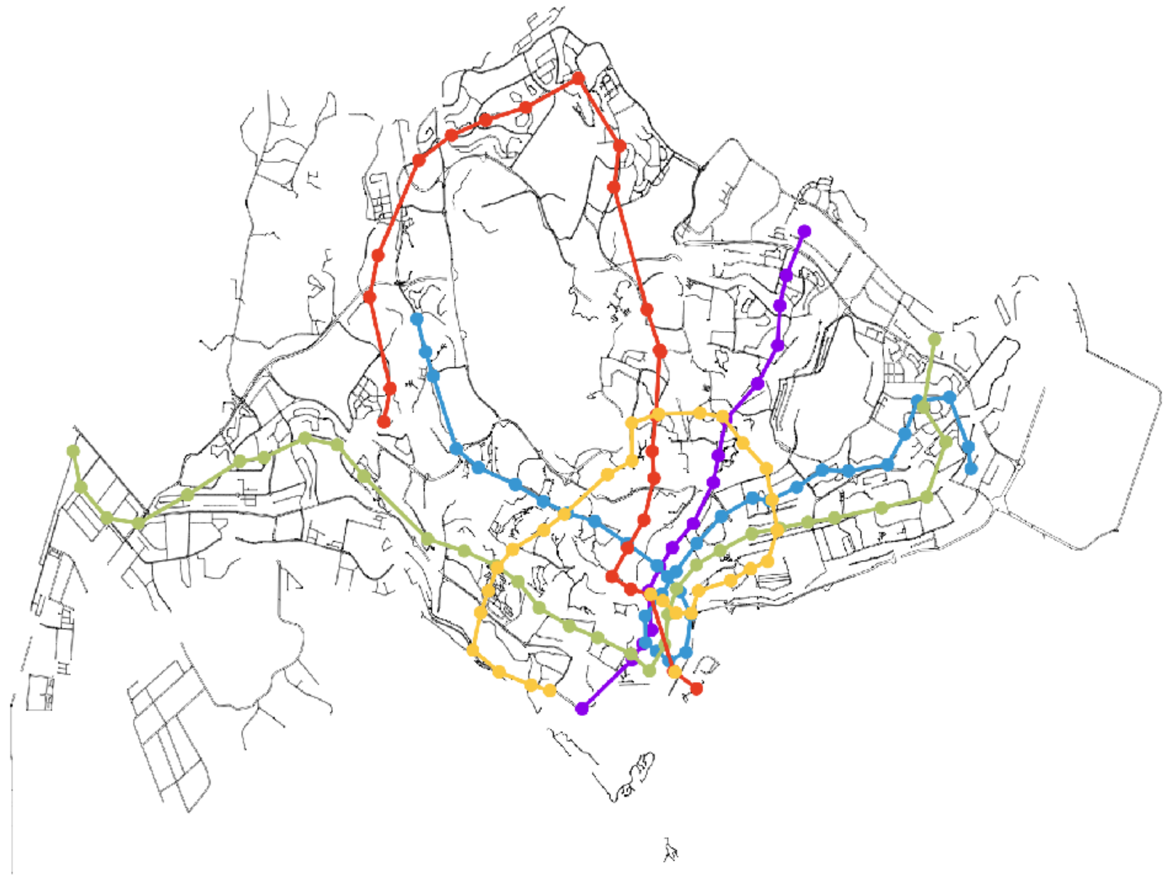
CSCI 357: Algorithmic Game Theory

Lecture 20: Selfish Routing

Shikha Singh

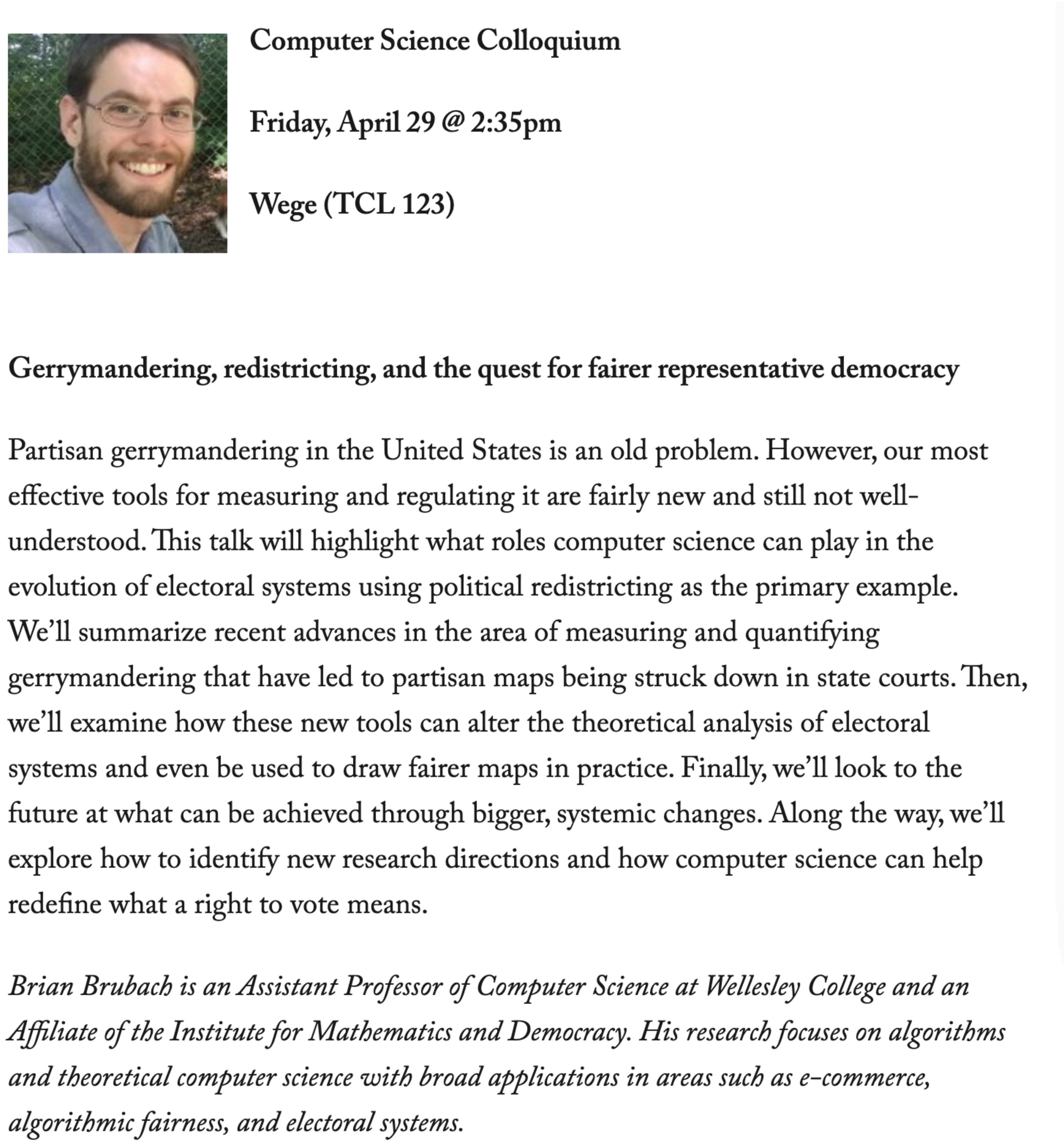
Announcements and Logistics

- No more homework or exams 🎉
- Working on Midterm 2 grading
  - Will return feedback soon
- 1-page project abstract due **via Github** tomorrow (April 29) 5 pm
  - LaTeX template linked on website/GLOW
- Project meetings instead of office hours now
  - Sign up ahead of time https://tinyurl.com/357projectmeet
- 2-page report due next week
- Student presentations last week of classes

Questions?

