# Applied Algorithms Lec 2: Meet in the Middle

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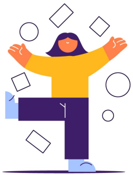
September 9, 2025

Williams College

# Admin



- Assignment 0 out!
- Please clone your git repo and fill in the questions
- If you are off-campus, ssh to `lohani.cs.williams.edu`, and then ssh to a lab computer
- I'll post the lab computer addresses once they are set up. Right now, can use Ward lab computers:
  `cow-i23-nuc23.cs.williams.edu`
  `cow-i23-nuc24.cs.williams.edu`
  `...`
  `cow-i23-nuc40.cs.williams.edu`

# Finishing C Review

# Memory Allocation

- `malloc` and `free`
  - Also use `calloc` and `realloc`
  - Need `stdlib.h`
- If you call `C++` code, be careful with mixing `new` and `malloc`
- Use useful library functions like `memset` and `memcpy`
- Example: `memory1.c`

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- Keep an eye out for unsigned vs signed.
- Quick example: `variabletypes.c`
- `printf` does expect primitive types

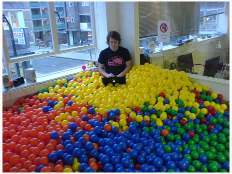
# Variable types cont.

- `int` (etc.) is OK for things like small loops
- If you care *at all* about size you should use the type explicitly
- Up to you when and where you use unsigned
  - Controversial in terms of style
  - Can help with overflow; often changes shift behavior

# List of particularly useful integer variable types

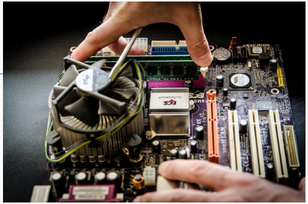
- `int64_t, int32_t`: signed integers of given size
- `uint64_t, uint8_t`: unsigned integers of given size
- `INT64_MAX` (etc.): maximum value of an object of type `int64_t`

# Sorting in C



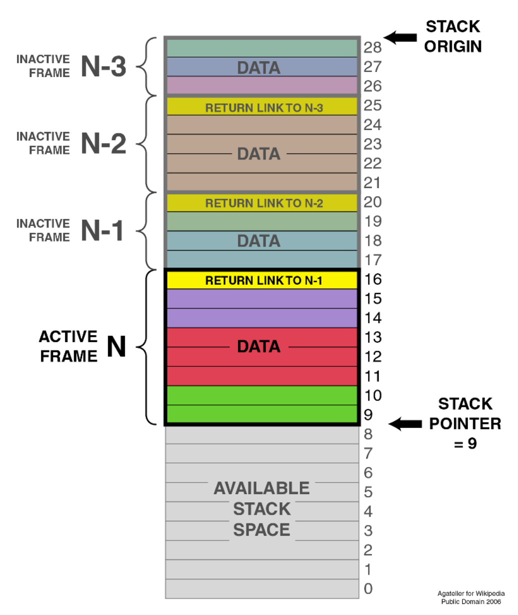
- `qsort()` from `stdlib.h`
- Takes as arguments array pointer, size of array, size of each element, and a comparison function. Let's look at an example of how it works
- What's a downside to this in terms of efficiency?
- Many ways to get better sorts in C:
  - Nicely-written homemade sort
    - Real world: Get an LLM to write one for you. But make sure it works
    - Reminder: *not allowed* in this class; I'll give you a sort for Assignment 1
  - `C++ boost` library
  - Third-party code

# Architecture this Semester



- x86 architecture (not AMD, not M2 etc.)
- Intel i7; run `lscpu` on a lab computer for details
- This *is* likely to have an effect on performance in some cases
- Your home computers are acceptable (but not recommended) for correctness and coarse optimization; use lab computers for fine-grained optimization
- If I ask you to do a performance comparison, you should do it on lab computers. (In the rare case where you don't, you should always write down exactly what machine it was done on.)

