit: how to get $O(2^{n/2})$ *operations*?
  - How many cache misses does this incur?

# What is Slow on Modern Computers Beyond Cache Misses?

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    - Any of you have ideas to start?

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- To *know* if something is fast: run an experiment!

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- Float add, multiply?
    - Pretty fast on x86; almost as fast as integers

# Experiments

- Let's run some (really rough) experiments: `timetests.c`
- Unroll loops to minimize loop overhead; compile with optimizations off
- Why is this important? Let's look at the assembly
- Compiler explorer: recent, *super cool* tool to look at assembly for C code
  - `godbolt.org`
  - Awesome for people (like me) who aren't assembly experts but sometimes care about what exactly the computer is doing

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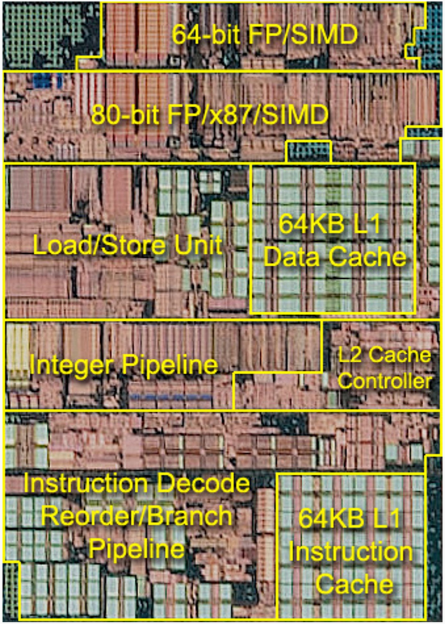
- memory allocation in bytes? `timetests2.c`
- memory allocation in megabytes? (So: fewer allocations, but each of larger size, for same total size)
- how does it grow as we increase the number of operations?
  - Cache efficiency is the problem here, not the memory call itself
  - (For what it's worth: `malloc` really is $O(1)$)

# Latency and Throughput

Latency vs throughput:

- Latency: time it takes for a sequence of *data-dependent* operations of a given type
- Throughput: time after a previous operation when a new operation of the same type can begin.
- Let’s look at an example: `latencythroughput2.c`
- When designing code, be careful of data dependencies; they can significantly affect running time
- (As an aside: this is one reason why statements like “additions take 1 clock cycle” aren’t really possible anymore)

# Modern processors

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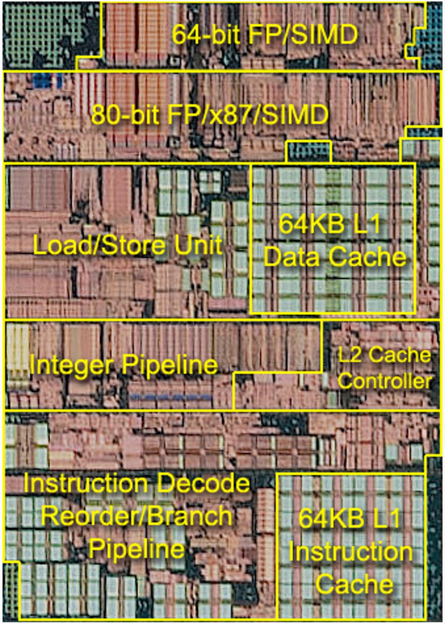


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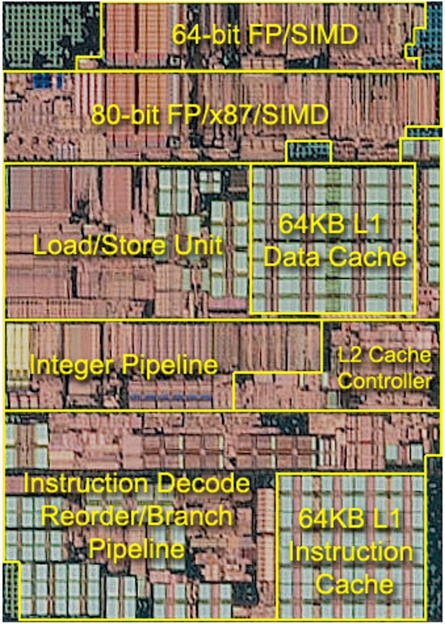


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- Cache costs are the most extreme example of this! But not the *only* example.

