Lecture 18: Integer Linear Programming Practice

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Williams College

Admin

- Assignment 3 due *Saturday* (basically: built-in 2 day extension for everyone). Get started if you haven't!
- Homework 5 and Assignment 2 back
- Today: two problems to be done in groups.
- Friday: we'll mostly talk about the project; we'll start the final part of the course next week. We'll likely finish early and I'll stick around for anyone with questions.
- Questions?

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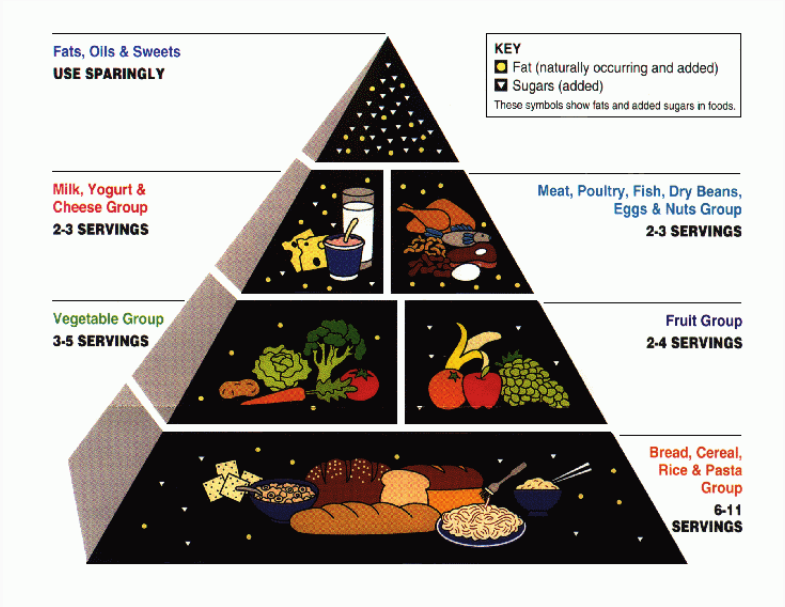
- Simple example: optimal eating while being able to choose your diet

Food Pyramid



Choice of diet

- You need to satisfy one of the three following diet goals:
  - 46 grams of protein and 130 grams of carbs every day; or
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- 100g Chicken: 13.5g protein, 0g carbs, $.70

What is the cheapest way you can hit one of these diet goals?

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- $x_i \in \{0, 1\}$
- $x_1 + x_2 + x_3 = 1$

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- Hint: if $x_1 = 0$, I want to do something to these constraint so that they're *always* satisfied
- $25.8p + 2.5r + 13.5c + 46(1 - x_1) \geq 46$

Choice of diet LP

- Diet options:
  - 46 g protein; 130 g carbs; or
  - 20 g protein; 200 g carbs; or
  - 100 g protein; 30 g carbs
- 100g Peanuts: 25.8g protein, 16.1g carbs, $1.61
- 100g Rice: 2.5g protein, 28.7g carbs, $.79
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$\min 1.61p + .79r + .7c$

- $25.8p + 2.5r + 13.5c + 46(1 - x_1) \geq 46;$
- $16.1p + 28.7r + 130(1 - x_1) \geq 130$
- $25.8p + 2.5r + 13.5c + 20(1 - x_2) \geq 20;$
- $16.1p + 28.7r + 200(1 - x_2) \geq 200$
- $25.8p + 2.5r + 13.5c + 100(1 - x_3) \geq 100;$
- $16.1p + 28.7r + 30(1 - x_2) \geq 30$
- $x_1 + x_2 + x_3 = 1$
- $p, r, c \geq 0; p, r \in \mathbb{Z}; x_i \in \{0, 1\}$

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- Need to be able to bound the constraint to do this!
- What happens with rounding when you use this technique?

Let's Code Up the Diet Problem Solution

$\min 1.61p + .79r + .7c$

- $25.8p + 2.5r + 13.5c + 46(1 - x_1) \geq 46;$
- $16.1p + 28.7r + 130(1 - x_1) \geq 130$
- $25.8p + 2.5r + 13.5c + 20(1 - x_2) \geq 20;$
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- Each student must be partnered with a student sitting adjacent to them: either next to each other, or in front of each other
- Goal: create a seating chart, and assign partners, to minimize the largest difference (absolute value) in level of interest between any partners.