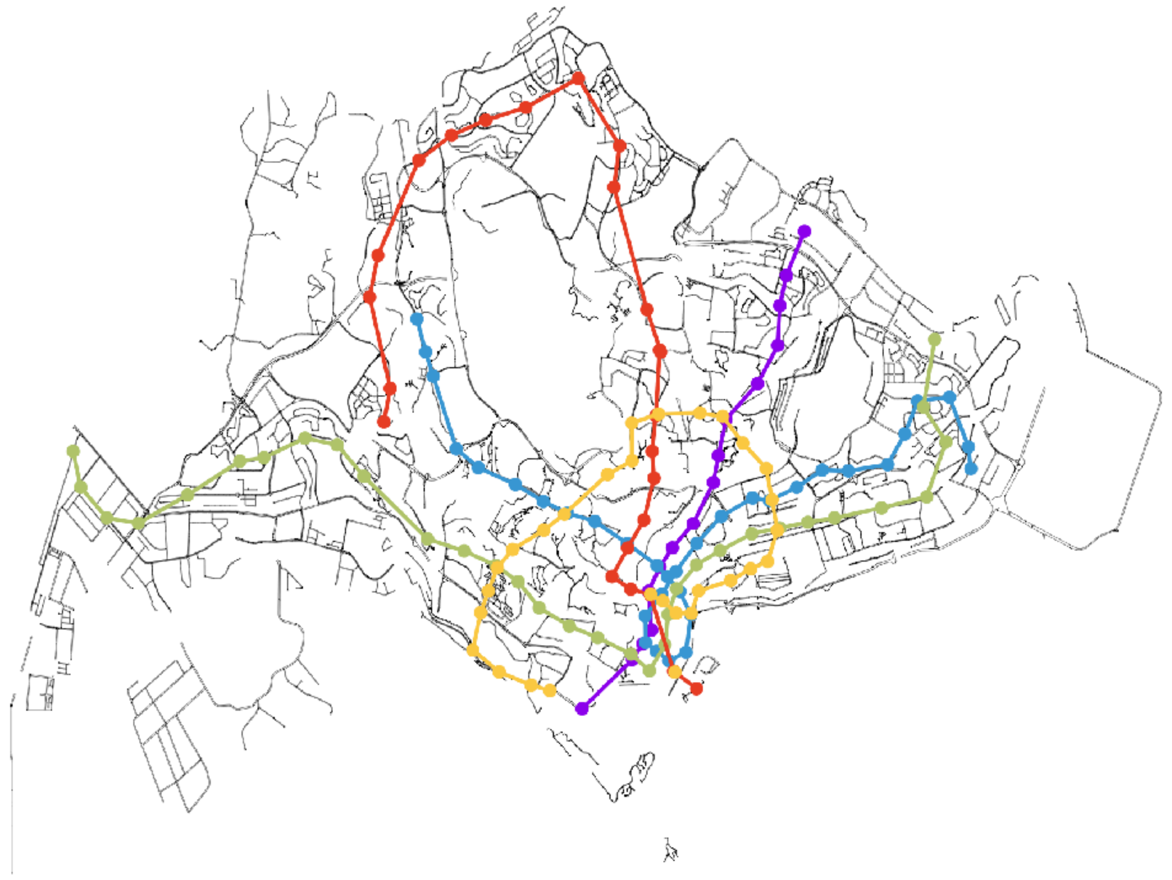
# CSCI 357: Algorithmic Game Theory

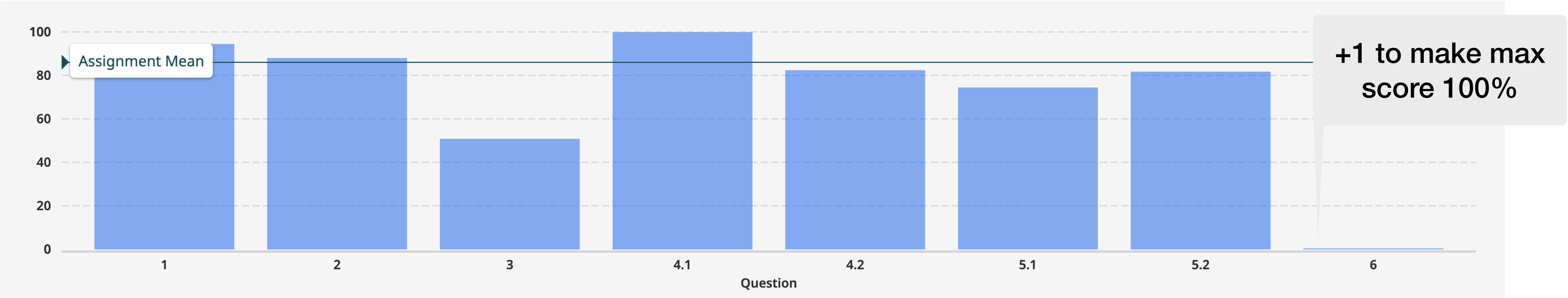
## Lecture 18: Incentives in Decentralized Systems

Shikha Singh

# Announcements and Logistics

- **Midterm # 2** graded feedback returned
    - Median: **88.9%** and mean: **85%**
    - Improvement over Midterm 1 (Median 87%, Mean 82%)
- Let's talk about Problem 3 and 5a



# Problem 3: Revenue Eq

- Homework 7 Problem 1
  - $v_1 = v_2 = e_1 + e_2$ where $e_1$ and $e_2$ are drawn i.i.d. from $U[0,1]$
  - Bidder $1$ only knows $e_1$ and Bidder $2$ only knows $e_2$
  - BNE of second-price auction in this setting is each bidder bids twice their estimate
- BNE of first-price auction?
  - Revenue equivalence says expected payment should be same in both auctions

# Problem 5a

- Based on Assignment 4 Problem 5
  - Any max-weight matching $M$ can be paired with any market-clearing price $\mathbf{p}$ vector to form a competitive equilibrium $(M, \mathbf{p})$
- If $(M, p)$ and $(M', p')$ are both competitive equilibria, we know
  - $M, M'$ are max-weight matching (from first-welfare theorem)
  - $p, p'$ are market-clearing prices
  - Thus, can $(M, p')$ and $(M', p)$ are also competitive equilibria

# Project Deadlines

# Project Ideas and Timeline

- 2-page report due this Friday (May 9) at noon
    - Cover background (paper or exposition of topic most related to your project)
- If project involves simulation, start making progress on the implementation!
- Another check in next week:  sign up for meetings to discuss progress
    - Incentive to make progress early:
    - 1/2 projects will be nominated for the Ward prize (nominations due Thur May 15)
- Project presentations in class next Friday (May 16)
- Final project report due Wed May 21st

# Ward Prize Talks

- A project that stands out in terms of creativity, contribution and results will be nominated for the Ward Prize talk
    - Nominations are due to the dept by Thurs, March 15
    - Presentations are during colloquium on Friday March 16
- It is an honor to win the Ward prize, esp for graduating seniors

# Expectations from Project Report

- Check out example project reports on GLOW
    - https://glow.williams.edu/courses/4311932/files/folder/Project%20Files
- Get a sense of expectations from examples
- Read the project rubric to understand how it will be graded
    - https://docs.google.com/document/d/1FS8HjeGNSDKFKpDzzSrJyEAjB0M9_pr9cHyPM0eGpX4/edit?tab=t.0

# Project Github

- Need Github usernames from everyone
  - Fill this out by tomorrow noon:  https://tinyurl.com/357sheet
- Will create a project repository and share with you
- All project documents submitted there
  - Project abstract:  ADD
  - Project 2-page report
  - Presentation PDF
  - Final report
  - Supplementary materials:  code, README files, figures, plots, etc

# End of Semester Gathering

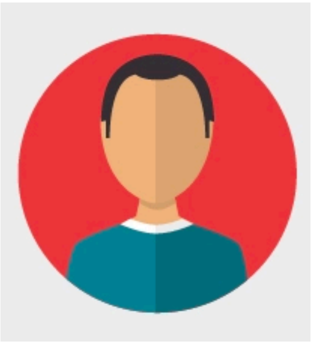
# Incentives in Decentralized Systems

# Last Time: Recap

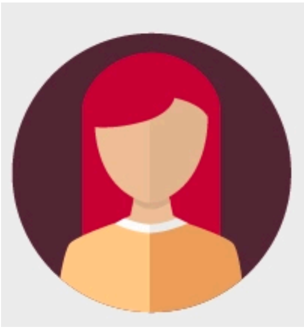
- Nash equilibrium is not a good predictor of outcome in sequential or repeated games
- File sharing games in P2P systems occur repeatedly over time
- Split or steal/ bargaining game can be sequential
    - One person proposes a split
    - Second person says yes or no



Aamir



Beth

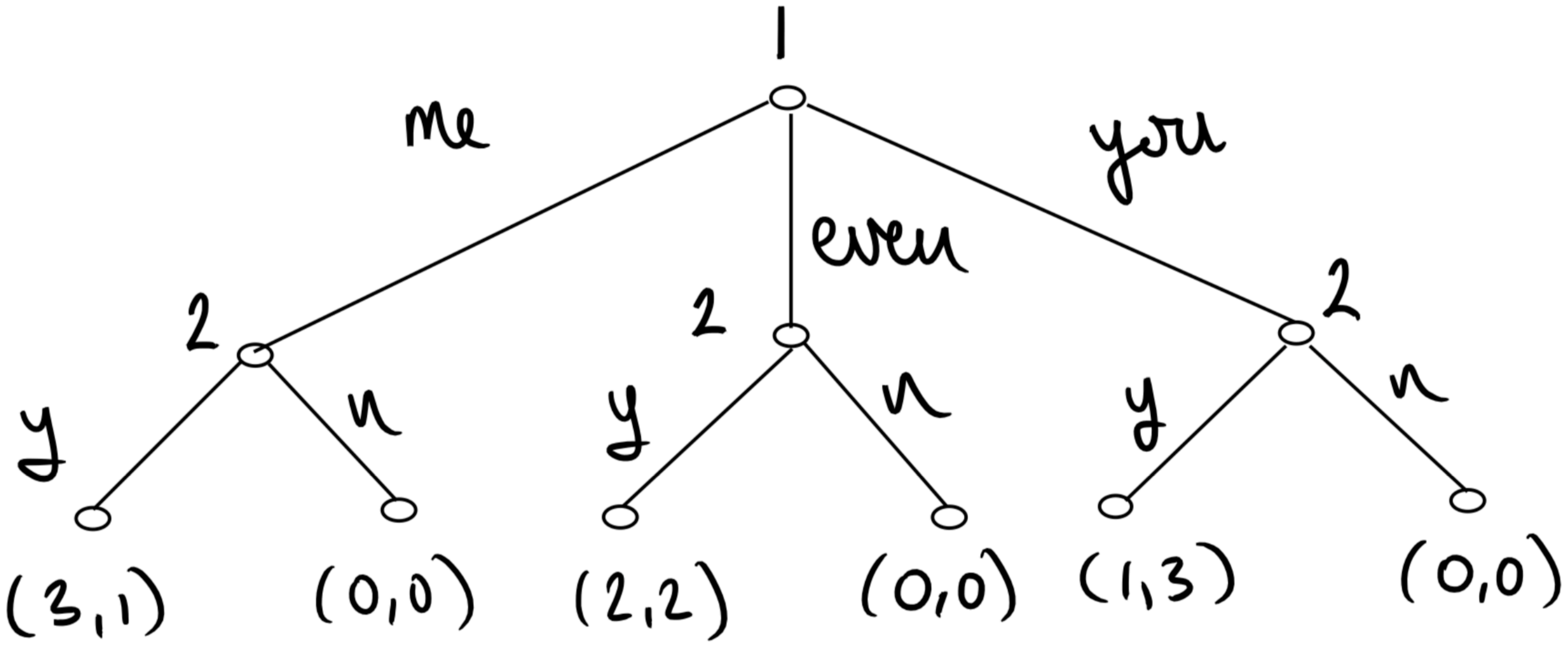


| | Upload | Don't Upload |
|---|---|---|
| Upload | $2, 2$ | $-1, 3$ |
| Don't Upload | $3, -1$ | $0, 0$ |

| | Split | Steal |
|---|---|---|
| Split | 1/2, 1/2 | 0, 1 |
| Steal | 1, 0 | 0, 0 |

