CSCI 357: HW 7 (Practice) **Your Name:** _____

More Practice with Bayes Nash Equilibrium

Problem 1. (Verifying Bayes Nash)

A local business *Desperados* is being auctioned off. Two competing businesses *Tonys* and *Tres Ninos* would like to acquire it. Desperados has two locations: one in Williamstown and one in North Adams. (Note that the business is being auctioned off as a whole, that is, both locations comprise of a single item and that there are only two bidders.)

Tonys knows the profitability estimate e_1 of the business in its Williamstown location, having operated in the town for decades. Tres Ninos, a North Adams fixture, knows the profitability estimate of the business e_2 in its North Adams location. Assume that both e_1 and e_2 are distributed uniformly and independently over the interval $[0, 1]$.

The total value of the Desperados business to both Tony and Tres Ninos is $v_1 = v_2 = e_1 + e_2$, but because each of them keep their portion of the estimate secret from the other, they each only know their own estimate. The only thing they know about the other's estimate is that it is drawn uniformly and independently from the unit interval.

Suppose the auction being held is a *second-price auction*. Show that in a Bayes' Nash equilibrium both Tonys and Tres Ninos bid twice their estimate. In particular, suppose we fix Tres Ninos' strategy as $b_2 = 2e_2$, then show that Tonys' best response is to bid $b_1 = 2e_1$. (*Hint. Since Tonys does not know e_2, it must compute the expected value of e_2 given the information that is available.*)

Problem 2. (Solving for Bayes Nash Equilibrium using revenue equivalence)

Consider a single-item auction with two buyers where the buyer's valuation are drawn i.i.d. from a uniform distribution on $[0, 1]$. Suppose we run an all-pay auction: each bidder submits a bid. The bidder with the largest bid wins and each bidder is charged their bid. (Notice that the bidder who loses ends up with a negative utility.)

What is the bidding strategy at the Bayes Nash equilibrium of such an auction? Use Myerson's revenue equivalence (steps in HW 4) to solve for a symmetric Bayes Nash equilibrium strategy in an all-pay auction.