Reminder: Colloquium Tomorrow

- Strategic gerrymandering
    - Paper in Projects page
- A bit of extra credit for attending!
- Brian will join the second half of the class and talk about liquid democracy



Computer Science Colloquium

Friday, April 29 @ 2:35pm

Wege (TCL 123)

Gerrymandering, redistricting, and the quest for fairer representative democracy

Partisan gerrymandering in the United States is an old problem. However, our most effective tools for measuring and regulating it are fairly new and still not well-understood. This talk will highlight what roles computer science can play in the evolution of electoral systems using political redistricting as the primary example. We'll summarize recent advances in the area of measuring and quantifying gerrymandering that have led to partisan maps being struck down in state courts. Then, we'll examine how these new tools can alter the theoretical analysis of electoral systems and even be used to draw fairer maps in practice. Finally, we'll look to the future at what can be achieved through bigger, systemic changes. Along the way, we'll explore how to identify new research directions and how computer science can help redefine what a right to vote means.

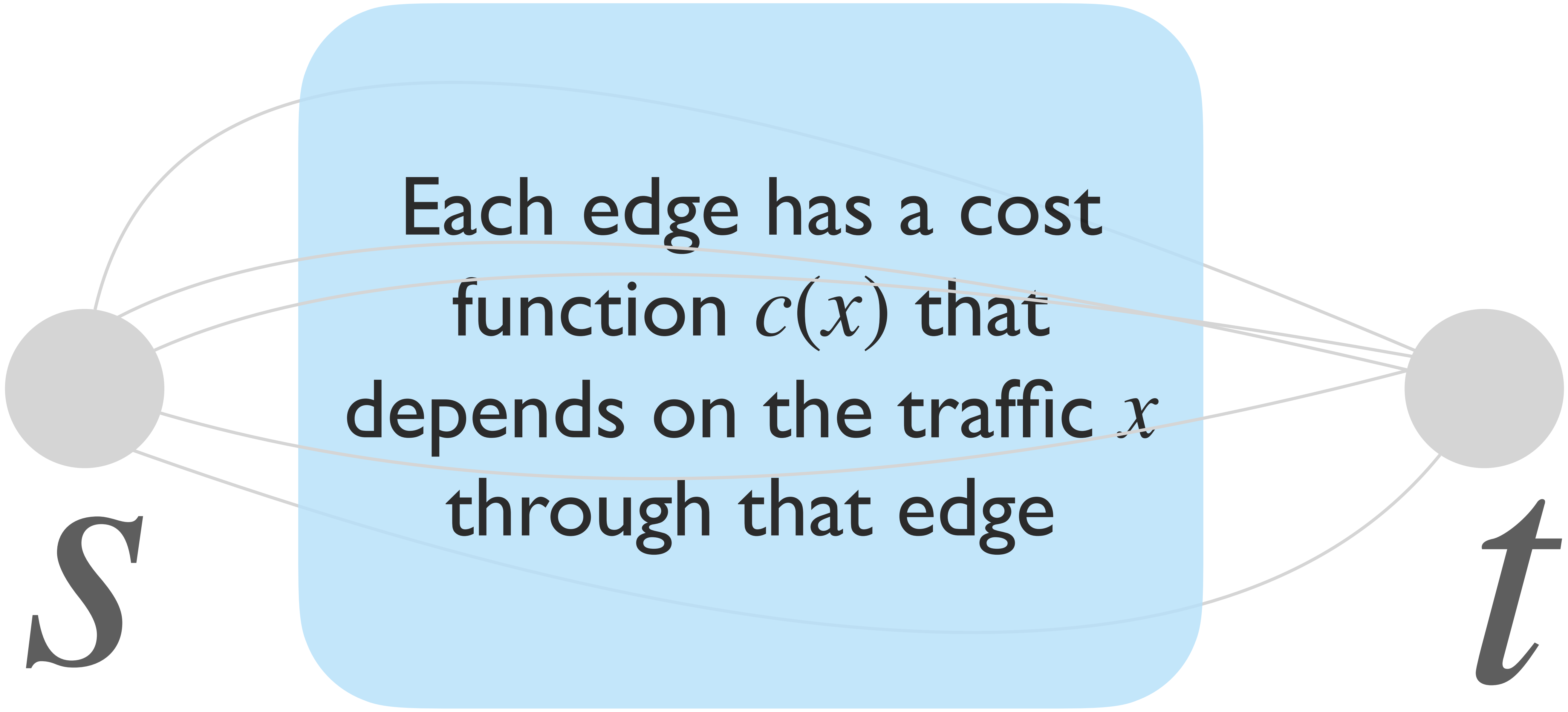
Brian Brubach is an Assistant Professor of Computer Science at Wellesley College and an Affiliate of the Institute for Mathematics and Democracy. His research focuses on algorithms and theoretical computer science with broad applications in areas such as e-commerce, algorithmic fairness, and electoral systems.