# Where are things stored?

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    - Temporary variables like loop indices
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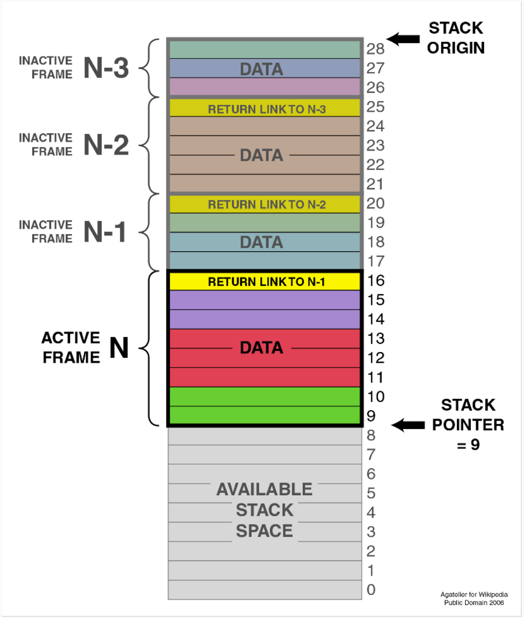


- In CPU register (never touching memory)
  - Temporary variables like loop indices
  - Compiler decides this
- Call stack
  - Small amount of dedicated memory to keep track of current function and *local* variables
  - Pop back to last function when done
  - **temporary**

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- Very large amount of memory (basically all of RAM)
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- Need `stdlib.h` to use `malloc`

# How to decide stack vs heap?

- Java rules work out well:
    - “objects” and arrays on the heap
    - Anything that needs to be around after the function is over should be on the heap
    - Otherwise declare primitive types and let the compiler work it out
    - Keep scope in mind!

# Makefile

- Each time we change a file, need to recompile that file
- Need to build output file (but don't need to recompile other unchanged files)
- Makefile does this automatically

# In this class

- I'll give you a makefile
- You don't need to change it unless you use multiple files or want to set compiler options
  - Probably don't need to use multiple files in this class
  - (Some exceptions for things like wrapper functions.)

# Let's look quickly at the default Makefile

- `make`, `make clean`, `make debug`

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  - `-O0` doesn't optimize; `-Og` is no optimization for debugging
  - Other flags to specifically take advantage of certain compiler features (we'll come back to this)
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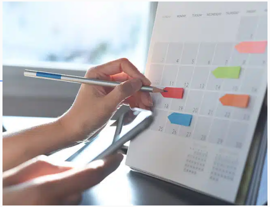
  - Also: "Compiler Explorer" online

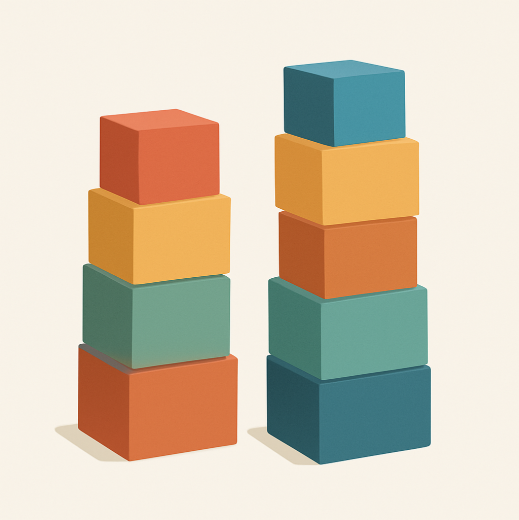
# Meet in the Middle

# Plan for today



- This part of the course: how time and space interact
- Today: using space to make things run faster
- Specifically, store results of frequently-computed values to save time

# Two towers reminder (?)



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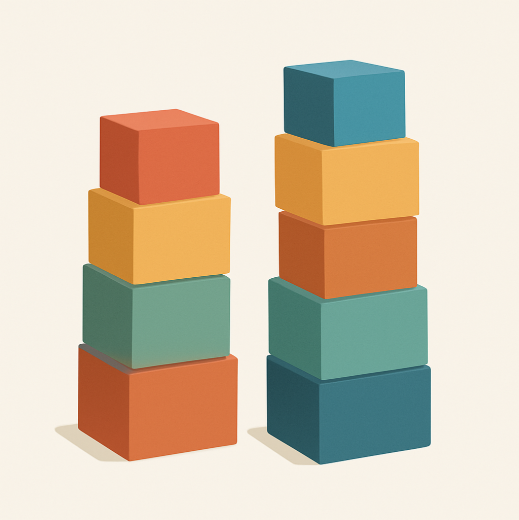