# Splitting Game

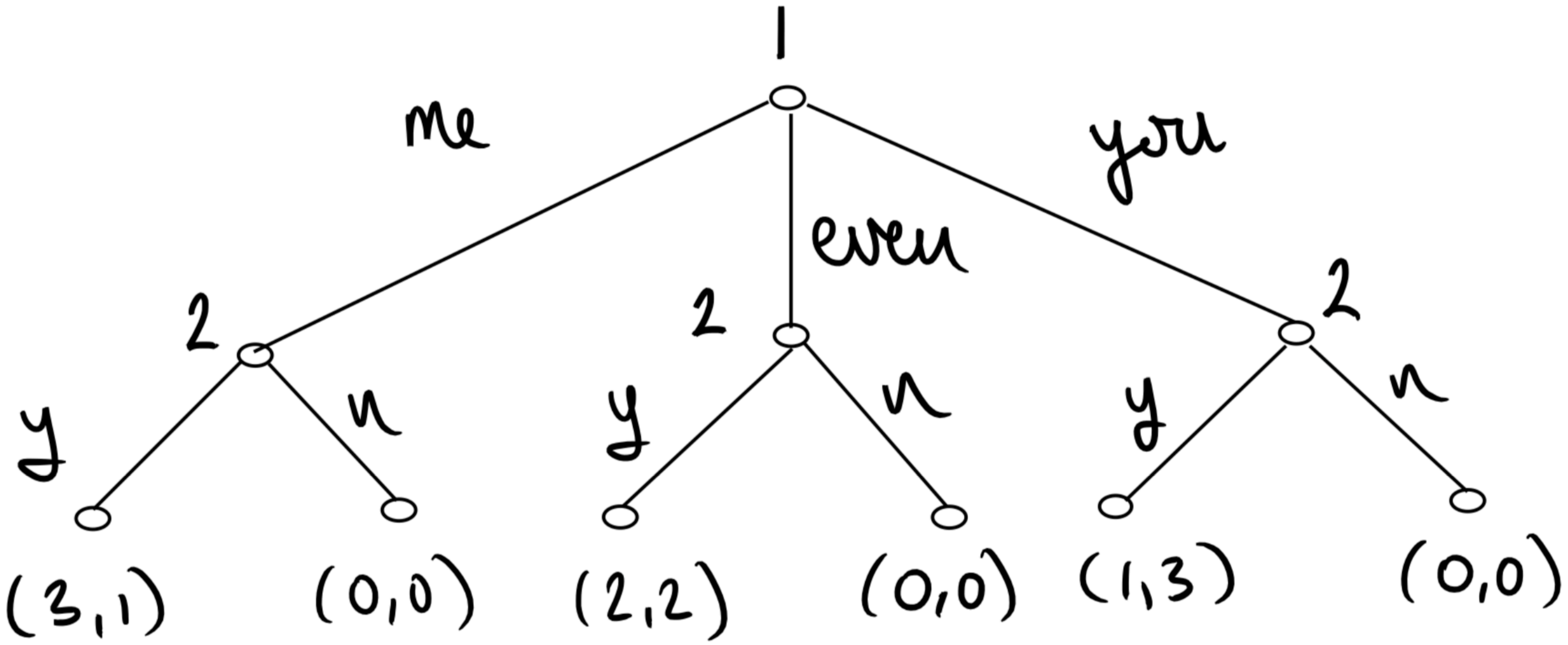
- Two players are deciding over how to divide $4
- If they do not agree, no one gets the money
- Player 1 **goes first** and can propose: **me (3, 1)**, **even (2,2)** or **you (1,3)**




Credit: Textbook by Parkes and Seuken

# Bargaining Game

- Seeing this, player 2 can respond by either accept (**y**) or decline (**n**)
- Game tree below shows the utilities of the players at the leaves

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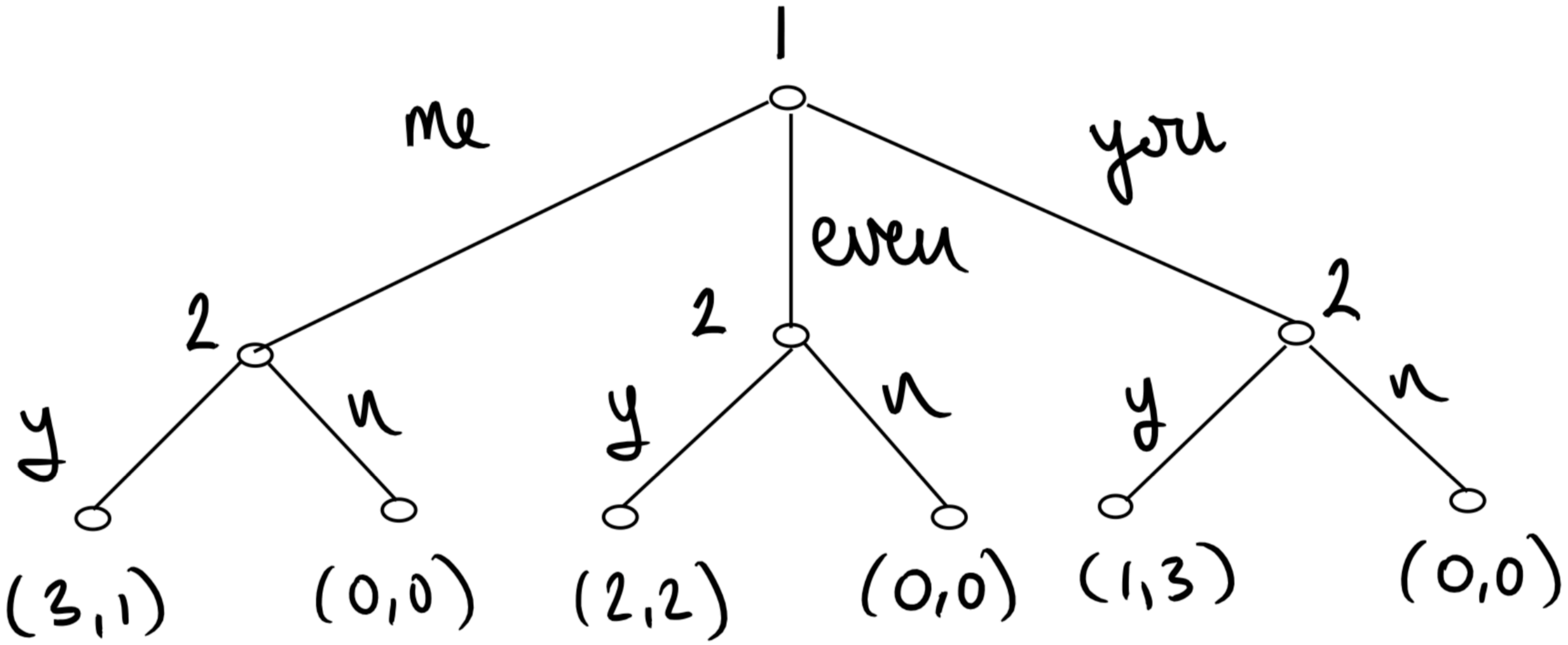
# Bargaining Game

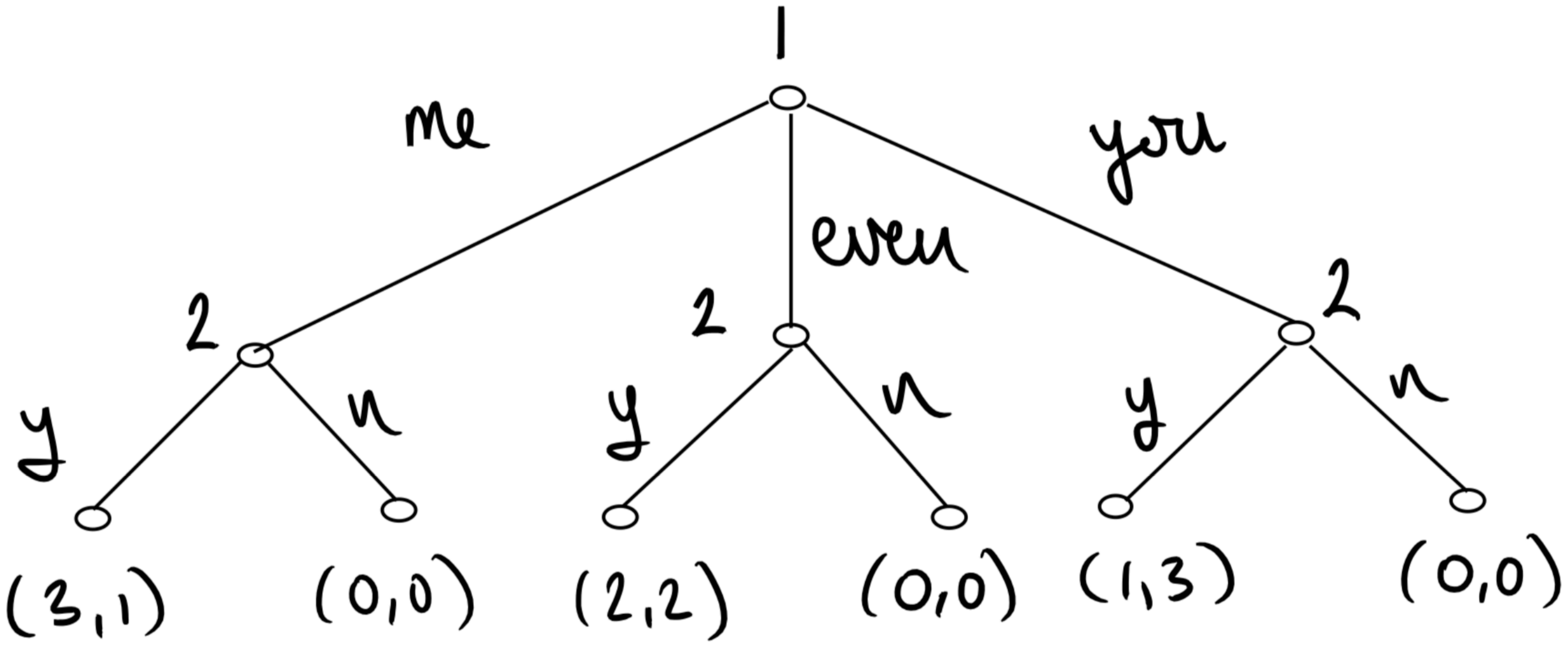
- Strategies in an extensive-form game must specify a complete plan of action
- Player 2 needs an action for all three nodes in the tree:  together they form player 2's strategy




Credit:  Textbook by Parkes and Seuken

# Bargaining Game

- Player 2's strategy thus needs to specify three actions
- For example, $(N, N, Y)$ represents the action plan to say **no** to **me**, **no** to **even**, and **yes** to **you**
- $2^3$ possible strategies

