CSCI 357: HW 3 (due 03/04/2025)

**Guideline:** These are short-answer questions that must be done individually. Write your answer in the space provided. No justification is required. The completed HW must be handed back in class on the due date.

## Myerson Payments for Knapsack Approximation

**Problem 1.** Consider the greedy-approximation algorithm to the Knapsack auction from lecture (or Chapter 4 Roughgarden). We will analyze this algorithm in Assignment 2 and show that it gives a monotone allocation. Here, we would like to apply it to example inputs.

Compute the greedy allocation and the Myerson payment of each bidder for the following two inputs. Recall that because this is a 0/1 allocation, by Myerson's lemma each agent pays their "critical bid"—the lowest bid at which their allocation jumpts from 0 to 1.

Set W = 5 for both cases below. Fill in the tables below with allocation ( $x_i = 0$  or 1) and payments  $p_i$ .

Bidder $\boldsymbol{i}$	Bid $b_i$	Size $s_i$	Allocation $x_i$	Payment $p_i$
1	5	2		
2	6	1		
3	6	3		
4	12	5		

Now suppose bidder 2 changes their bid to 8. Assuming the same capacity W = 5, what is the new allocation and corresponding payments.

Bidder <i>i</i>	Bid $b_i$	Size $s_i$	Allocation $x_i$	Payment $p_i$
1	5	2		
2	8	1		
3	6	3		
4	12	5		

## VCG Mechanism and Revenue Anomaly

**Problem 2.** Consider the VCG mechanism in a *combinatorial auction* with two items A and B, where the possible allocation that can be given to a single bidder are AB (both items), A, B or  $\emptyset$  (no item). Suppose we have 2 bidders whose valuations for each allocation as:

	Ø	A	В	AB
Bidder 1	0	2	0	2
Bidder 2	0	0	0	2

That is, bidder 1 only cares about receiving A and bidder 2 only cares about receiving the combination of A and B. Fill in the blanks below.



(b) One of the problems with the VCG mechanism is that it may generate worse revenue (sum of payments) when the competition increases. Show that adding a third bidder to this example (with appropriate valuations) may actually decrease the revenue generated by the mechanism. Add the valuation profile of Bidder 3 below that achieves this.

	Ø	A	В	AB
Bidder 1	0	2	0	2
Bidder 2	0	0	0	2
Bidder 3				



Note that the total revenue (sum of payments) must have gone down from part (a).

(c) Does this problem occur in a single-item Vickrey auction? State Yes or No: