Applied Algorithms Lec 24: Review

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Admin

- MM2 and MM3 back tonight. (MM2 is basically done.)
- Assignment "8" due tonight at 10 (will run test script at 7, 9, 10)
- Evaluations at the end of today
- Office hours today in my office from 2:30–4 (I may be a tad late)
- Please go to colloquium tomorrow! (It's in-person!)
    - I know you're all very busy. But we (and the speaker) will really appreciate it!
    - Cool topic: internet measurements. Combination of networking/security/privacy. Very relevant research, and from a technical point of view.

Looking Back at the Class

About the Class

- Goal: bridge the gap between theory and practice
- How can theoretical models better predict practice?
- Useful algorithms you may not have seen
- Using algorithmic principles to become better coders!

Pantry Algorithms

- Algorithms that you should always have handy because they are incredibly useful
- Bloom filters, linear programming, suffix arrays
- What drives the course
- Algorithmic understanding of these ideas!

Course topics

Throughout the course: efficient coding

- `gcc` and compiler decisions/flags
- `gprof` and `cachegrind`
- Making code more efficient
- Intrinsics
- SIMD operations

Course topics

Part 1: Time and space

- Meet in the middle
- Hirshberg's algorithm
- External memory model
- Blocking, external memory sorting

Course topics

Part 2: Randomization

- Basic probabilistic algorithm analysis
- Practical hashing
- Bloom Filters, cuckoo filters
- Count-Min Sketch, HyperLogLog counting
- Locality-sensitive hashing

Course topics

Part 3: Linear Programming

- LP
- ILP/MIP
- Solving algorithmic problems using LP, ILP, MIP
- Simplex solver (for LP)
- Branch and bound solver (for ILP/MIP)
- Using `GLPK` to solve LPs, ILPs, MIPs

Course topics

Part 4: Strings and Trees

- Burrows-Wheeler Transform
- Suffix array usage
- SA-IS algorithm

Review

Optimizing Code

- Cost of different operations
- Branch mispredictions
- Loop unrolling
- Making your intentions clear to the compiler (i.e. storing length of a string in a separate variable when you know the string doesn't change)

SIMD Instructions

- Operations on 256 bits at a time
- Can greatly speed up certain types of code
- Disadvantages:
  - Operations are a bit slower each (but they operate on more data)
  - Have to load and store data (expensive)
- Let's look at one example

SIMD Instructions

```
int simdFindFirst0(int* A, int size){
    __m256i vector0 = _mm256_set1_epi32(0);
    for(int i = 0; i < size/8; i++) {
        __m256i a = _mm256_loadu_si256((__m256i*)(A+8*i));

        //note that this works for comparing any two vectors
        //(there may be a better way to only look for 0s)
        __m256i res = _mm256_cmpeq_epi32(a, vector0);

        //res now has all 1s in locations where they are the same; all 0s in locations where they differ
        //(note that this may be counterintuitive)

        //movemask stores the first bit of each byte of res in the integer
        int integerResult = _mm256_movemask_epi8(res);

        if(integerResult != 0) {
            //found correct sequence of 8 bytes.  Now search for correct answewr
            for(int j = 8*i; j < 8*i + 8; j++) {
                if(A[j] == 0) return j;
            }
        }
    }
    return -1;
}
```

Locality-Sensitive Hashing for Searching for Close Points

- Reminder: a locality-sensitive hash is likely to hash close items together, but unlikely to hash far items together
- We saw: want expected size of a bucket to be $O(1)$
- Strategy: hash all items into buckets. Do all-compare all within each bucket
- If the close pair is not found, choose a new hash function and repeat

LSH with Repetitions

0	1	2	3	4
(101, 37, 65)	(103,37,64)	(91,84,3)	(100,18,79)	

0	1	2	3	4
	(101,37,65) (103,37,64)	(91,84,3)		(100,18,79)

0	1	2	3	4
(101, 37, 65)		(103,37,64)		(91,84,3) (100,18,79)

LSH Analysis

- Reminder: to make sure buckets are small, we concatenate multiple (k) minHashes hashes together
- These k minHashes will determine what bucket each item goes in
- Let's say that each pair of items (other than the close pair) has similarity $1/3$; this means they collide (under *one* hash, not all k) with probability $1/3$
- Let's look at some item x_i. How big is its bucket?