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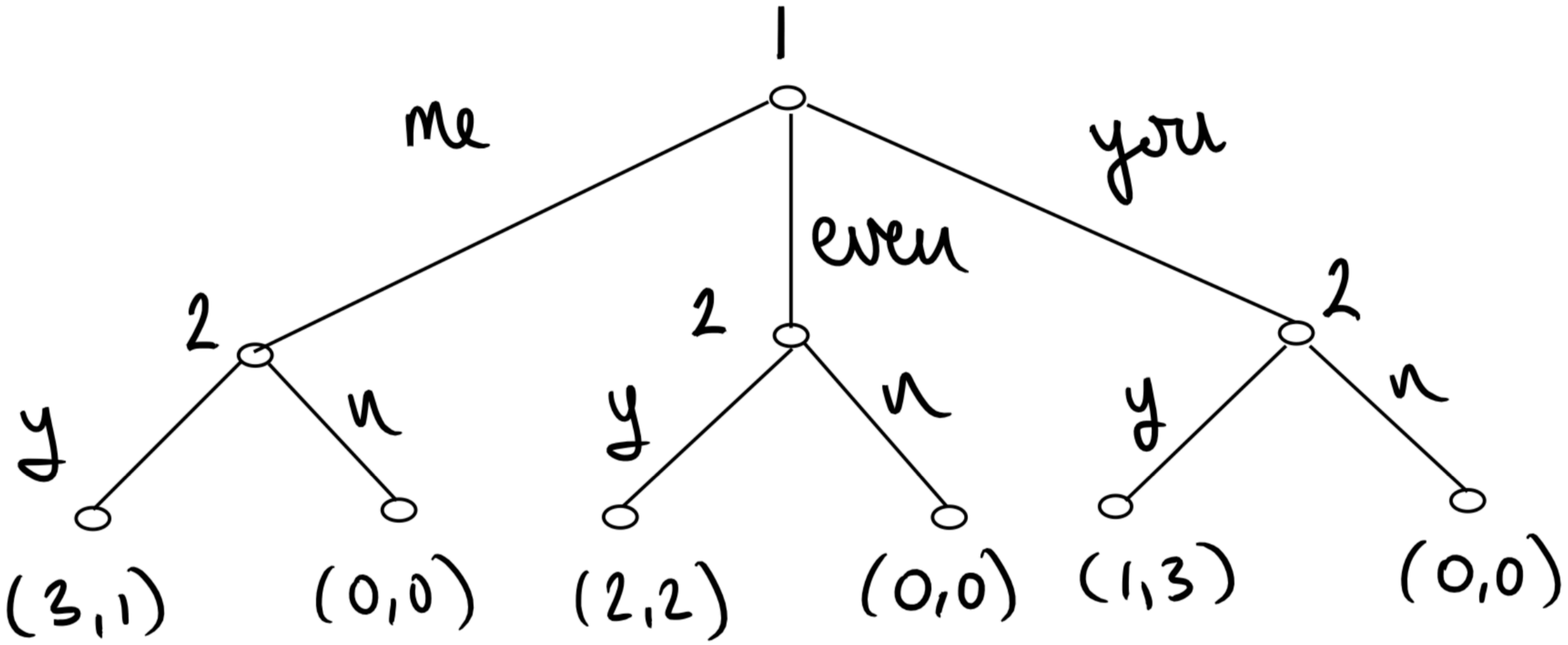
# Strategic-Form

- One can convert an extensive-form game into a strategic (normal-form)
- However, such a representation is far from ideal, and can be confusing

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# Strategic-Form

- One can convert an extensive-form game into a strategic (normal-form)
- However, such a representation is far from ideal, and can be confusing
- Strategic-form representation of our bargaining game:
- Can you identify some of the Nash equilibria?

Player 2

| | | $\langle N,N,N\rangle$ | $\langle N,N,Y\rangle$ | $\langle N,Y,N\rangle$ | $\langle N,Y,Y\rangle$ | $\langle Y,N,N\rangle$ | $\langle Y,N,Y\rangle$ | $\langle Y,Y,N\rangle$ | $\langle Y,Y,Y\rangle$ |
|---|---|---|---|---|---|---|---|---|---|
| | *me* | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 3, 1 | 3, 1 | 3, 1 | 3, 1 |
| Player 1 | *even* | 0, 0 | 0, 0 | 2, 2 | 2, 2 | 0, 0 | 0, 0 | 2, 2 | 2, 2 |
| | *you* | 0, 0 | 1, 3 | 0, 0 | 1, 3 | 0, 0 | 1, 3 | 0, 0 | 1, 3 |

Credit: Textbook by Parkes and Seuken

# Nash Equilibrium

- Lots of Nash equilibria of the extensive form game
  - Not meaningful as a predictor of what players will do
- Some of the Nash equilibria are not plausible
  - For example, the Nash equilibrium (you, $(N, N, Y)$) implies that player $2$ would decline $1\$$ or $2\$$ if offered
  - If player $1$ did offer it, this would not be rational to decline

| | | Player 2 | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | $\langle N,N,N\rangle$ | $\langle N,N,Y\rangle$ | $\langle N,Y,N\rangle$ | $\langle N,Y,Y\rangle$ | $\langle Y,N,N\rangle$ | $\langle Y,N,Y\rangle$ | $\langle Y,Y,N\rangle$ | $\langle Y,Y,Y\rangle$ |
| | *me* | 0, 0 | 0, 0 | 0, 0 | 0, 0 | 3, 1 | 3, 1 | 3, 1 | 3, 1 |
| Player 1 | *even* | 0, 0 | 0, 0 | 2, 2 | 2, 2 | 0, 0 | 0, 0 | 2, 2 | 2, 2 |
| | *you* | 0, 0 | 1, 3 | 0, 0 | 1, 3 | 0, 0 | 1, 3 | 0, 0 | 1, 3 |

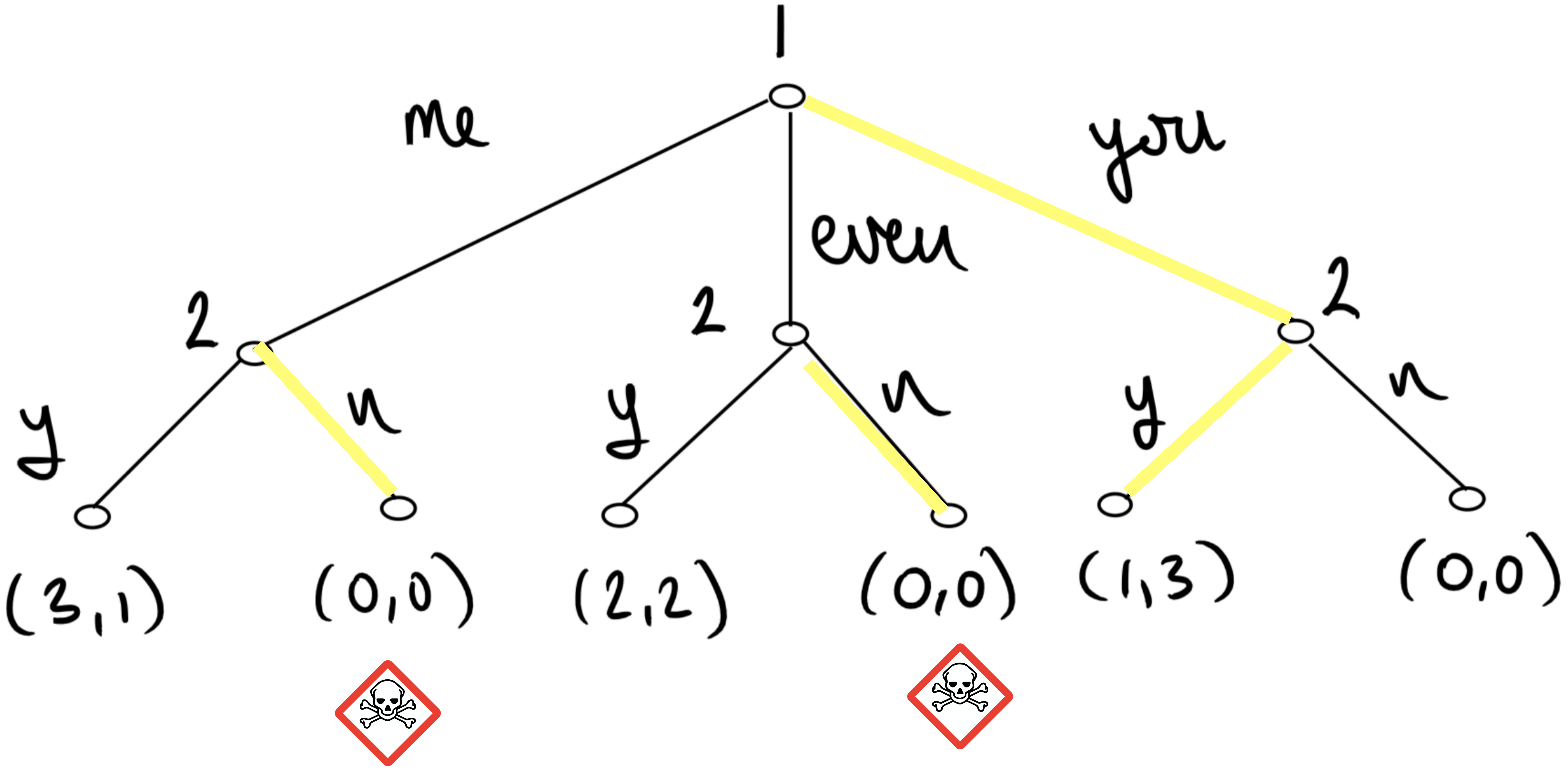
Credit: Textbook by Parkes and Seuken

# Empty Threats

- Nash equilibria as a solution concept for extensive-form games is susceptible to **empty threats** or **non-credit threats**
- An empty threat is when Player 2 who will move in a later round threatens to do something irrational
  - The threat is non-credible because it is not in the best interest of Player 1 to carry it out *if it comes to it*
- Player 1's goal is to convince Player 1, who is moving in an earlier round, to take an action that is favorable to Player 1

# Empty Threats

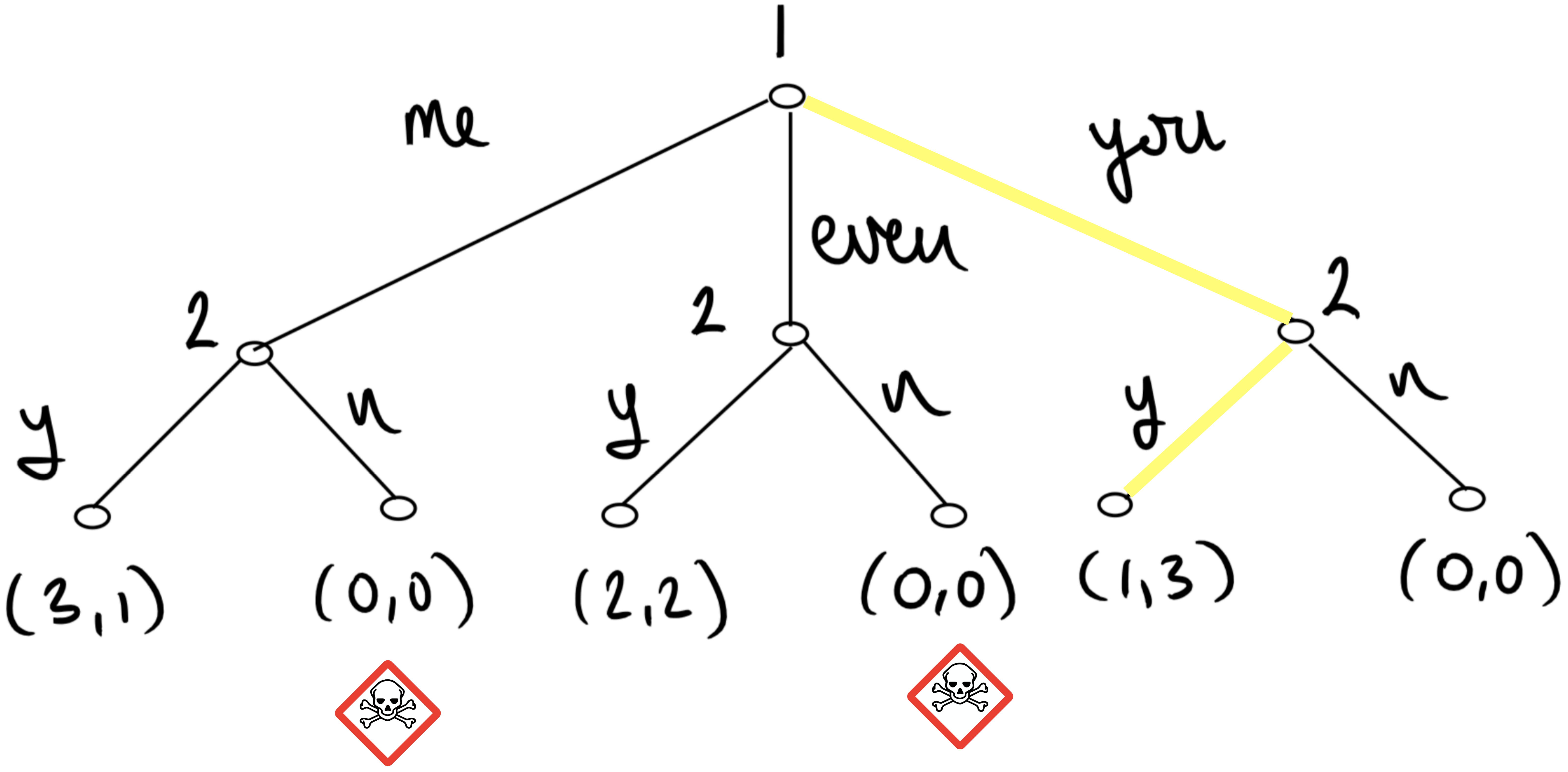
- $s_2$ playing $(N, N, Y)$ is part of a Nash equilibrium but this is an empty threat
- Aimed at deterring player $1$ to pick an option they prefer more