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- (Can be free if they don't)

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- “Branch misprediction:” 10-20 cycles to fetch the new instructions from memory
- Can have similar issues with calling non-inlined functions (compiler is very good at avoiding this)

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  - Predictors are so good that this rarely helps

# Avoiding branch mispredictions

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   int diff = a - b;
   int dsgn = diff >> 31;
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- Crazy tricks often not worth it nowadays—true in general; though some exceptions (again, check this only at the very end of optimizing; only for crucial operations)

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- `branchpredictions.c`

# Code Profiling

# Profiling code

- Why not just have your computer tell you what functions are called the most, or keep track of how long they run, or monitor specific high-cost operations?
- Lots of such tools! We'll look at a couple of them right now, and use them throughout the class.
    - `gprof`
    - `cachegrind`
    - We won't use `perf` (needs root access to do many of the interesting things)—but can do cachegrind-like things *without simulation*
    - We won't use Intel VTune either but seems very cool and powerful
- What do you think some advantages and disadvantages are of using profiling software?

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- Gives information about time as well as the call graph
- *Quite* limited. But in some circumstances gives good advice.
  - Recursion; function-level resolution; cannot optimize; overhead; sampling problems

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- Let's look at what it says about Assignment 1

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- Essentially runs the program on a simulated virtual machine
- Gives detailed information about costs you could not otherwise get, but VERY slow.

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- If you instead compile with debug information on, outputs a line-by-line analysis

# Optimization

With these costs in mind, how does one write code that runs quickly?

# Taking out expensive operations

```
for(int i = 0; i < strlen(str1); i++){
  str1[i] = 'a';
}
```

- What's wrong with this code? How long does it take?

[1] Of course, we know that we're never setting any values to 0 before checking them, but the compiler doesn't check for that.

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- It *can't*:[1] we're changing the array, which could change the first `0`.

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# More subtle issues

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- Why is that?
- Don’t need to look up value! (Compiler doesn’t know it doesn’t change after the first iteration)

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- Bear in mind: only common optimizations are implemented
- Opportunities for you: what do you know about your data, and about your methodology, that allows for further efficiency?

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- What are the costs of each iteration of a simple for loop?

```
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# Unrolled Loop

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for(int x = 0; x < 1000; x+=5){
   total += array[x];
   total += array[x+1];
   total += array[x+2];
   total += array[x+3];
   total += array[x+4];
}
```

- In short: repeat body of the loop multiple times.
- What does this gain us?

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- Branch every 5 loops
- Instruction pointer jump every 5 loops
- Compare every 5 loops
- Increment every 5 loops (?) Need to do some *extra* additions however

# What did we need to know to make this substitution?

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- Can get around this with some extra work

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- Can hurt performance if important parts of code no longer fit in cache
- *Fetching instructions* can require cache misses!
  - We saw this in the output of `cachegrind`

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- `-O3` does a specific kind of unrolling of nested loops

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- Many optimizations of code reduce the number of operations (or their total time), but increase the size of the code itself—potentially leading to cache misses

# Revisiting compiler flags

- `-O0`: No optimizations
- `-O1`: Some optimizations; may take longer to compile than `-O0`
- `-O2`: Turns on “nearly all” optimizations *that do not involve a space-time tradeoff*
- `-O3`: More optimizations. May lead to larger final programs
- `-Ofast`: Even more optimizations. Most notable is reordering floating point operations (can lead to correctness issues)

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- Not advised in general
- Example: Gentoo user manual. (Gentoo is a linux distribution in which *all* software is compiled from scratch. So this is advice for people compiling large software like the linux kernel, chromium, libreoffice, etc. (as well as, of course, very small utilities like git))

# Gentoo optimization advice

- `-O1`: the most basic optimization level. The compiler will try to produce faster, smaller code without taking much compilation time. It is basic, but it should get the job done all the time.
- `-O2`: A step up from `-O1`. The *recommended* level of optimization unless the system has special needs. `-O2` will activate a few more flags in addition to the ones activated by `-O1`. With `-O2`, the compiler will attempt to increase code performance without compromising on size, and without taking too much compilation time. SSE or AVX may be utilized at this level but no YMM registers will be used unless `-ftree-vectorize` is also enabled.
- `-O3`: the highest level of optimization possible. It enables optimizations that are expensive in terms of compile time and memory usage. Compiling with `-O3` is not a guaranteed way to improve performance, and in fact, in many cases, can slow down a system due to larger binaries and increased memory usage. `-O3` is also known to break several packages. Using `-O3` is not recommended. However, it also enables `-ftree-vectorize` so that loops in the code get vectorized and will use AVX YMM registers.

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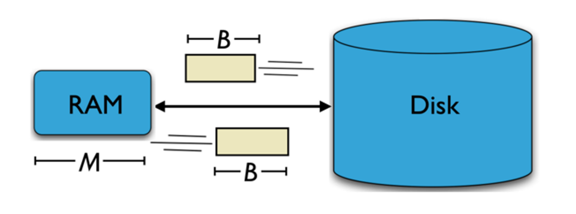
  - Probably want to always use `static inline`
- Can also use `__attribute__((always_inline))` which really forces it to inline even if optimizations are turned off

# One more optimization flag

- `-march=native`
- tells `gcc` to use instructions specific to this processor. May increase speed
- Only disadvantage: your compiled binary may not run on other computers unless they have an identical processor (this is not a problem for us!)