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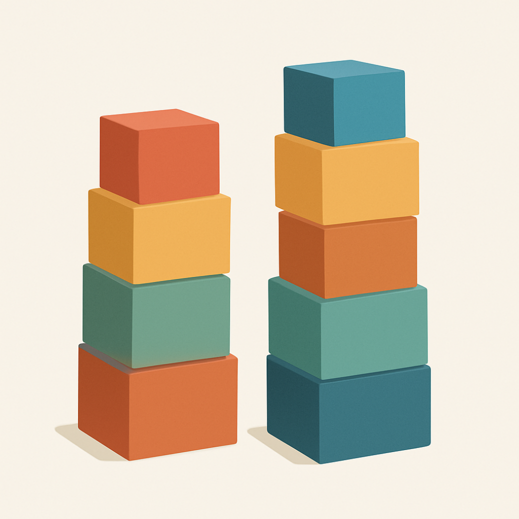

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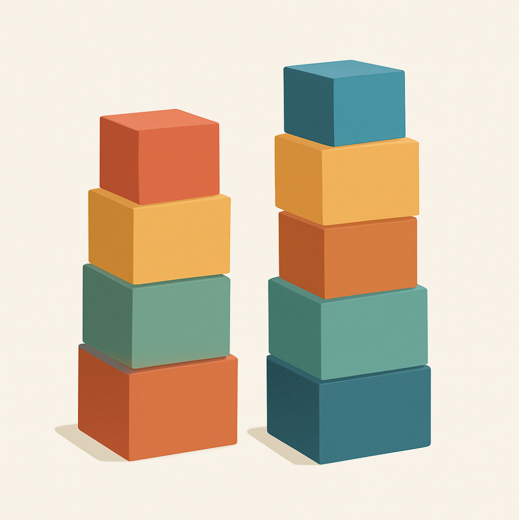

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- This is the problem we will solve on Assignment 1

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- How can we solve this? (Let's go to the board.)
  - Try all subsets as the smaller tower; keeping track of largest seen
- Running time? Space (what do we need to store)?
  - Time: $O(2^n)$. Space: Only need to store current subset; best height seen so far

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- For each subset, calculate the height by going through the bits and adding when you see a 1. Keep the heights as an array of floats.

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- There must be SOME subset of $S_1$ in the correct final smaller tower.
- Let's make an algorithm on the board based on this observation. (It won't be faster yet.)

# Meet in the middle

- Divide $S$ into two sets: $S_1$ and $S_2$.

For any set $S'$, let $h(S')$ be the height of all elements in $S'$. Then:

```
for each subset A1 of S1:
    s1 ← h(A1)
    for each subset A2 of S2 :
        if h(A2) + s1 ≤ h(S)/2 :
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- Running time? $2^{n/2} \cdot O(2^{n/2}) = O(2^n)$. Same as before!

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- Answer: we're looking for the largest subset of $S_2$ with height at most $h(S)/2 - s_1$.
- Sort all subsets of $S_2$. Then can answer this query using binary search!

# Meet in the Middle

```
Fill array P with all subsets of S_2
Sort P by height
for each subset A_1 of S_1:
    s_1 ← h(A_1)
    binsearch(P, h(S)/2 − s_1)
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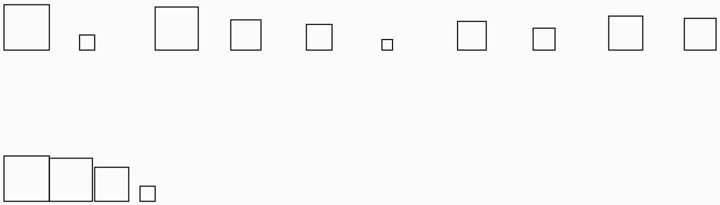
- Each binary search takes $O(n)$ time; perform $O(2^{n/2})$ of them
- Total: $O(n2^{n/2})$ space, $O(n2^{n/2})$ time