Credit: Textbook by Parkes and Seuken

# New Equilibrium

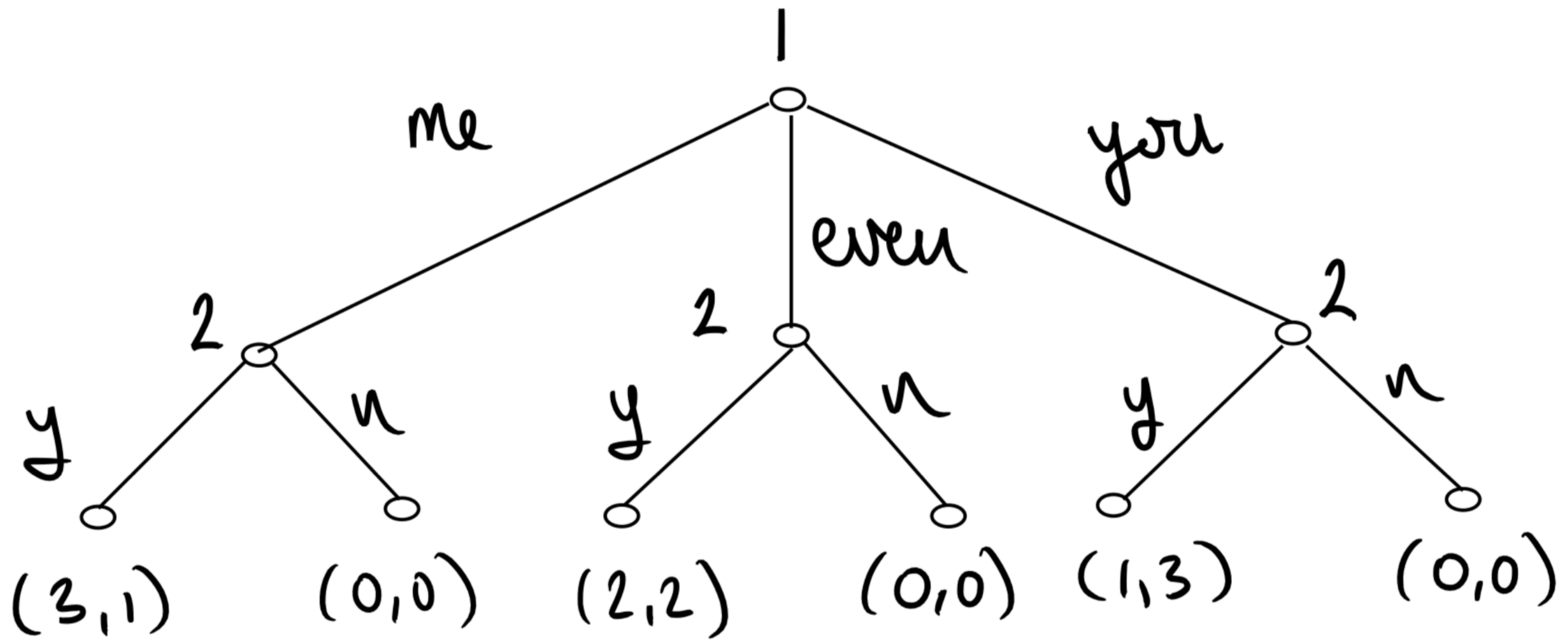
- We need a new equilibrium concept for sequential form games which takes the sequential nature in account and avoids empty threats
- A "refinement" of Nash equilibrium in such games




Credit: Textbook by Parkes and Seuken

# Extensive Form Model

- Game tree representation: a path from root to any node is a **history** $h$
- Player utilities are specified for all the leaves of the tree (terminal histories)
- A player function $P(h) \in N$ specifies which player plays at each node
- Action set $A_i(h)$: set of actions available to player $i$ at non-terminal history $h$

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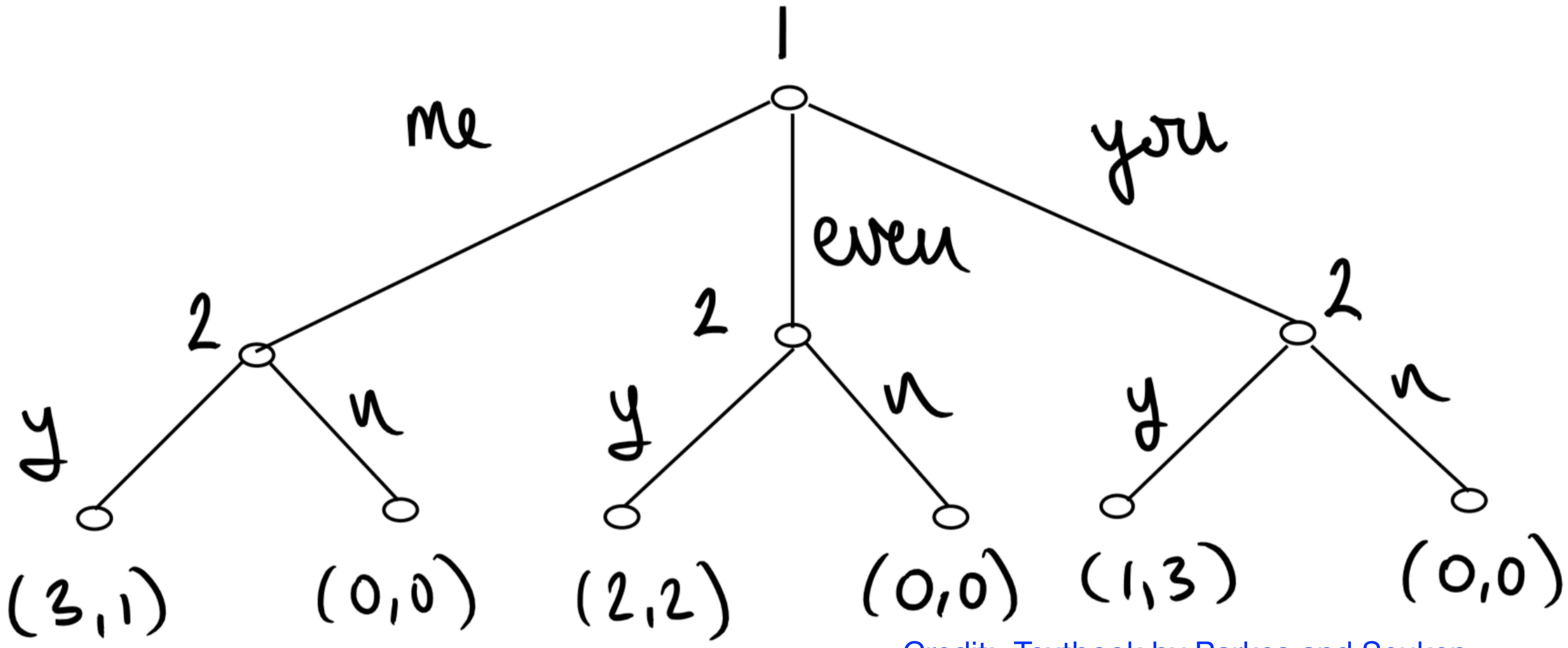
# Subgames

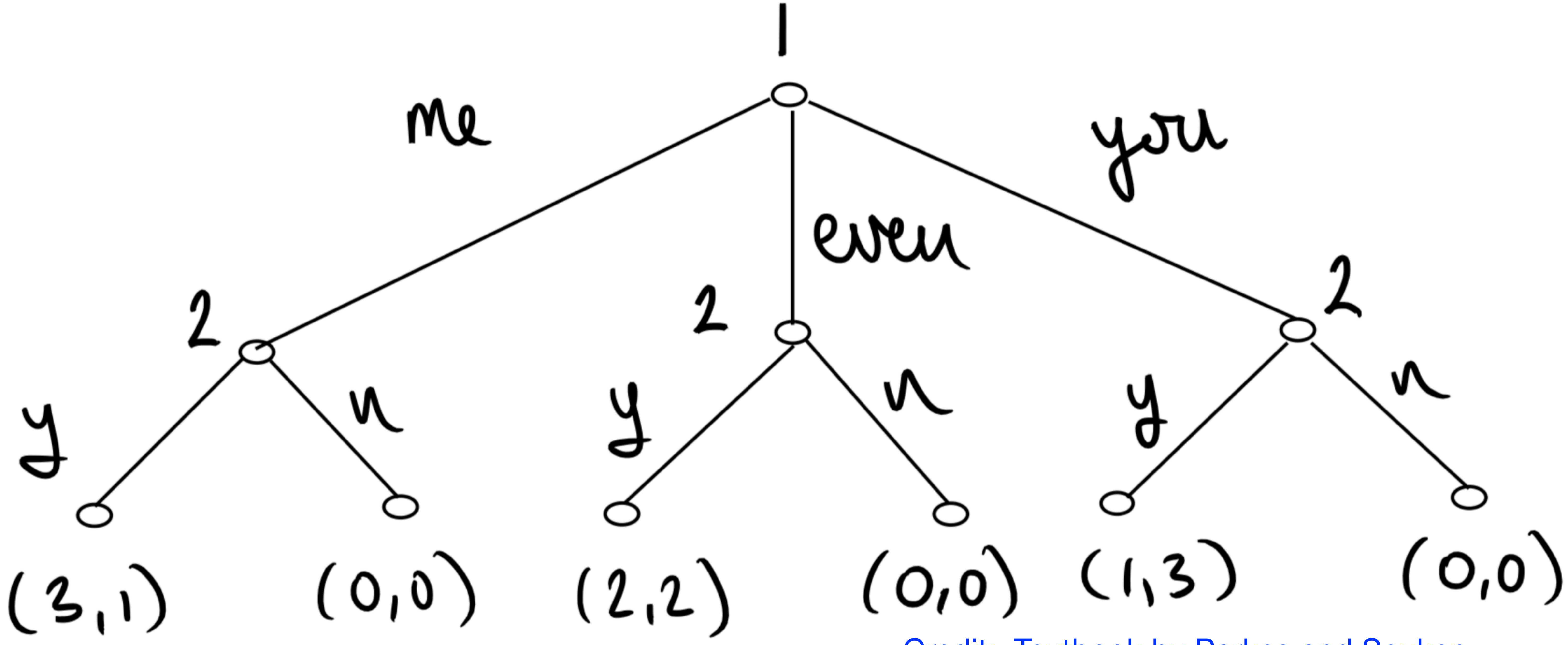
- We define a new solution concept for extensive-form games
- **(Definition).** The subgame starting at history $h$ of an extensive-form game is the extensive-form game rooted at the decision node that corresponds to history $h$
- Can you identify all subgames in this game?