Incentives: Network Routing

- Last week we discussed incentives in P2P systems
- Today I want to talk about incentives when it comes to routing protocols in computer networks
- Two types of routing:
    - Selfish routing in local area networks
    - Inter-domain routing in the Internet

Routing Games

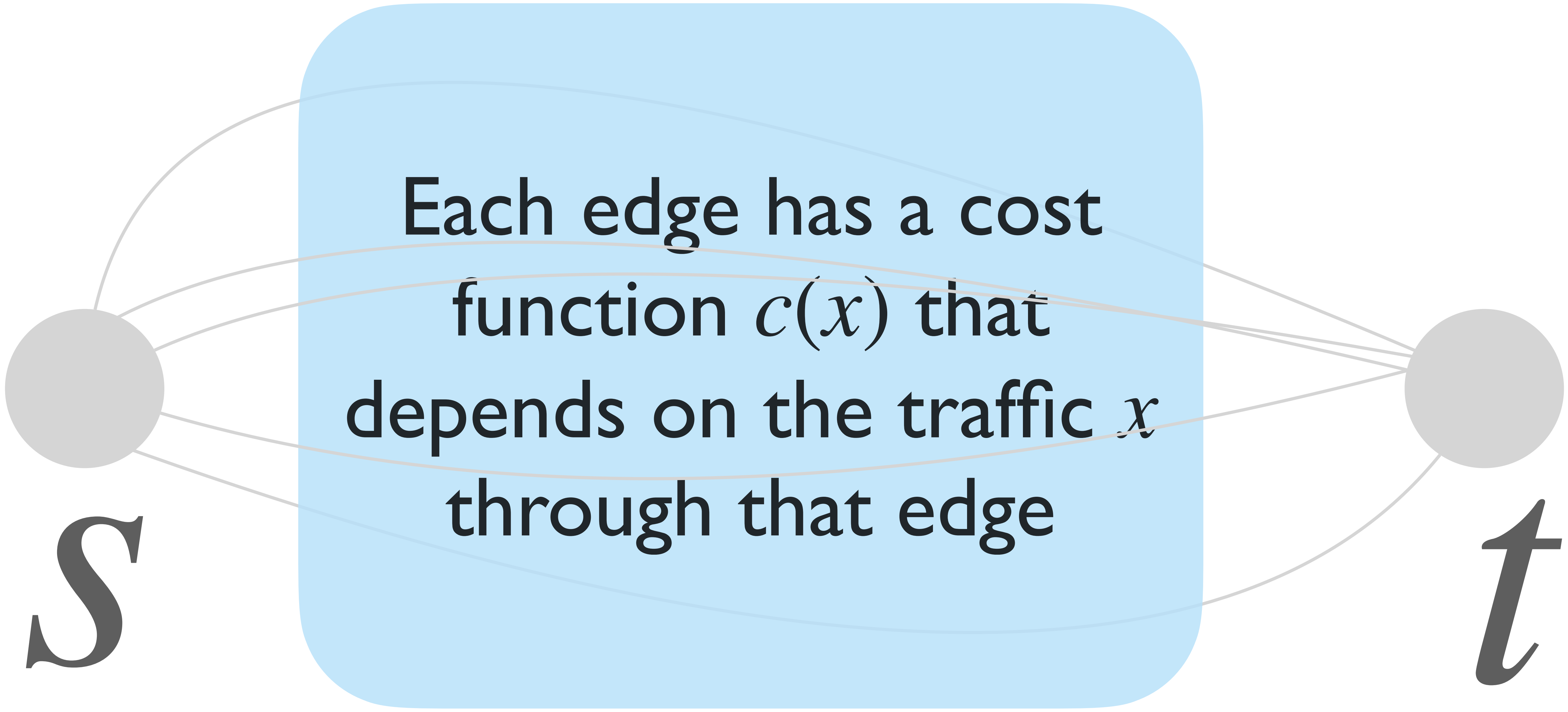
- Also called congestion games
- Simple model that captures many routing applications:
- Routing in traffic networks, routing in local-area-networks, communication networks, etc



