CSCI 357: HW 6 (due 04/15/2025) **Your Name:** ____________________

Problem 1. The *veto rule* is the following social-choice rule:

- Every voter names their least favorite alternative.
- The rule then selects the alternative that is named the least number of times.

Show that the veto rule is not Condorcet consistent by giving a counterexample with 3 alternatives.

Problem 2. Show that the Borda rule is not Condorcet consistent by giving a counter example.

Problem 3. (Independence of Clones Criterion) The *independence of clones* criterion in voting measures a voting mechanism's robustness to strategic nomination. Voting rules, such as plurality, are not robust against strategic nomination because the addition of similar candidates can divide the support among them, which can cause them to lose. We formalize this criterion next.

A set $C \subseteq A$ of alternatives are clones for alternative a if the alternatives in $C \cup \{a\}$ are consecutive in the preference ranking of every voter (they do not need to be in the same order). For example, consider $A = \{A, B, B_2, B_3, C\}$ as the set of alternatives and consider the following preference rankings:

(2 voters) B, B_3, B_2, A, C	(2 voters) B_2, B_3, B, C, A	(3 voters) C, A, B_2, B, B_3

Then, candidates $\{B_2, B_3\}$ are clones of candidate B.

A social-choice function f satisfies *independence of clones* criterion if whenever $f(L) = a$ on a preference ranking L, then $f(L') = a$ (or clone of a) whenever one or more clones are introduced, that is, L' is formed by introducing one of more clones to L.

To see why plurality rule does not satisfy independence of clones criterion, consider the example profiles above with B_2 and B_3 removed. Then, plurality would select B as the winner. However, with the addition of the clones, plurality would select C as the winner.

Show that the Borda rule does not satisfy the independence of clones criterion by giving a counterexample.