Credit: Textbook by Parkes and Seuken

# Subgame-Perfect Equilibrium

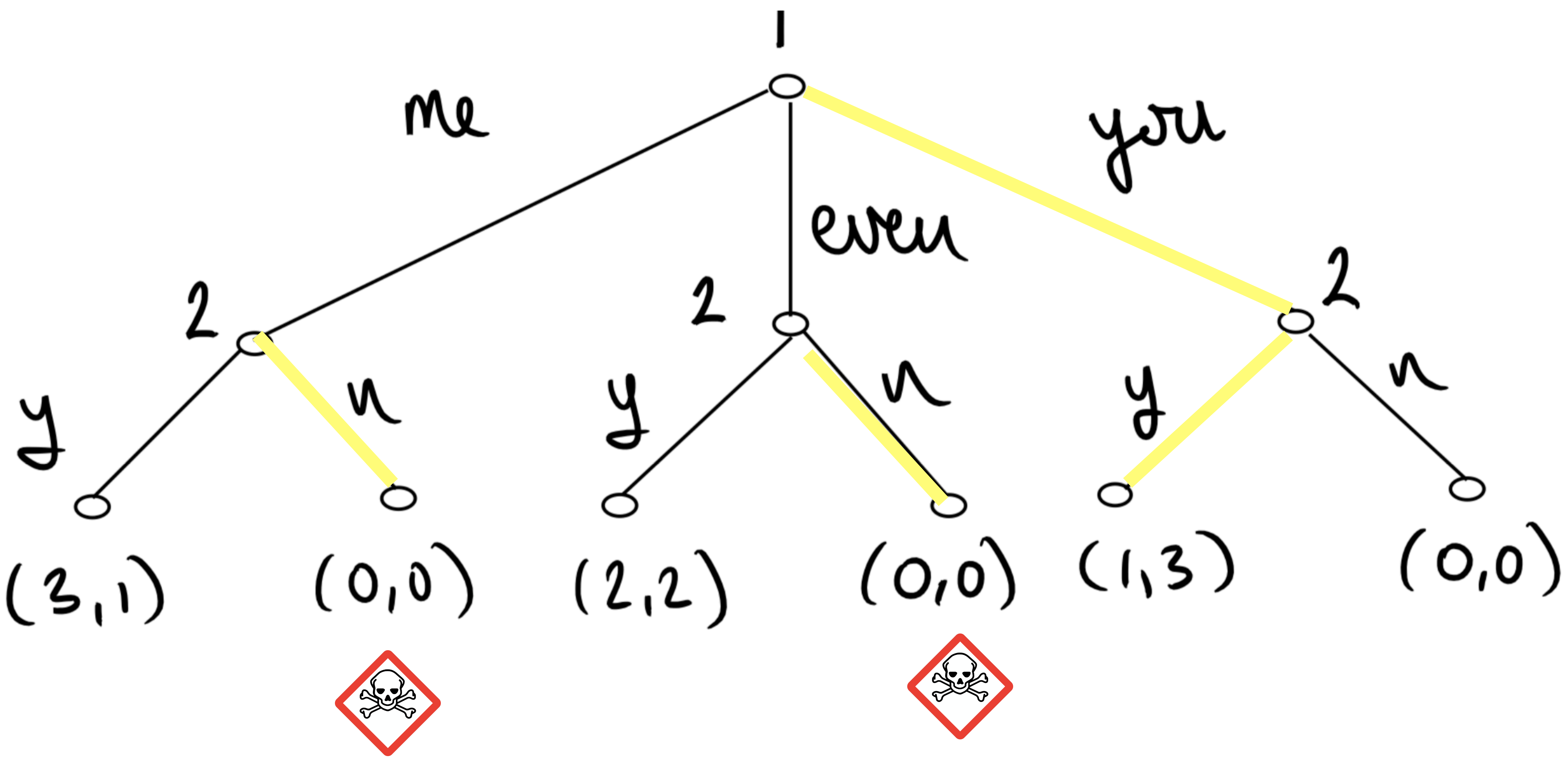
- **(Definition)** A strategy profile $s = (s_1, \ldots, s_n)$ is a subgame-perfect equilibrium of an extenstive form game if the strategy profile is a **Nash equilibrium in every subgame** of the game starting at a non-terminal history
- Enforce that players should play their best responses after each history of the game




Credit: Textbook by Parkes and Seuken

# Subgame-Perfect Equilibrium

- Is the strategy $(\text{you}, (N, N, Y))$ a subgame perfect equilibrium?

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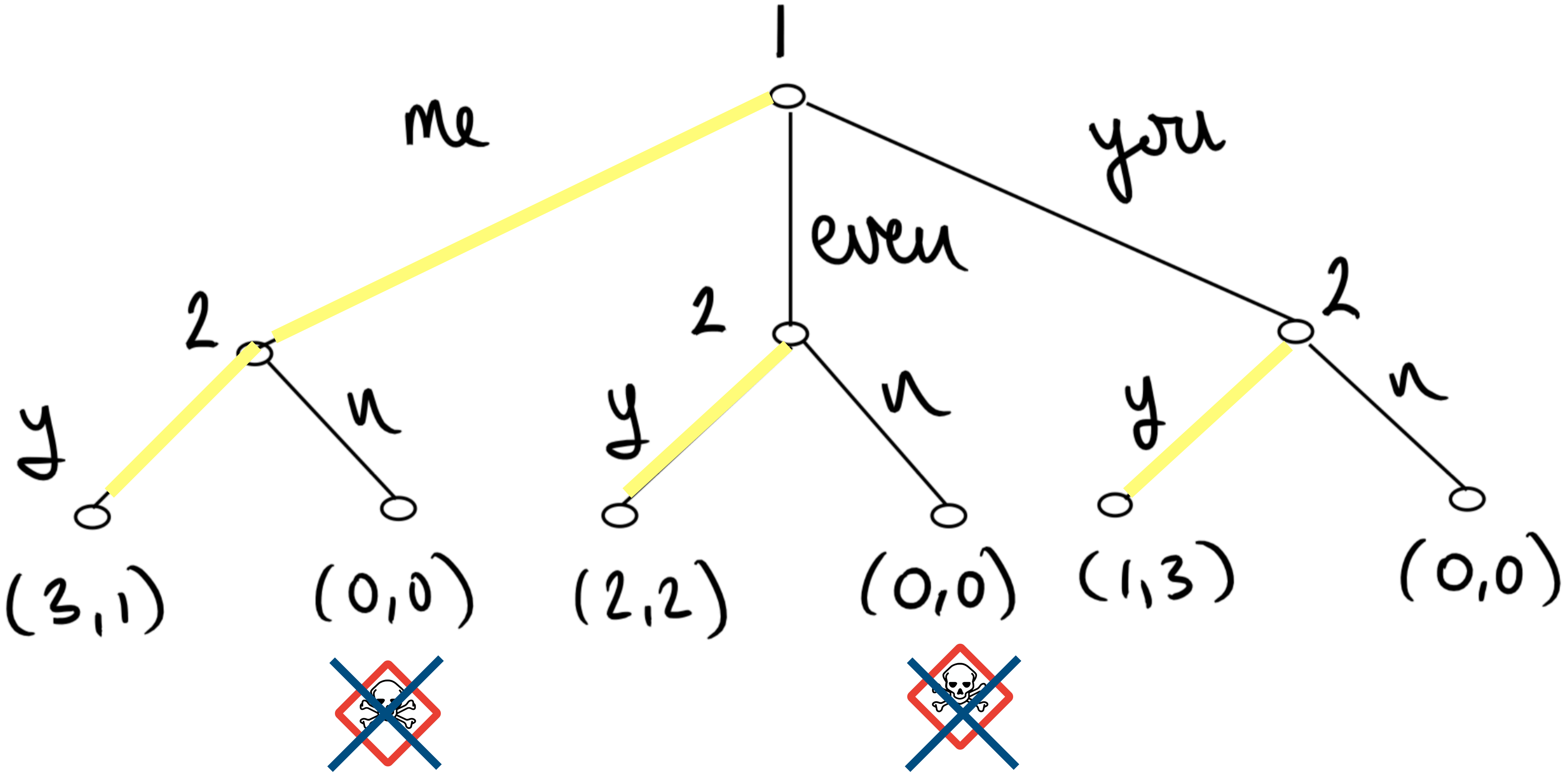
# Subgame-Perfect Equilibrium

- "Conditioned on reaching" any history where player 2 must act, saying no is never a best response

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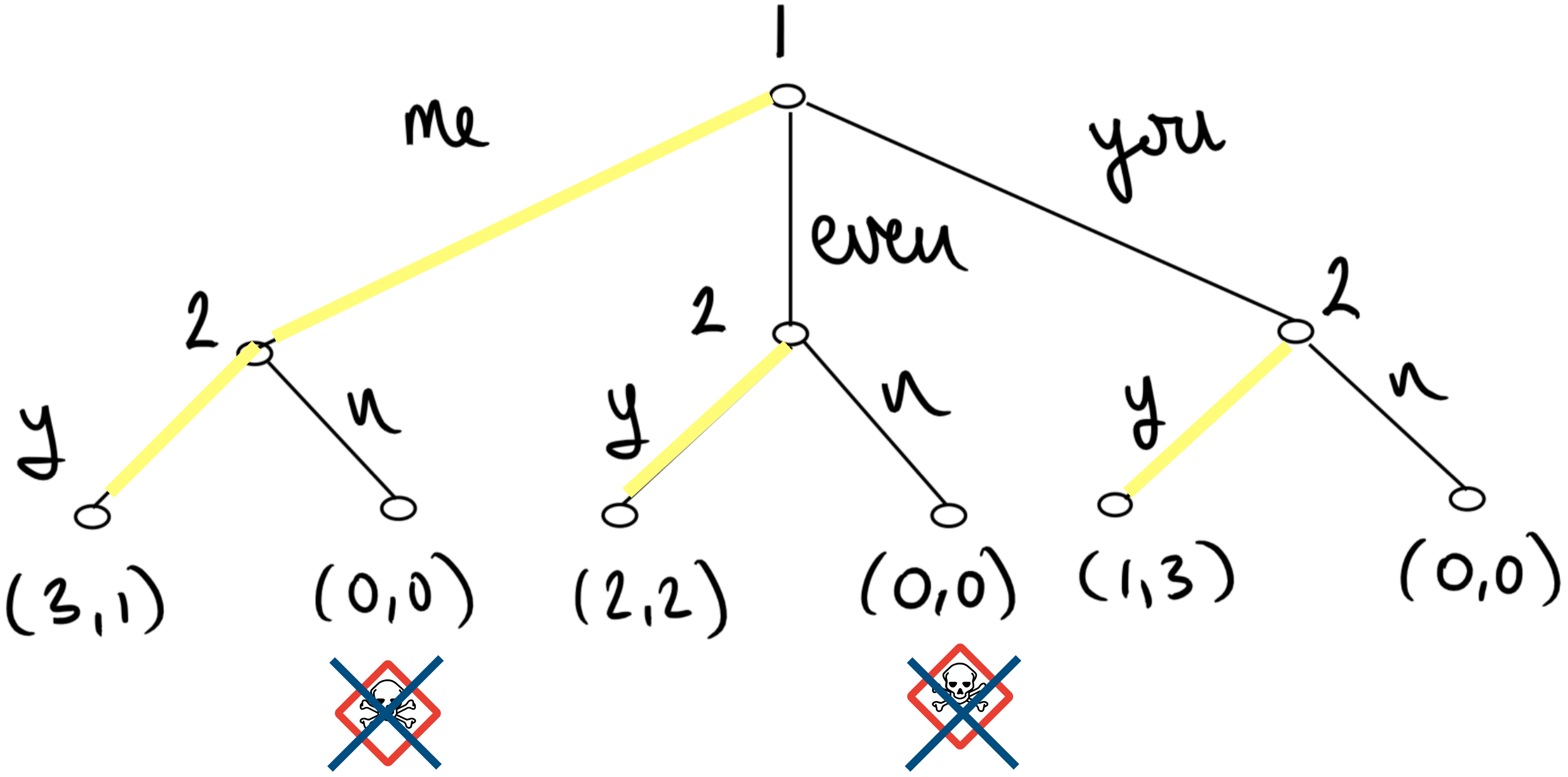
# Subgame-Perfect Equilibrium