# Sorting in External Memory

# External Memory Model Reminder




- Cache line of size *B*
- Cache can hold at most *M* elements
- Computing on elements in cache is free! Only cost is to bring things in and out of cache

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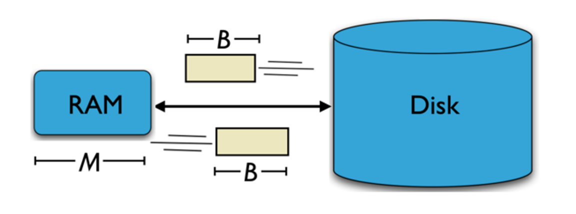

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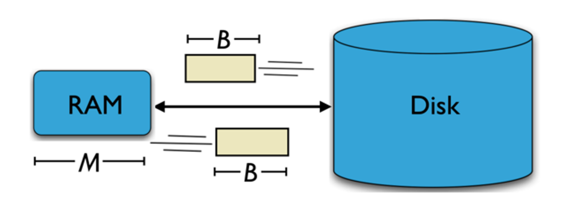


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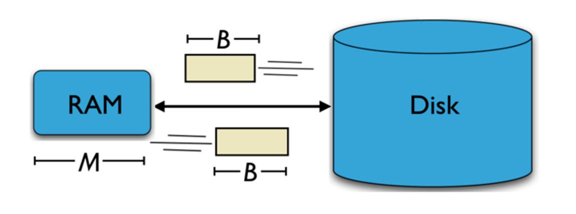


- Let's say I want to find the maximum element in an array using a linear scan. How much time does that take?
    - $O(n/B)$ cache misses: if I have a cache miss accessing $A[i]$, my next cache miss is accessing $A[i+B]$.

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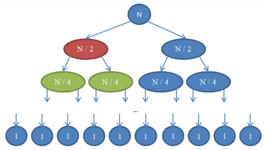


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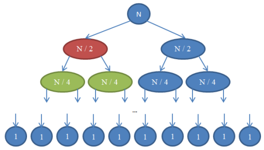


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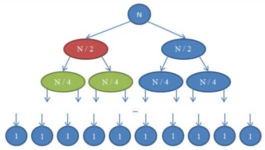


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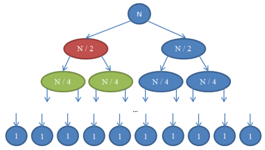


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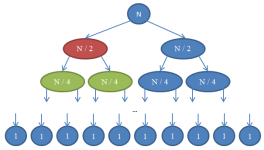


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- Seems pretty good! Can we do better?

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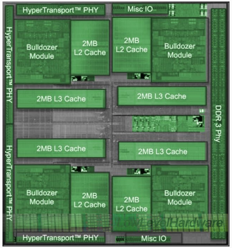


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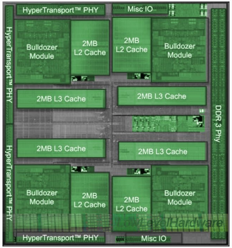


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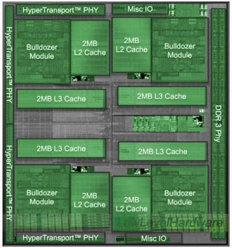


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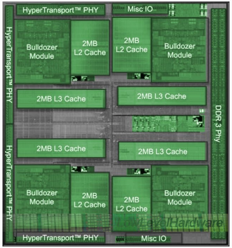


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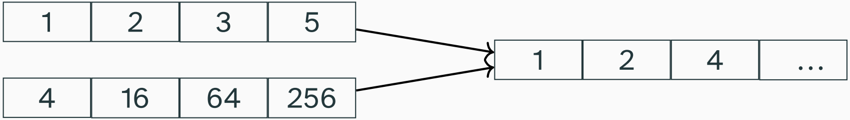
- Each time we bring a cache line into cache, how many more things can we compare it to?

