Applied Algorithms Lec 3: Cache Efficiency

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September 12, 2025

Williams College

Admin

- Career fair in Chandler today
- Assignment 1 out! Start early!!
    - I'll get gradescope set up soon
- Don't do extra credit until you're done with the rest of the assignment
- 312 Lab computers are now accessible via ssh; link sent and also available on assignments page
- Meet and greet colloquium today. Next week: an (applied?!) algorithms colloquium

Wrapping up MITM

Meet in the Middle

```
1  for each subset A_1 of S_1:
2      s_1 ← h(A_1)
3      for each subset A_2 of S_2 :
4          if h(A_2) + s_1 ≤ h(S)/2 :
5          updateMax(h(A_2) + s_1)
```

- What is this inner poriton doing?
- Finds the set A_2 with height closest to $h(S)/2$
- How can we preprocess S_2 to answer these queries quickly? Let's split into pairs and discuss for a few minutes.
- Answer: we're looking for the largest subset of S_2 with height at most $h(S)/2 - s_1$.
- Sort all subsets of S_2. Then can answer this query using binary search!

Meet in the Middle

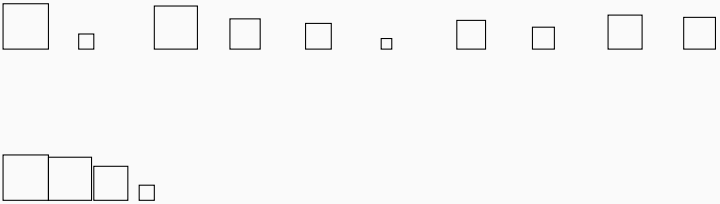
```
1  Fill array P with all subsets of S_2
2  Sort P by height
3  for each subset A_1 of S_1:
4      s_1 ← h(A_1)
5      binsearch(P, h(S)/2 − s_1)
6      updateMax(h(A_2) + s_1)
```

- Analysis?
- P has length $O(2^{n/2})$. Sorting it takes $O(n2^{n/2})$
- Each binary search takes $O(n)$ time; perform $O(2^{n/2})$ of them
- Total: $O(n2^{n/2})$ space, $O(n2^{n/2})$ time

Meet in the Middle

- Before we go forward, let’s go over the high level strategy

Meet in the Middle



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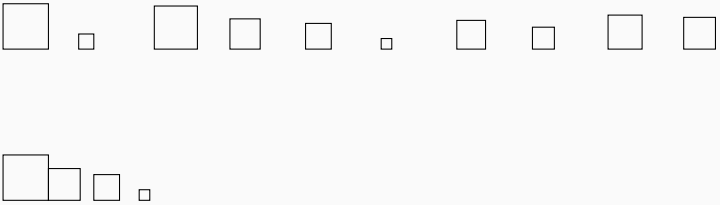

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Partition the blocks into two equal-sized sets. Question: what subset of the *yellow* blocks is used in the correct solution?

Meet in the Middle

0.0	00000
7.2	00001
5.1	00010
12.3	00011
9.8	00100
17.0	00101

...

First, let's do some brute force preprocessing on the blue blocks. Go through all subsets of the blue blocks, and store their heights in a table.

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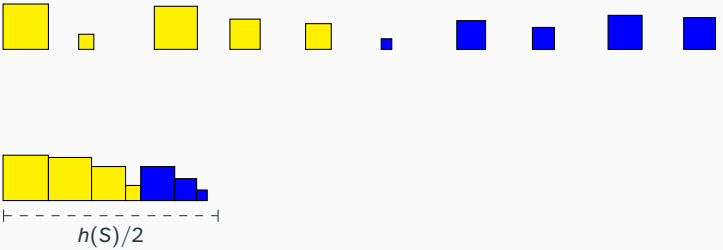

Now, go through every possible subset of yellow blocks. We want blue blocks with height as close to $h(S)/2 - h(A_1)$ as possible.

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How quickly can we find the *best* set of blue blocks with height at most $h(S)/2 - h(A_1)$? Why don't we need to check any other subsets of blue blocks?

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- "Meet in the middle"—rather than considering all subsets, we break into two halves. We search in the yellow and blue halves *one at a time*, then combine them to get one solution.
- Very wide uses: optimization problems, cryptography, etc.