- Given player $2$ plays their best response at every node, Player $1$ must choose **me**
- This is the unique SPE of this game




Credit: Textbook by Parkes and Seuken

# Perfect Information

- We will assume the extensive-form games are games of **perfect information**:
  - Each player $i$ knows the complete history $h$ of the game whenever it is $i$'s turn to act $P(h) = i$
  - The structure of the and utilities are common knowledge
- Example of perfect-information sequential game:
  - Chess
- Example of imperfect-information sequential game:
  - Poker
- Extensive-form games with imperfect information are more complicated:
  - Have “information sets” & players’ probabilistic beliefs on histories

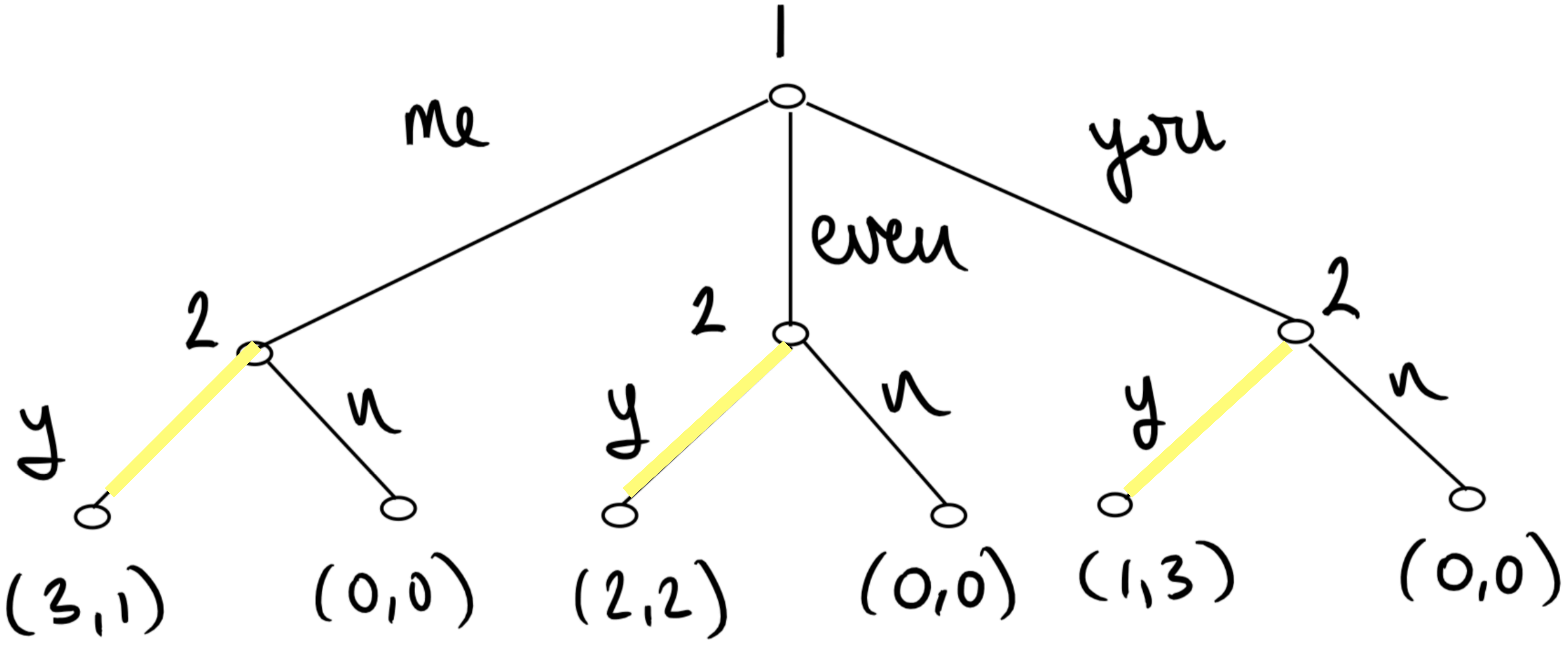
Credit: Textbook by Parkes and Seuken

# Backward Induction

- Approach to compute a SPE of an extensive-form game
- Start at the bottom (say at depth $k$) and look at the player who acts last $P_\ell$
- Conditioned on reaching their decision nodes, figure out $P'_\ell s$ best response
- Fixing the best response of $P_\ell$ at depth $k$, we know have a tree of depth $k-1$
- Continue applying this logic until we reach the root:
  - The resulting strategy profile must be a SPE
  - We need to prove this

# Backward Induction

- Backward's induction is essentially "dynamic programming"
- You keep track of the optimal moves as you go up the tree




Credit: Textbook by Parkes and Seuken

# Backward Induction

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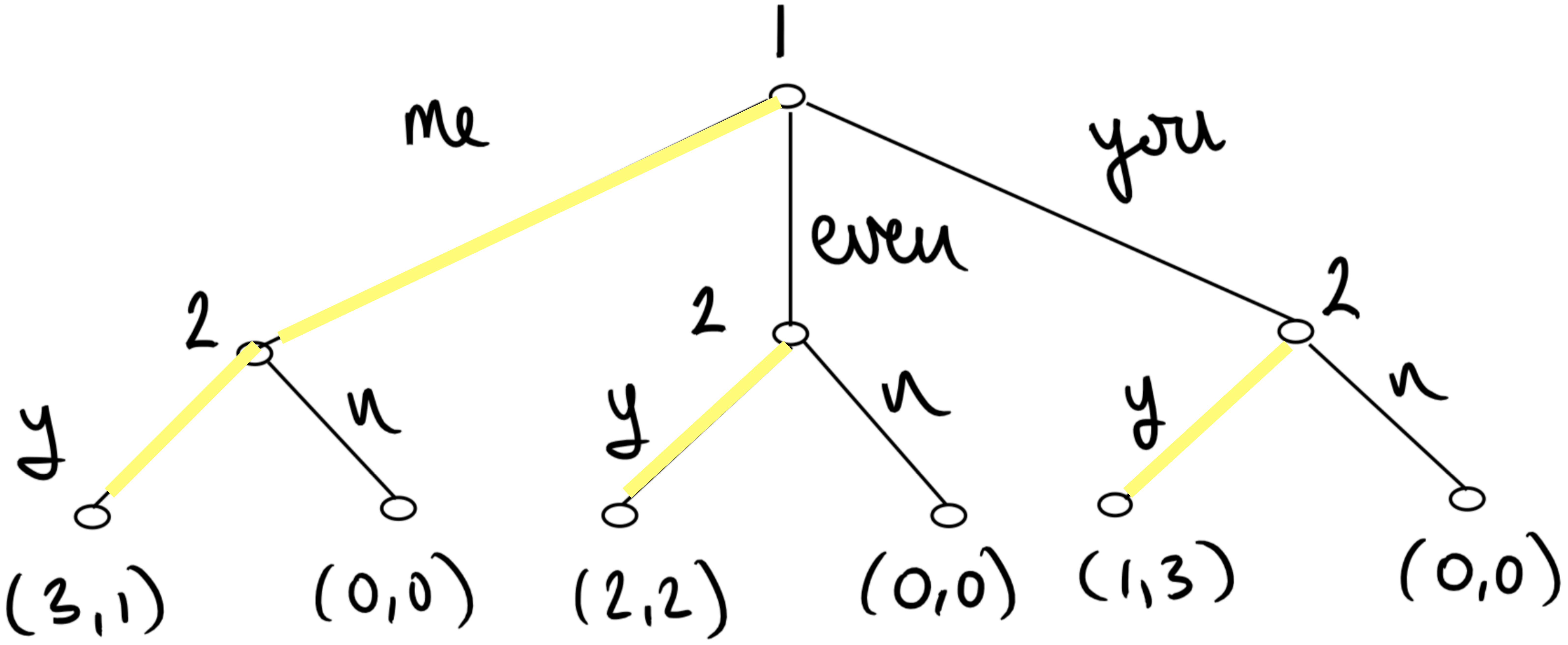
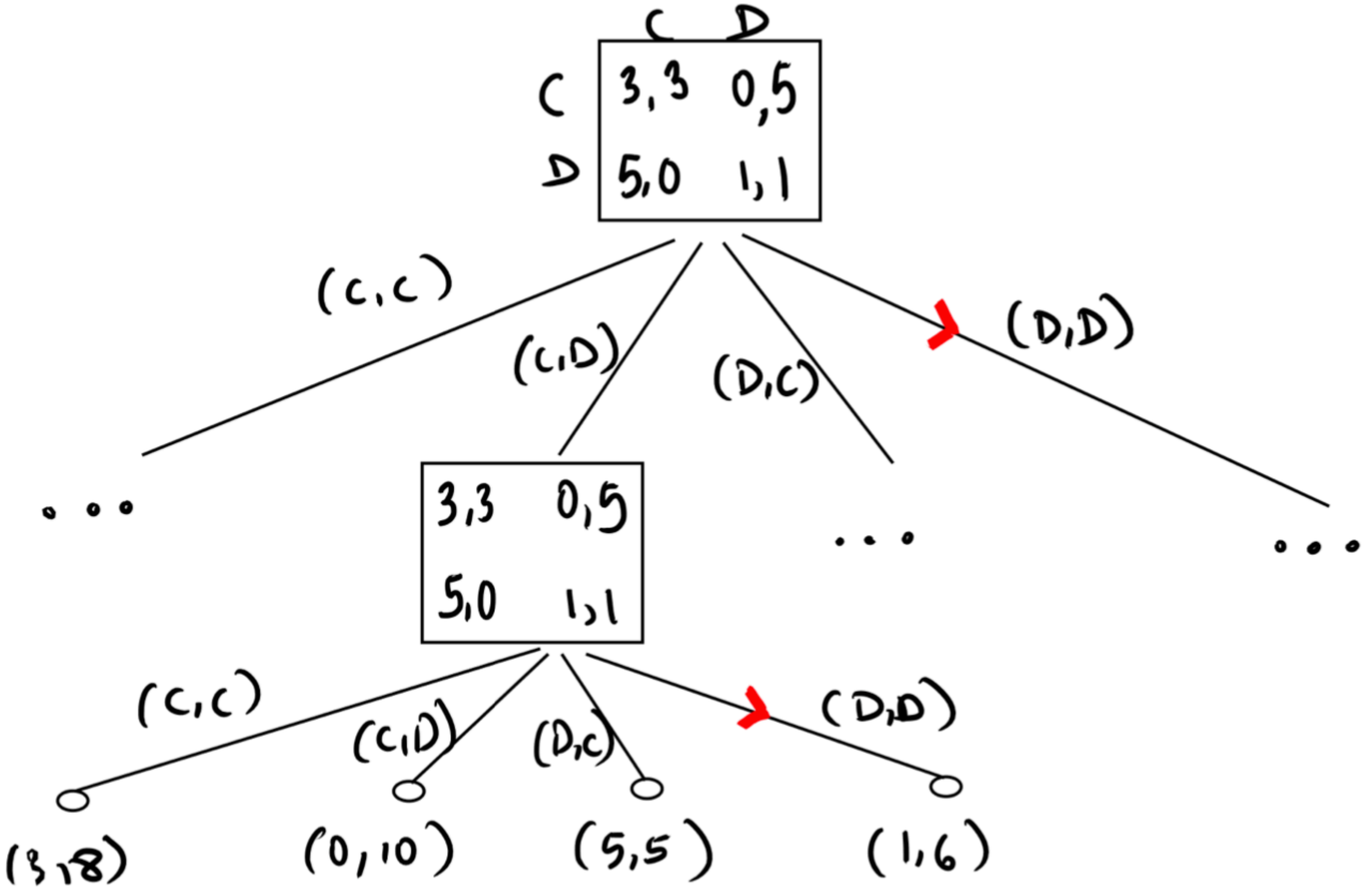