Each edge has a cost function $c(x)$ that depends on the traffic x through that edge

Routing Games

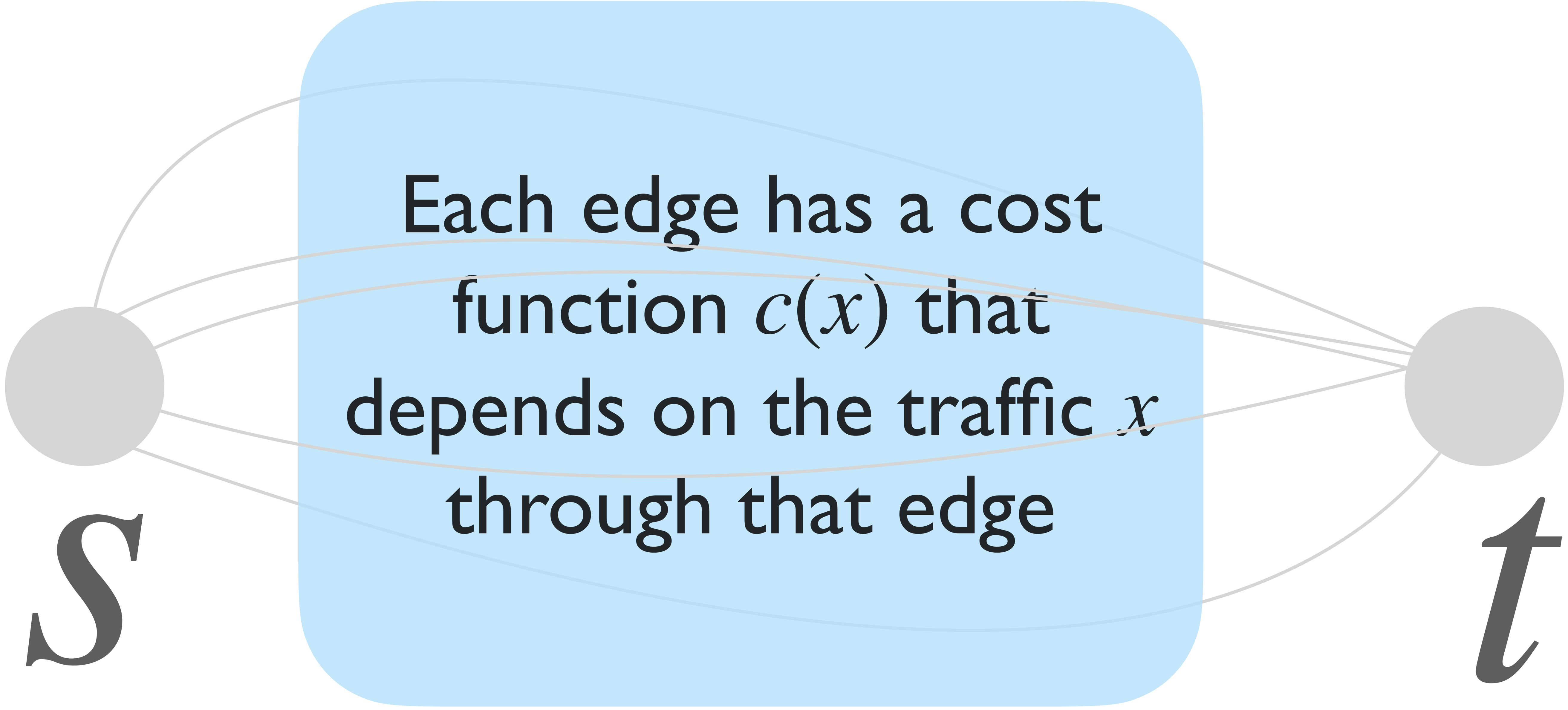
- Directed graph (edges have a direction: think of one-way streets)
- Single source s and destination t (can be generalized)
  - All traffic originates at s and is going to t
- Assume there is some fixed number of drivers n (say 100 or 1000)