LSH Analysis

- Let's look at some item x_i. How big is its bucket?
- Let's use linearity of expectation! For any $j \neq i$, x_i and x_j wind up in the same bucket only if all k hashes have the same value
- $\Pr(h(x_i) = h(x_j)) = 1/3^k$.
- Let B_i be a random variable denoting the number of items that hash to the same bucket as x_i. Then $X_i = X_{i1} + X_{i2} + \ldots + X_{in}$, where $X_{ij} = 1$ if i and j hash to the same bucket
- By linearity of expectation, $E[X_i] = \sum_{j \neq i} E[X_{ij}]$
- X_{ij} is a 0-1 random variable, so $\mathrm{E}[X_{ij}] = \Pr(X_{ij} = 1) = 1/3^k$
- Therefore, $E[X_i] = (n-1)(1/3)^k$.

Analysis from Lecture 12

- We want buckets of size $O(1)$
- So we solve $n(1/3)^k = O(1)$, which implies $k = \log_3 n$
- Many people found that smaller k led to better running time. Why is that?
- Each time we go to a bucket, need to hash the item; need to access the bucket (probably a cache miss); some other smaller overheads
- So the $O(1)$ is actually pretty big; leading to a smaller k
- Cache misses are a big part of this: for practice, let's look at how to optimize with cache misses in mind.

What about cache misses?

- Let's analyze this algorithm in external memory
- How many cache misses does it take for a bucket of size X?
- Are there assumptions about our cache parameters that will affect this analysis?
- First, let's say that $X \leq M$. How many cache misses does it take?
    - Can bring the entire bucket into cache and do all-compare all. $O(1 + X/B)$ cache misses.
    - Where is the 1 from?? Is there a case where we don't have that extra 1?

What about cache misses?

- Let's analyze this algorithm in external memory
- How many cache misses does it take for a bucket of size X?
- Are there assumptions about our cache parameters that will affect this analysis?
- Now, let's say that $X \gg M$. How many cache misses does it take?
- (Don't worry about the case where $X \approx M$. Just deal with the cases where it's significantly larger or smaller.)
    - For each item in X, we do a linear scan through the bucket
    - $X \cdot O(\frac{X}{B})$ cache misses $= O(X^2/B)$

Intuition: How big do we want buckets to be?

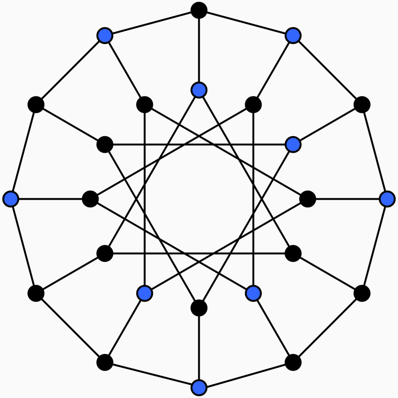
- It always takes at least one cache miss per bucket if $n \gg M$
- So let's shoot for $O(1)$ cache misses per bucket: expected size of a bucket is $O(B)$
- We set k:
  - $n(1/3)^k = O(B)$, so $k = \log_3 \frac{n}{B}$

Finishing the Analysis

- We have $k = \log_3 \frac{n}{B}$
- What is the expected number of repetitions?
- Probability that close pair is in the same bucket? (Let's say the close pair has similarity 3/4)
- $(3/4)^k$.
- So the expected number of repetitions until the close pair winds up in the same bucket is $1/(3/4)^k = (4/3)^k = (4/3)^{\log_3 \frac{n}{B}}$
- Let's simplify this on the board
- Hopefully we got: $(n/B)^{\log_3 4/3} \approx (n/B)^{.26}$

Questions about probability or external memory?

Independent Set (ILP practice)



- Given a graph G with nodes V and edges E
- Find the largest collection of vertices such that no two vertices in the collection are adjacent

Setting up the ILP

- Variables?
- Let $x_i = 1$ if vertex i is in the independent set; $x_i = 0$ otherwise. (Binary variables.)
- Objective?
- Maximize $\sum x_i$

Setting up the ILP

- Constraint?
- Need to make sure that no two vertices share an edge
- For every $e \in E$ (where $e = (v_i, v_j)$), have a constraint:
- $x_i + x_j \leq 1$.

ILP Wrapup

- Any questions about ILPs?
- May be an LP on the exam. Could we write an LP for this problem?
- No obvious way to do so; need x_i to be binary variables for $x_i + x_j \leq 1$ to force choosing one vertex
- (As you may know, this problem is NP hard, so it's likely impossible to write an LP for this problem.)

Review: Requested Topics

Bookshelf/Hirschberg's

- I was requested to go over how to use a Hirshberg's-like approach for other problems
- I started making some slides to go over the bookshelf problem
- They would definitely take at least 20 minutes to go over properly
- It's not on the final. So I don't want to take that time.