Credit: Textbook by Parkes and Seuken

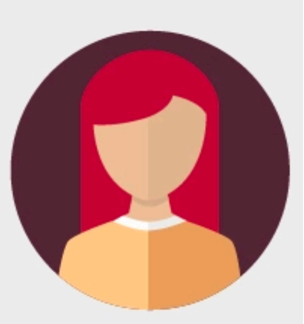
# Repeated Games

# Repeated Prisoner's Dilemma

- To model repeated interactions in a P2P system such as file sharing, consider a prisoners dilemma game repeated $n$ times



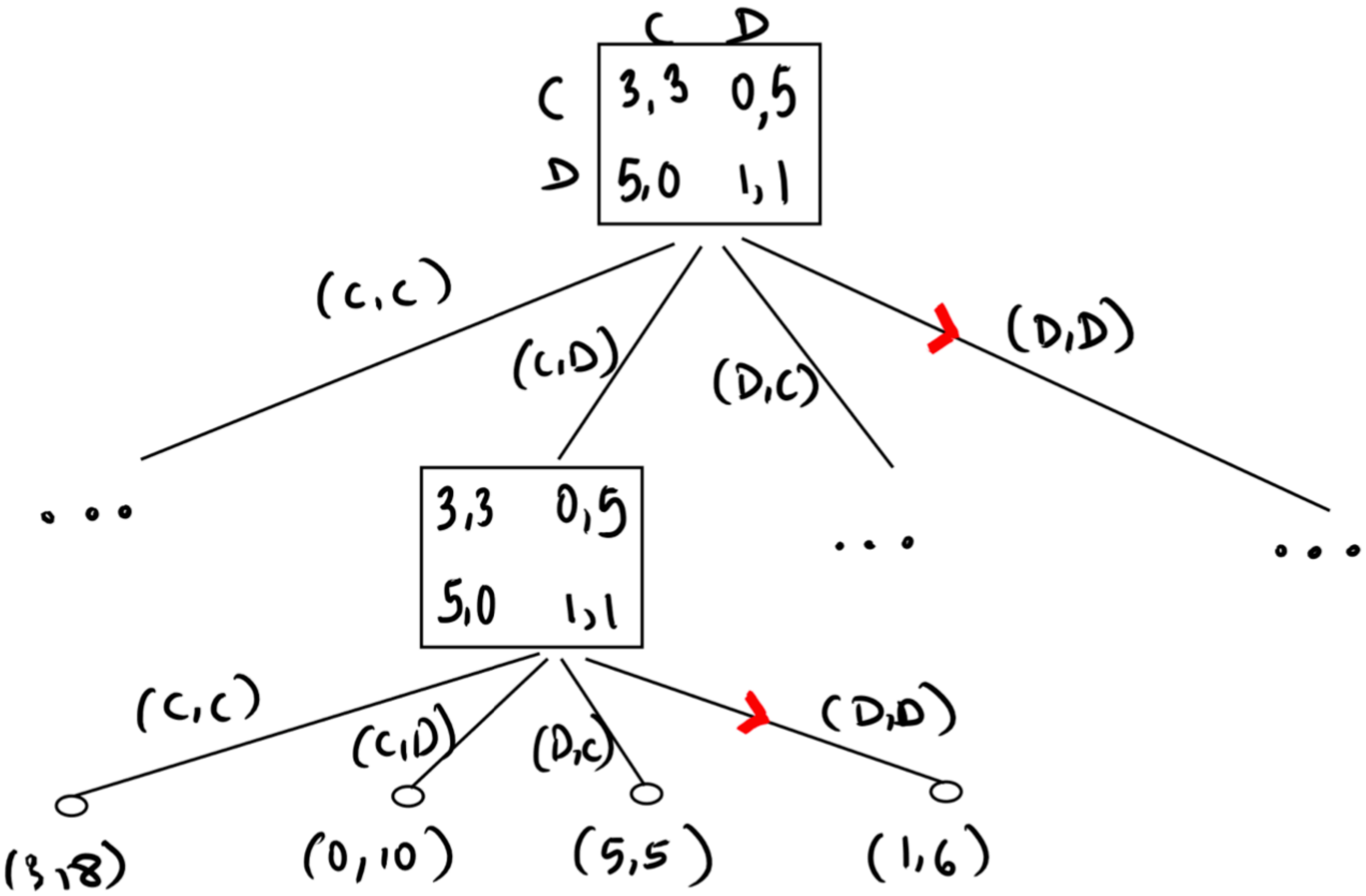





| Beth \ Aamir | $C$ | $D$ |
|---|---|---|
| $C$ | $a, a$ | $b, c$ |
| $D$ | $c, b$ | $d, d$ |

$$c > a > d > b$$

# Example: Repeated PD




Credit: Textbook by Parkes and Seuken

# Infinitely Repeated Games

- If PD game is repeated finitely many times,   what does backward induction tell us about the SPE?
  - Only best response in final round is to play the unique Nash equilibrium
  - In second-last round, the actions of the players does not effect the payoff in the last round, so best response is to play the unique Nash in this round and so on
- To model reality,  need to introduce "uncertainty" about future
  - Done so by introducing "infinitely repeated games"
  - Amir and Beth play the one-shot simultaneously move Prisoner's dilemma game
    - With probability $\delta$ (where $0 < \delta < 1$) the game continues for another round
    - With probability $1 - \delta$ the game ends at this round
- When modeling it this way, can show $(C, C)$ as a SPE of the game

# What Strategy is BitTorrent Based On?

- **Tit-for-tat strategy**:
    - Start by cooperating
    - Do in stage $i$ whatever the opponent does in stage $i-1$
- Thus, tit-for-tat starts optimistically, punishes immediately and forgives quickly
- Turns out to be a good strategy in repeated prisoner's dilemma
    - Also perhaps in life?

# Repeated Prisoner’s Dilemma (Empirically)

# Alexrod IPD Tournaments

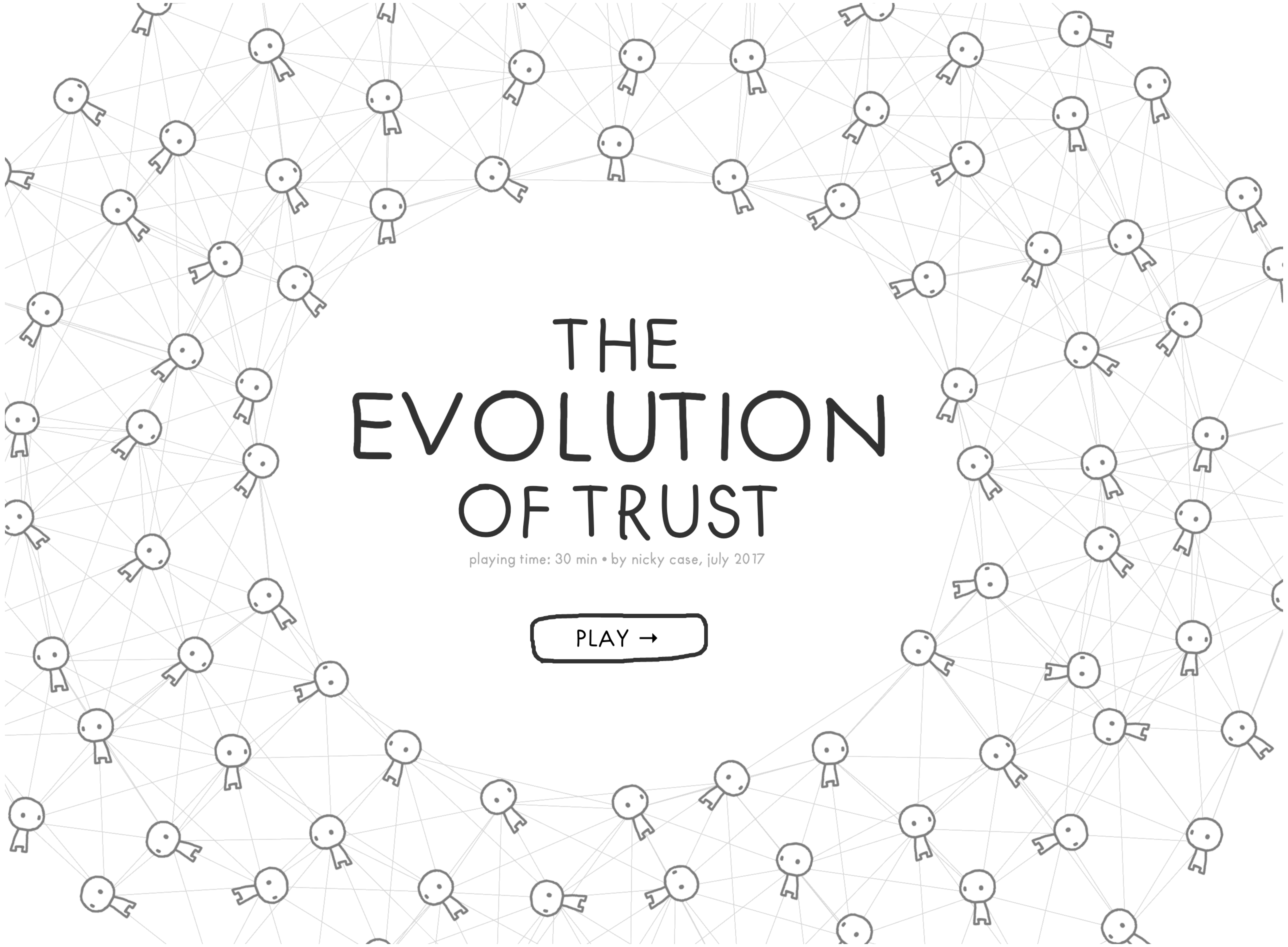
- In ~1980, Robert Axelrod invited colleagues to enter into a tournament for computer programs playing the repeated Prisoner’s dilemma
- There were **15 contestants** and each program played the other 14 in a repeated PD game with 200 stages (so, a round-robin tournament)
- The payoff of a program was averaged over all 200 stages of all 14 matches
- The winning strategy (submitted by Anatol Rapoport) was **Tit-for-Tat**!
- Tit-for-Tat was the shortest entry and many other programs were (trying to be) sophisticated
- What makes this even more surprising
    - TfT cannot win a head-to-head match with any opponent!