# Merge sort reminder

- Divide array into two equal parts
- Recursively sort both parts
- Merge them in $O(n)$ time (and $O(n/B)$ cache misses)



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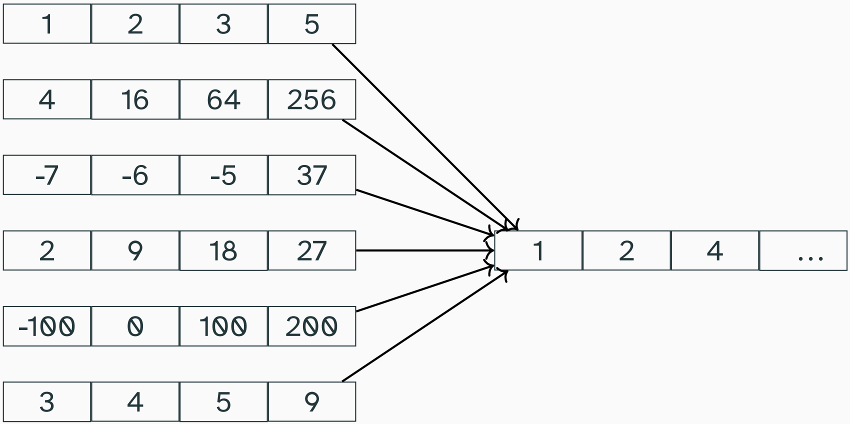

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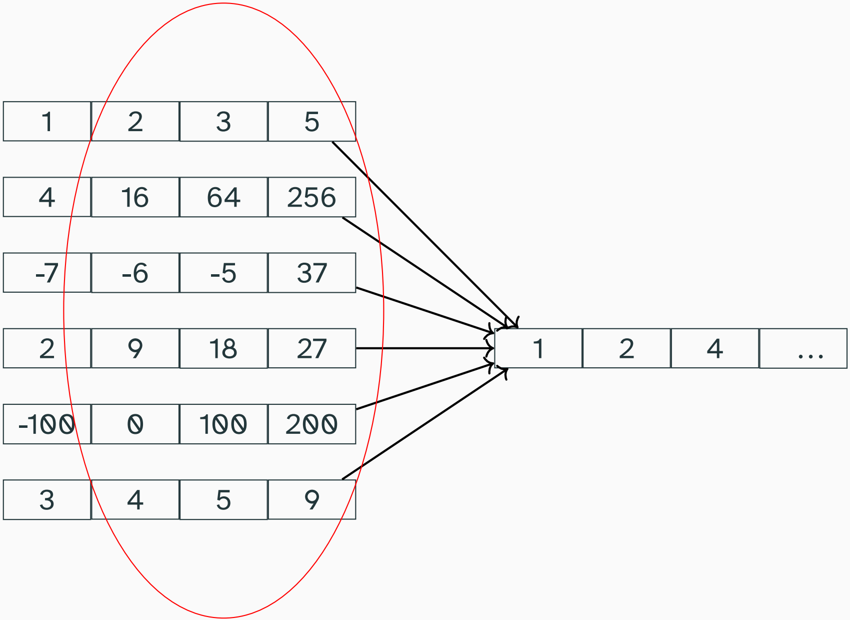

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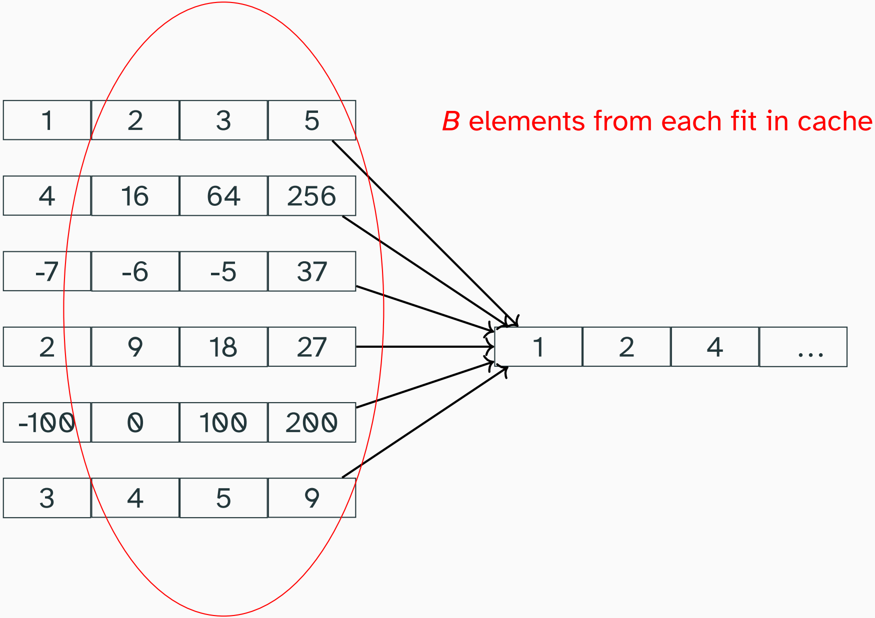

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- When you call sort in python, it is either Timsort or Powersort