Hirschberg's takeaway

- (It is a cool algorithm that I do want you to know about in general)
- Also: Hirschberg's took more operations than normal edit distance, and required less space. But it ran faster. Why?
- Answer: cache-efficiency!
- Our Hirchberg's operations fit in cache, which sped things up
- Naive dynamic programming approach didn't fit in cache. Cache misses slowed us down.

Assgn 4 Prob 2

- Stream of 1,001,000 elements
- 1,000,000 elements only appear once $(a_1, \ldots, a_{1000000})$
- One element, q, appears 1000 times
- Goal for the next questions: determine number of columns, number of rows, number of bits for a Count-min sketch
- First: how does a count min sketch work? (Let's do it on the board)
- What are the guarantees?

Assign 4 Prob 2 par (a)

- Recall: 1,000,000 different a_i appear once; 1000 instances of q
- Let's say I query some a_i for an answer o_i, and some q for an answer o_q. How do I set my parameters so that 90% of the time, $o_q \geq o_i$?
- We know that $o_q \geq 1000$ by the CMS guarantees. So if we can make sure that 90% of the time, $o_i \leq 1000$ we are done
- CMS guarantees give: with probability $1 - \delta$, $o_i \leq 1 + 1001000\varepsilon$
- So we set $\delta = .1$, and $\varepsilon = 999/1001000$
- Number of rows $= \lceil \ln 1/\delta \rceil = 10$. Number of columns $= \lceil e/\varepsilon \rceil = 2724$. (Can tighten this a bit, but this is fine)
- Number of bits per element: $\lceil \log_2 1001000 \rceil = 20$

Assign 4 Prob 2 part (b)

- Recall: 1,000,000 different a_i appear once; 1000 instances of q
- Let's say I query *all* a_i for an answer o_i, and some q for an answer o_q. How do I set my parameters so that 90% of the time, $o_q \geq \max o_i$?
- Hint was to use the union bound

Assign 4 Prob 2 part (a)

- Recall: 1,000,000 different a_i appear once; 1000 instances of q
- CMS guarantees give: with probability $1 - \delta$, $o_i \leq 1 + 1001000\varepsilon$
- We still set $\varepsilon = 999/1001000$. But now we want this to happen more often
- We want the probability that $o_1 > o_q$, OR $o_2 > o_q$, OR $\ldots$ OR $o_{1000000} > q$ is at most .1
- By union bound: the probability that any of these happen is the sum of the probability that each happens
- So want $1000000\delta < .1$; $\delta = 1/10000000$.
- Number of rows $= \lceil \ln 1/\delta \rceil = 17$.

Assign 4 Prob 4 part (b)

- Have a stream of items with a majority item m that appears more than half the time
- How many rows do we need so that the count-min sketch is correct with constant probability?
- In other words: if we make any query q, we should be able to determine if $q = m$ or $q \neq m$ with constant probability.
- Let's go back to the count min sketch and see what this means

Looking at the CMS

- Our CMS is going to have one cell containing m
- This cell will have size at least $n/2$, where n is the length of the stream
- All other cells have size $< n/2$
- So on a query q: if $h(q)$ has size $> n/2$ we return $q = m$; otherwise we return $q \neq m$.

Analysis

- Let's say we have c entries in a row.
- If the correct answer is $q = m$, how does our CMS perform?
- Always gives the correct answer.
- If the correct answer is $q \neq m$, how does our CMS perform?
- Answers correctly if $h(q) \neq h(m)$
- So: answers correctly with probability $1/c$
- $c = O(1)$ is sufficient to obtain constant probability. (In fact $c = 2$ is enough.)

Any Questions about Assignment 4? Or CMS?

Any Other Questions?

Course Evaluations!

Course Evaluations

- Please do fill them out :)
- They're on Glow; course titled "Course Evaluations"
- Two kinds: main course evaluation that many people see (me, senior faculty in the department, admin); also "blue sheets" that only I see
- Hopefully clearly labelled

Course Evaluations Spiel

OPTIONAL SCRIPT FOR PROMOTING THE STUDENT COURSE SURVEY

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You may skip questions that you don't wish to answer, and there is no penalty for choosing not to participate. All of your answers are confidential and I will only receive a report on your responses after I have submitted all grades for this course. While evaluations are open, I will receive information on how many students have filled out the evaluations, but I won't be told which of you have and haven't completed them. I won't know which responses are associated with which student unless you identify yourself in the comments.

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