# Alexrod IPD Tournaments

- Axelrod circulated the results of the first tournament and solicited entries for a second tournament with the same rules
- This time there were **62 entries**
- Rapoport resubmitted **Tit-for-Tat**, completely unchanged, and won again!
    - This happened even though other programs were explicitly tailored to exploit Tit-for-Tat
    - Turns out they imploded against each other!
- **Python Axelrod library** (https://axelrod.readthedocs.io/en/stable/) has implementations of all the entries in the tournament
- Their Github has extensive documentation and is useful for potential project on the topic of Iterated Prisoner's Dilemma

# Evolution of Trust

- Simulation that compares different strategies in a visually appealing format: https://ncase.me/trust/
    - Adds the notion of error



# Strategic Behavior in BitTorrent

# BitTorrent

- Inspired by repeated prisoner's dilemma and tit-for-tat strategy, Bram Cohen introduced the BitTorrent protocol

## Incentives Build Robustness in BitTorrent

Bram Cohen
bram@bitconjurer.org

May 22, 2003

**Abstract**

The BitTorrent file distribution system uses tit-for-tat as a method of seeking pareto efficiency. It achieves a higher level of robustness and resource utilization than any currently known cooperative technique. We explain what BitTorrent does, and how economic methods are used to achieve that goal.

Cohen, B. (2003, June). Incentives build robustness in BitTorrent. In *Workshop on Economics of Peer-to-Peer systems* (Vol. 6, pp. 68-72).

# Strategic Behavior on BitTorrent

- There are many places where strategic behavior is involved in P2P file-sharing
  - Piece-revelation:  peers can be strategic about which pieces to reveal to others
  - Upload Bw:  how to allocate upload bandwidth across peers by choosing number of upload slots/ how to distribute bandwidth
  - What pieces to allow an uncooked peer to download
  - What pieces to try to download, etc.
- Variations on how to handle each decision leads to different strategic clients, often designed to “game” BitTorrent

# Strategic Behavior on BitTorrent

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SCOTT GILBERTSON 01.04.2007 12:03 PM

## BitTorrent Bullies: BitTyrant and BitThief

Impatience doesn’t get you anywhere. It causes stress and frustration and it usually gives everyone around you the impression that you’re a total jerk. And as these two new P2P clients demonstrate, impatience also has the potential to ruin BitTorrent. Yesterday, TorrentFreak alerted us to a new BitTorrent client with a "selfish" anti-social streak. BitTyrant, [...]

https://www.wired.com/2007/01/bittorrent-bullies-bittyrant-and-bitthief/

# Bit Thief

- The unapologetic named BitThief client is to **leech off** a BitTorrent reference client
  - Download without ever uploading anything just like free riders in Gnutella
- The goal is to exploit the “optimistic unchokes”
- BitThief does this by pestering the tracker incessantly, asking for more peers to grow its neighborhood
- Downloads are slower than the reference client (because you don’t get the reciprocation advantage) but around 5x
- How to mitigate against such an incentive attack?
  - Have tracker ignore repeated requests in some window

Free Riding in BitTorrent is Cheap

Thomas Locher[1], Patrick Moor[2], Stefan Schmid[1], Roger Wattenhofer[1]
[1] Computer Engineering and Networks Laboratory (TIK), ETH Zurich, 8092 Zurich, Switzerland
{lochert, schmiste, wattenhofer}@tik.ee.ethz.ch
[2] Google Inc., Mountain View, CA 94043, USA
pmoor@google.com

# Bit Tyrant

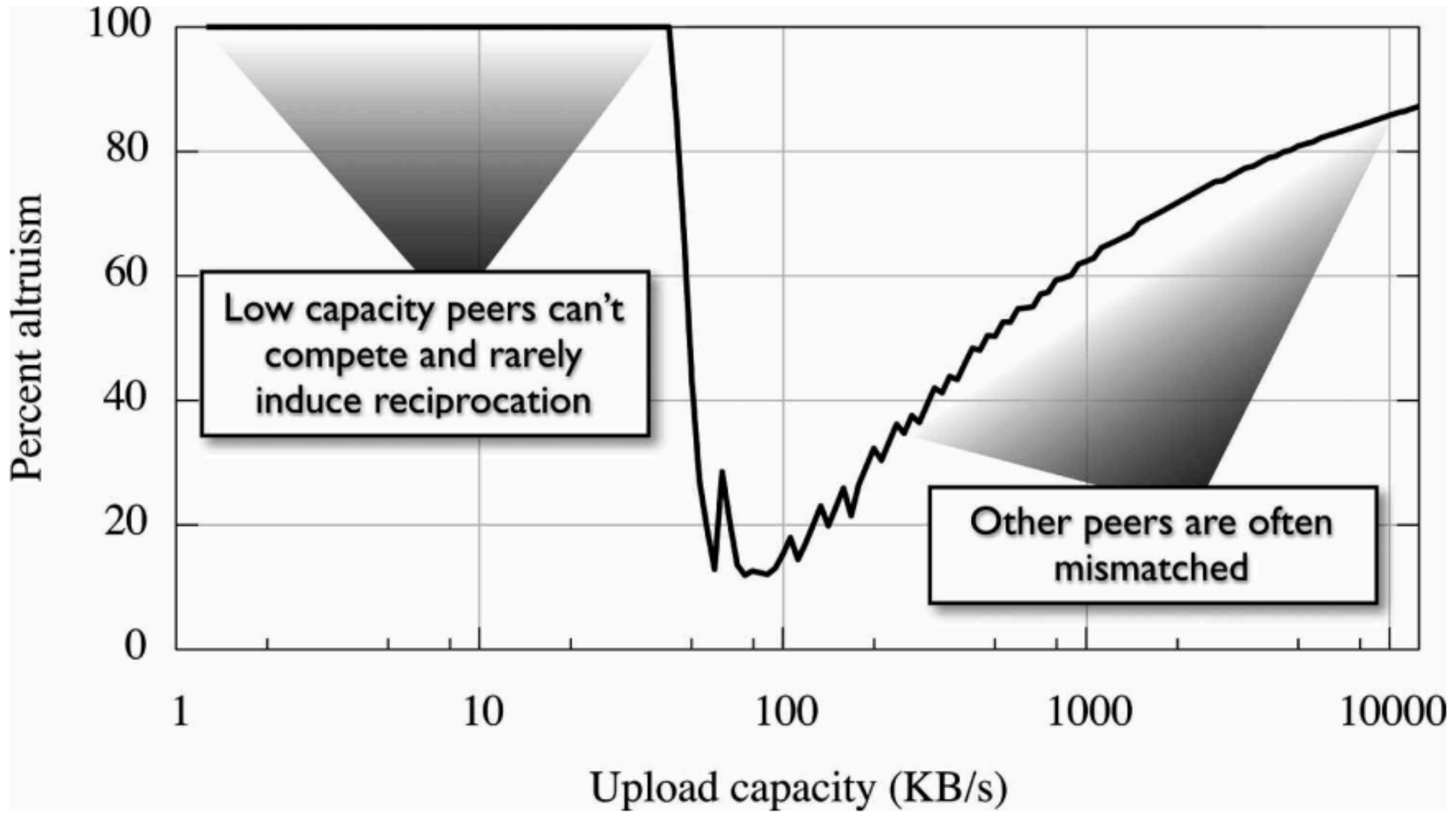


FIGURE 2: PERCENTAGE OF ALTRUISTIC UPLOAD CONTRIBUTION AS A FUNCTION OF CAPACITY


MICHAEL PIATEK, TOMAS ISDAL, TOM ANDERSON, ARVIND KRISHNAMURTHY, AND ARUN VENKATARAMANI


## building BitTyrant, a (more) strategic BitTorrent client


Michael Piatek is a graduate student at the University of Washington. After spending his undergraduate years working on differential geometry, his research interests now include incentive design in distributed systems, network measurements, and large-scale systems building.

*piatek@cs.washington.edu*

Tomas Isdal graduated with a MSc in Computer Science and Engineering from the Royal Institute of Technology, Stockholm, Sweden, and is currently a graduate student in the Department of Computer Science and Engineering at the University of Washington. His interests include peer-to-peer and distributed systems, Internet measurements, and network security.

*isdal@cs.washington.edu*

Tom Anderson is a Professor in the Department of Computer Science and Engineering at the University of Washington. He is an ACM Fellow and a winner of the ACM SIGOPS Mark Weiser Award, but he is perhaps best known as the author of the Nachos operating system.

*tom@cs.washington.edu*

Arvind Krishnamurthy is an Assistant Research Professor at the University of Washington. His research interests are primarily at the boundary between the theory and practice of distributed systems. He has worked on automated mechanisms for managing overlay networks and distributed hash tables, network measurements, parallel computing, techniques to make low-latency RAID devices, and distributed storage systems that integrate the numerous ad hoc devices around the home.

*arvind@cs.washington.edu*

Arun Venkataramani has been an Assistant Professor at the University of Massachusetts Amherst since 2005, after receiving his Ph.D. from the University of Texas at Austin by way of the University of Washington. His research interests are in the practice and theory of networking and distributed systems.

*arun@cs.umass.edu*


PEER-TO-PEER SYSTEMS OFTEN APPEAL to scalability as a motivating feature. As more users request data, more users contribute resources. Scaling a service by relying on user contributions—the P2P approach—depends on providing incentives for users to make those contributions. Recently, the popular BitTorrent file distribution tool has emerged as the canonical example of an incentive-aware P2P design. Although BitTorrent has been in widespread use for years and has been studied extensively, we find that its incentive strategy is not foolproof. This article describes BitTyrant, a new, strategic BitTorrent client. For users interested in faster downloads, BitTyrant provides a median 70% performance improvement on live Internet swarms. However, BitTyrant also demonstrates that selfish users can improve performance even while reducing upload contribution, circumventing intended incentives.


Piatek, M., Isdal, T., Anderson, T., Krishnamurthy, A., & Venkataramani, A. (2007, April). Do incentives build robustness in BitTorrent. In *Proc. of NSDI* (Vol. 7).

# Bit Tyrant

- Goal: use upload capacity strategically
- Each user $i$ maintains two estimates about other peers:
  - $d_{ij}$ : download capacity expected to get from peer $j$ if $i$ uploaded to $j$
  - $u_{ij}$ : amount of upload $i$ would need to get $j$ to reciprocate back
    - If $j$ is getting uploads from a lot of people, it may not be worth it
- Based on these estimates, BitTyrant uses a simple greedy strategy
  - Get most bang-for-buck (like in Knapsack)
  - Sort everyone based on ratio $d_{ij}/u_{ij}$ and upload in this order until capacity runs out
- Unlike reference client, does not split bandwidth equally

Levin, Dave, et al. "Bittorrent is an auction: analyzing and improving bittorrent's incentives." *Proceedings of the ACM SIGCOMM 2008 conference on Data communication*. 2008.

# BitTorrent is an Auction: Analyzing and Improving BitTorrent's Incentives

Dave Levin
University of Maryland
dml@cs.umd.edu

Katrina LaCurts
University of Maryland
katrina@cs.umd.edu

Neil Spring
University of Maryland
nspring@cs.umd.edu

Bobby Bhattacharjee
University of Maryland
bobby@cs.umd.edu

## ABSTRACT

Incentives play a crucial role in BitTorrent, motivating users to upload to others to achieve fast download times for all peers. Though long believed to be robust to strategic manipulation, recent work has empirically shown that BitTorrent does not provide its users incentive to follow the protocol. We propose an auction-based model to study and improve upon BitTorrent's incentives. The insight behind our model is that BitTorrent uses, not tit-for-tat as widely believed, but an auction to decide which peers to serve. Our model not only captures known, performance-improving strategies, it shapes our thinking toward new, effective strategies. For example, our analysis demonstrates, counter-intuitively, that BitTorrent peers have incentive to intelligently under-report what pieces of the file they have to their neighbors. We implement and evaluate a modification to BitTorrent in which peers reward one another with proportional shares of bandwidth. Within our game-theoretic model, we prove that a proportional-share client is strategy-proof. With experiments on PlanetLab, a local cluster, and live downloads, we show that a proportional-share unchoker yields faster downloads against BitTorrent and BitTyrant clients, and that under-reporting pieces yields prolonged neighbor interest.

# Takeways

- P2P provides a rich landscape to analyze how strategic behavior appears in the wild
- Provides useful case study when designing similar systems

# Remaining Topics