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- $O(n2^{n/2})$ space is a lot. Is this worth it?
- Wait, can we do better than this?

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  - No: need the two halves to be *independent*. (We build the table on the blue half once. That table needs to work for every query.)
  - For example, 3SAT doesn't work here.

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- But they all have the same asymptotic running time!
- Topic for next couple lectures: why is that??

Any lingering questions about Assignment 1 or MITM?

Optimization

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  - May depend on the compiler used
  - Oftentimes: not obvious what causes the improvement!

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  - CPU vs wall clock time shouldn't make much difference for us
  - Parallelism doesn't help
- Costs of specific operations are sometimes given using number of "CPU cycles"
- Not-really-accurate-anymore definition of a cycle: time to perform one basic operation

Easiest way to measure time: just time it using built-in tools!

Easy, probably reflective of what you want.

But some things to bear in mind:

- Make sure your timing is macroscopic.
  - No timing is exact.
  - CPU clocks usually only have a resolution of $\approx$ 1 million ticks per second (sometimes less)
  - Minimize issues with overhead, external factors
  - Rule of thumb: ideally an experiment will take $\approx$ 1 second
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- Can also use unix `time` function

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- Look for *significant* improvement: usually you want experiments where the running time is close to a second, or more
    - Very difficult to draw conclusions from “Program A ran in 10 milliseconds; Program B ran in 15 milliseconds.”

Let’s say I have a piece of code, and I want to make it faster. How should I do that?

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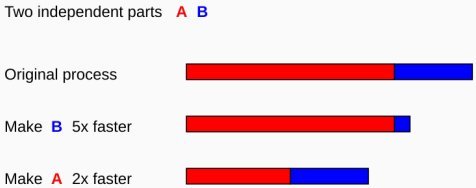
  - May not be the slowest function—in fact, it's often a very fast but very frequently-used function
- Probably need to take into account potential to speed it up as well—I want the function that takes up the most time that I can save.

Amdahl's Law



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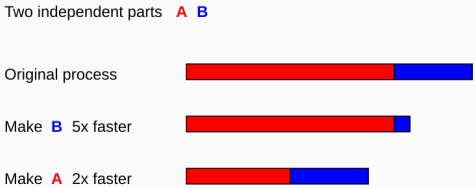

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I want you to know the principle here, but not memorize the formula.

Amdahl’s Law and Asymptotics

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- Example: Where to improve Dijkstra's algorithm?