# Meet in the Middle

- Before we go forward, let’s go over the high level strategy

# Meet in the Middle



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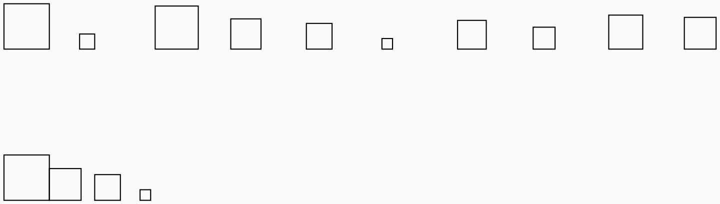

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Partition the blocks into two equal-sized sets.

# Meet in the Middle

Partition the blocks into two equal-sized sets. Question: what subset of the *yellow* blocks is used in the correct solution?

# Meet in the Middle

| | |
|---|---|
| 0.0 | 00000 |
| 7.2 | 00001 |
| 5.1 | 00010 |
| 12.3 | 00011 |
| 9.8 | 00100 |
| 17.0 | 00101 |

...

First, let's do some brute force preprocessing on the blue blocks. Go through all subsets of the blue blocks, and store their heights in a table.

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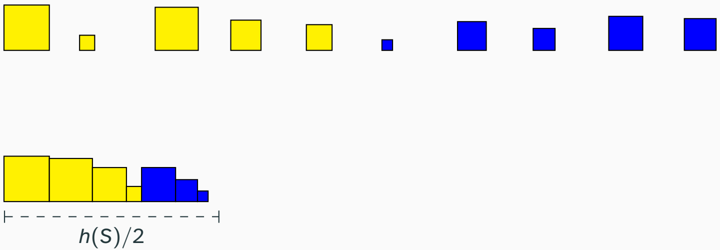


Now, go through every possible subset of yellow blocks. We want blue blocks with height as close to $h(S)/2 - h(A_1)$ as possible.

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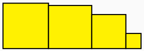

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...

How quickly can we find the *best* set of blue blocks with height at most $h(S)/2 - h(A_1)$? Why don't we need to check any other subsets of blue blocks?

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# What we get

- $O(n2^{n/2})$ space, $O(n2^{n/2})$ time. (Everyone remember how?)
- "Meet in the middle"—rather than considering all subsets, we break into two halves. We search in the yellow and blue halves *one at a time*, then combine them to get one solution.
- Very wide uses: optimization problems, cryptography, etc.

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- What is $O(n2^n)$ vs $O(n2^{n/2})$ time? Do they differ by more than a constant?
- $O(n2^{n/2})$ space is a lot. Is this worth it?
- Wait, can we do better than this?

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- How else can we store solutions from the blue subproblems? What does this data structure need to support?
  - Needs to support predecessor queries!
- What if we wanted to search for two towers that were exactly equal? Would our strategy change? Could we get improved running time?
- What property must a problem have for MitM to work?
  - Can *all* brute force search problems with $N$ solutions be solved in something like $O(\sqrt{N})$ time?

# Some questions about meet in the middle

- How else can we store solutions from the blue subproblems? What does this data structure need to support?
    - Needs to support predecessor queries!
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  - No: need the two halves to be *independent*. (We build the table on the blue half once. That table needs to work for every query.)
  - For example, 3SAT doesn't work here. On Assignment 1 you'll look into this further

# Any lingering questions about Assignment 1 or MITM?

# Optimization

# Making Software Fast

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  - Can be impacted by other software running on the computer
  - May depend on the compiler used
  - Oftentimes: not obvious what causes the improvement!

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- Costs of specific operations are sometimes given using number of "CPU cycles"

- Not-really-accurate-anymore definition of a cycle: time to perform one basic operation

# Easiest way to measure time: just time it using built-in tools!

Easy, probably reflective of what you want.