Dijkstra's Algorithm

```
function Dijkstra(Graph, source):
    create vertex set Q
    for each vertex v in Graph:
        dist[v] ← INFINITY
        prev[v] ← UNDEFINED
        add v to Q
    dist[source] ← 0
    while Q is not empty:
        u ← vertex in Q with min dist[u]
        remove u from Q
        for each neighbor v of u still in Q:
            alt ← dist[u] + length(u, v)
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The inner for loop (blue part) is, at first glance, by far the most important part to optimize.

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2. Another option: Run same code with and without subroutine
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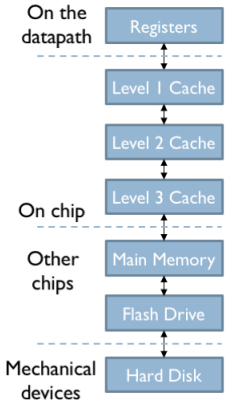
3. Profiling! Tools that will time your functions for you.

We'll come back to this with some examples in the next couple lectures. Bear in mind: benchmarking itself is an entire area of computer science.

Today: Cost to access data frequently dominates running time

A Typical Memory Hierarchy

- Everything is a cache for something else…



Access time	Capacity	Managed By
1 cycle	1 KB	Software/Compiler
2-4 cycles	32 KB	Hardware
10 cycles	256 KB	Hardware
40 cycles	10 MB	Hardware
200 cycles	10 GB	Software/OS
10-100us	100 GB	Software/OS
10ms	1 TB	Software/OS

Takeaway: if I access RAM one extra time, that is the same as doing (very roughly) *two hundred* extra operations.

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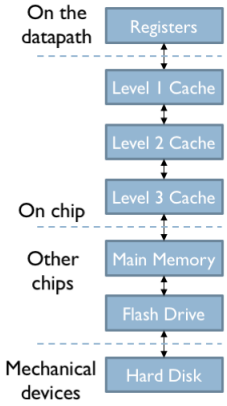

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Tradeoff: lower-level caches are much faster, but much smaller. Only store a small amount of data can be accessed quickly.

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- Example: `sortedlinkedlist.c` `unsortedlinkedlist.c`

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- Virtual machine: not 100% accurate; *slow*

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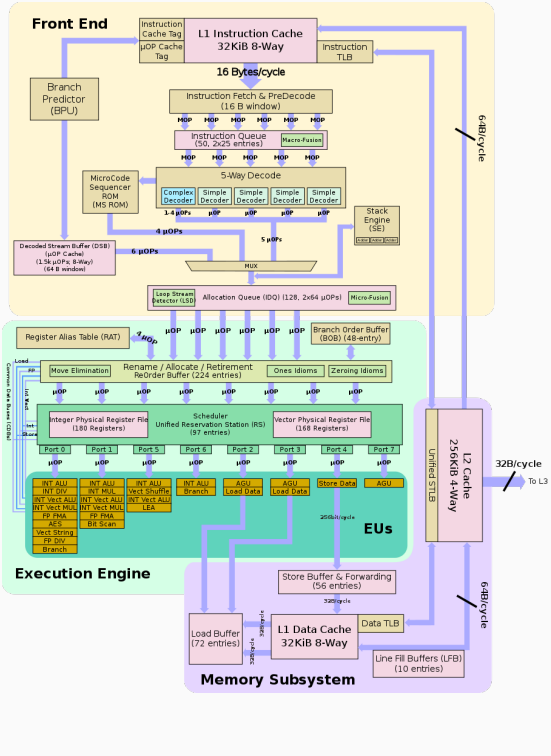
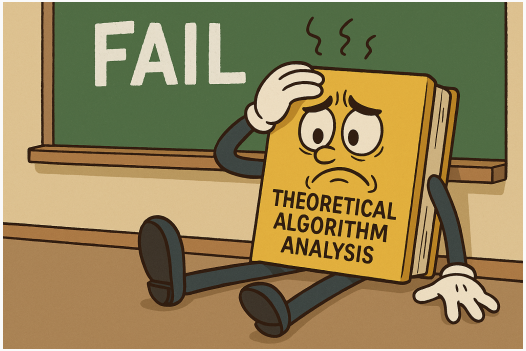
- But: bear in mind that modern caches are very complicated; interact nontrivially with other costs (branch mispredictions; expensive operations; etc.)
- I should at least mention *prefetching*: if your computer thinks it can get a head start on fetching your data, it will
- Model things the best you can, but always use experiments when you're not sure


Front End
Instruction Cache Tag
µOP Cache Tag
L1 Instruction Cache 32KiB 8-Way
Instruction TLB
16 Bytes/cycle
Branch Predictor (BPU)
Instruction Fetch & PreDecode (16 B window)
Instruction Queue (50, 2x25 entries)
Macro-Fusion
MicroCode Sequencer ROM (MS ROM)
5-Way Decode
Complex Decoder
Simple Decoder
Simple Decoder
Simple Decoder
Simple Decoder
1-4 µOPs
µOP
µOP
µOP
µOP
Stack Engine (SE)
6-8B/cycle
4 µOPs
5 µOPs
Decoded Stream Buffer (DSB) (µOP Cache) (1.5k µOPs, 8-Way) (64 B window)
6 µOPs
MUX
Loop Stream Detector (LSD)
Allocation Queue (IDQ) (128, 2x64 µOPs)
Micro-Fusion
Register Alias Table (RAT)
4 µOPs
Branch Order Buffer (BOB) (48-entry)
Load
RF
Move Elimination
Rename / Allocate / Retirement ReOrder Buffer (224 entries)
Ones Idioms
Zeroing Idioms
Store
Int Vect
Integer Physical Register File (180 Registers)
Scheduler Unified Reservation Station (RS) (97 entries)