But some things to bear in mind:

- Make sure your timing is macroscopic.
  - No timing is exact.
  - CPU clocks usually only have a resolution of $\approx$ 1 million ticks per second (sometimes less)
  - Minimize issues with overhead, external factors
  - Rule of thumb: ideally an experiment will take $\approx$ 1 second
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- Let's look at how `test.c` times your code on Homework 1
- Can also use unix `time` function

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- Look for *significant* improvement: usually you want experiments where the running time is close to a second, or more
    - Very difficult to draw conclusions from “Program A ran in 10 milliseconds; Program B ran in 15 milliseconds.”

**Let’s say I have a piece of code, and I want to make it faster. How should I do that?**

# Thought question

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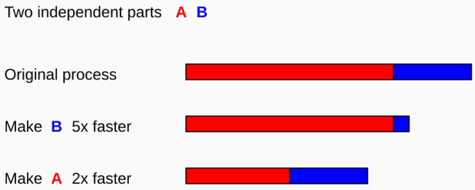
- Probably need to take into account potential to speed it up as well—I want the function that takes up the most time that I can save.

# Amdahl's Law

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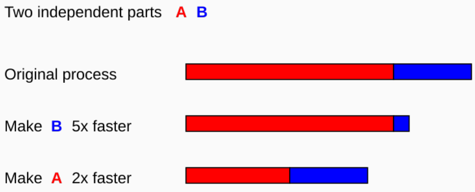


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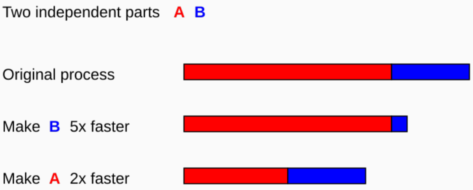


If a function takes up a fraction of the entire program's runtime and you speed it up by a factor s, then the overall program speeds up by a factor

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I want you to know the principle here, but not memorize the formula.

# Amdahl's Law and Asymptotics

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- Can *estimate* the total time of an algorithm asymptotically
- Example: Where to improve Dijkstra's algorithm?

# Dijkstra's Algorithm

```
function Dijkstra(Graph, source):
    create vertex set Q
    for each vertex v in Graph:
        dist[v] ← INFINITY
        prev[v] ← UNDEFINED
        add v to Q
    dist[source] ← 0
    while Q is not empty:
        u ← vertex in Q with min dist[u]
        remove u from Q
        for each neighbor v of u still in Q:
            alt ← dist[u] + length(u, v)
            if alt < dist[v]:
                dist[v] ← alt
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The inner for loop (blue part) is, at first glance, by far the most important part to optimize.

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    - Need to get info on how often it’s called; simulate correct types of data.
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2. Another option: Run same code with and without subroutine
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3. Profiling! Tools that will time your functions for you.

We'll come back to this with some examples later this week. Bear in mind: benchmarking itself is an entire area of computer science.

# Optimization

## Code Profiling

# Profiling code

- Why not just have your computer tell you what functions are caused the most, or keep track of how long they run, or monitor specific high-cost operations?
- Lots of such tools! We'll look at a couple of them right now, and use them throughout the class.
  - `gprof`
  - `cachegrind`
  - We won't use `perf` but some people like it
  - We won't use Intel VTune either but seems very cool and powerful
- What do you think some advantages and disadvantages are of using profiling software?

# gprof

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- We’ll look at some examples later

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- `callgrind` gives `gprof`-like profiling
- `cachegrind` helps determine the cost of *moving* data: cache misses, branch mispredictions, etc.
- Essentially runs the program on a virtual machine
- Gives information about costs you could not otherwise get, but VERY slow.

# Amdahl's Law Takeaways

- When optimizing code, always think first about where your effort is best spent!
- Looking for the portion of the code with most total time
- Can estimate using asymptotics. And/or, run experiments.

# Thought Question in Pairs

We want to make our code faster. What makes code fast? What makes a specific function fast? What slows it down?

# Things to consider

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- Next topic: *cache efficiency*