Vector Physical Register File (168 Registers)
Port 0
Port 1
Port 5
Port 6
Port 2
Port 3
Port 4
Port 7
INT ALU
INT DIV
INT Vect ALU
INT Vect MUL
FP MAA
AES
Vect String
FP DIV
Branch
INT ALU
INT MUL
INT Vect ALU
INT Vect MUL
FP MAA
Bit Scan
INT ALU
Vect Shuffle
INT Vect ALU
LEA
INT ALU
Branch
AGU
Load Data
AGU
Load Data
Store Data
AGU
256bit/cycle
EUs
Execution Engine
Unified STLB
L2 Cache 256KiB 4-Way
32B/cycle
To L3
Store Buffer & Forwarding (56 entries)
32B/cycle
Load Buffer (72 entries)
64B/cycle
64B/cycle
L1 Data Cache 32KiB 8-Way
Data TLB
Line Fill Buffers (LFB) (10 entries)
6-8B/cycle
Memory Subsystem

Summary



- Algorithms has failed us
- Not all operations are equal in practice!
- Constants matter!

Return of Algorithms

A very current reference to a comeback story.

- Can we take cache misses into account?
- What if rather than measuring operations (as we did in CS 256), we analyze only *cache misses*?
- Next topic: the External Memory Model

External Memory Model

What do we want out of this model?

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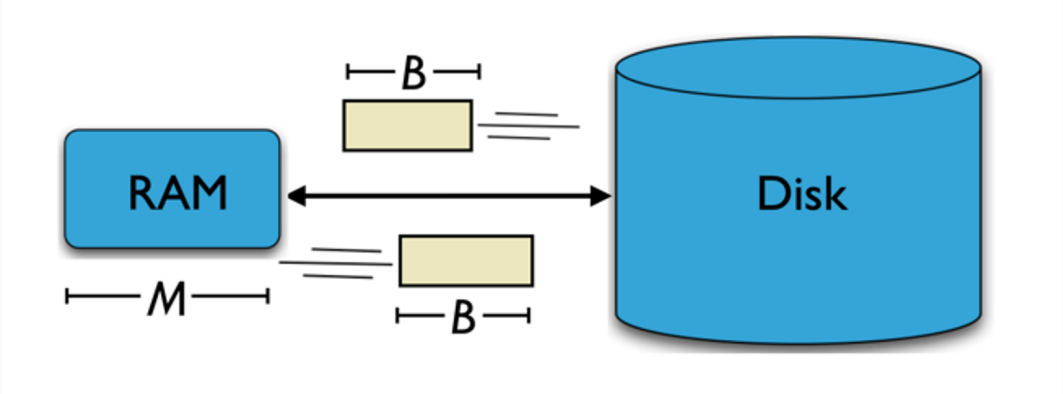
- We will say something like "$O(n/B)$ cache misses" rather than "$O(n)$ operations" to emphasize the model.

External Memory Model Basics



Transferring *B* *consecutive* items to/from the disk costs 1. Can only store *M* things in cache.

Memory Evictions

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Memory Evictions

- Can only hold M items in cache!
- So when we bring B in, need to write B items back to disk. (We can bring them in later if we need them again)
- Assume that the computer does this optimally.
  - Reasonable; it's really good at it. Very cool algorithms behind this!

Vocabulary

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Vocabulary

- “Cache” of size *M*; “disk” of unlimited size
- With the cost of one “cache miss” can bring in *B* consecutive items
  - (Sometimes called “memory access” or “I/Os” but I will try not to use those terms.)
- These *B* items are called a “block” or a “cache line”.

Let's revisit `sortedlinkedlist.c`

- What is the cost of our algorithm in the external memory model if the items are stored in order?

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- The external memory model predicts (roughly) the real-world slowdown of this process.

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- Total: $O(\log_2(n/B))$ cache misses.

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- If you have a sequence of operations on a dataset of size at most M, there is no further cost so long as they all stay in cache!
- $O(M/B)$ to load the items into cache, then all computation is free
- Real-world time: what if instead of a linked list of 100 million items, we repeatedly access a linked list of 100 thousand items?

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- Simple model that captures *one level* of the memory hierarchy
- Idea: usually one level has by far the largest cost.
  - Small programs may be dominated by L1 cache misses
  - Larger programs it may be by L3 cache misses
- External memory model zooms in on one crucial level of the memory hierarchy (with particular B, M); gives asymptotics for how well we do on that level.

Question about External Memory Model Basics?