Each edge has a cost function $c(x)$ that depends on the traffic x through that edge

Routing Games

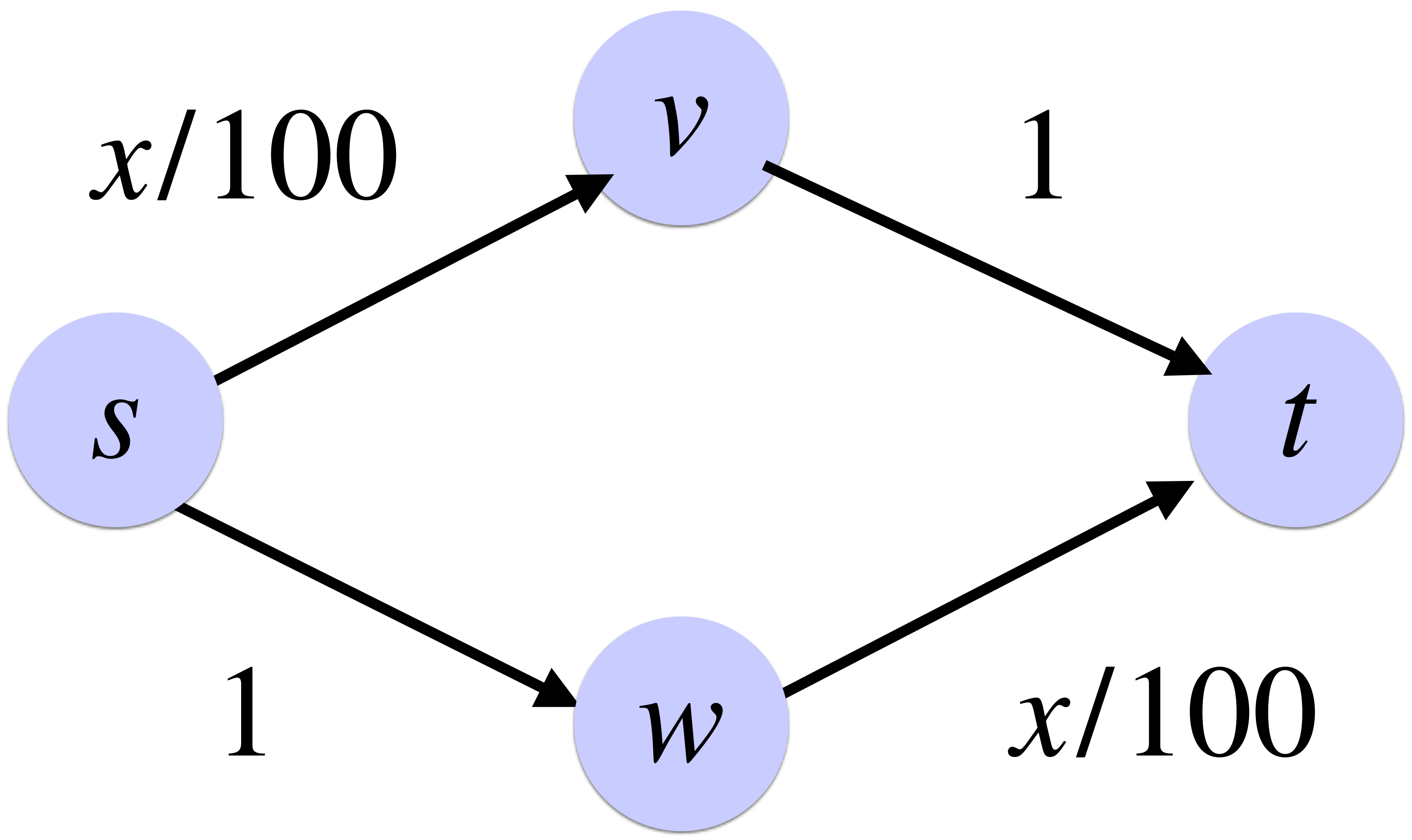
- **Driver's goal:** minimize their own commute time, defined as sum of costs of edges in their s to t path
- **Non-cooperative game:** your commute time depends on what path other drivers are choosing



Each edge has a cost function $c(x)$ that depends on the traffic x through that edge

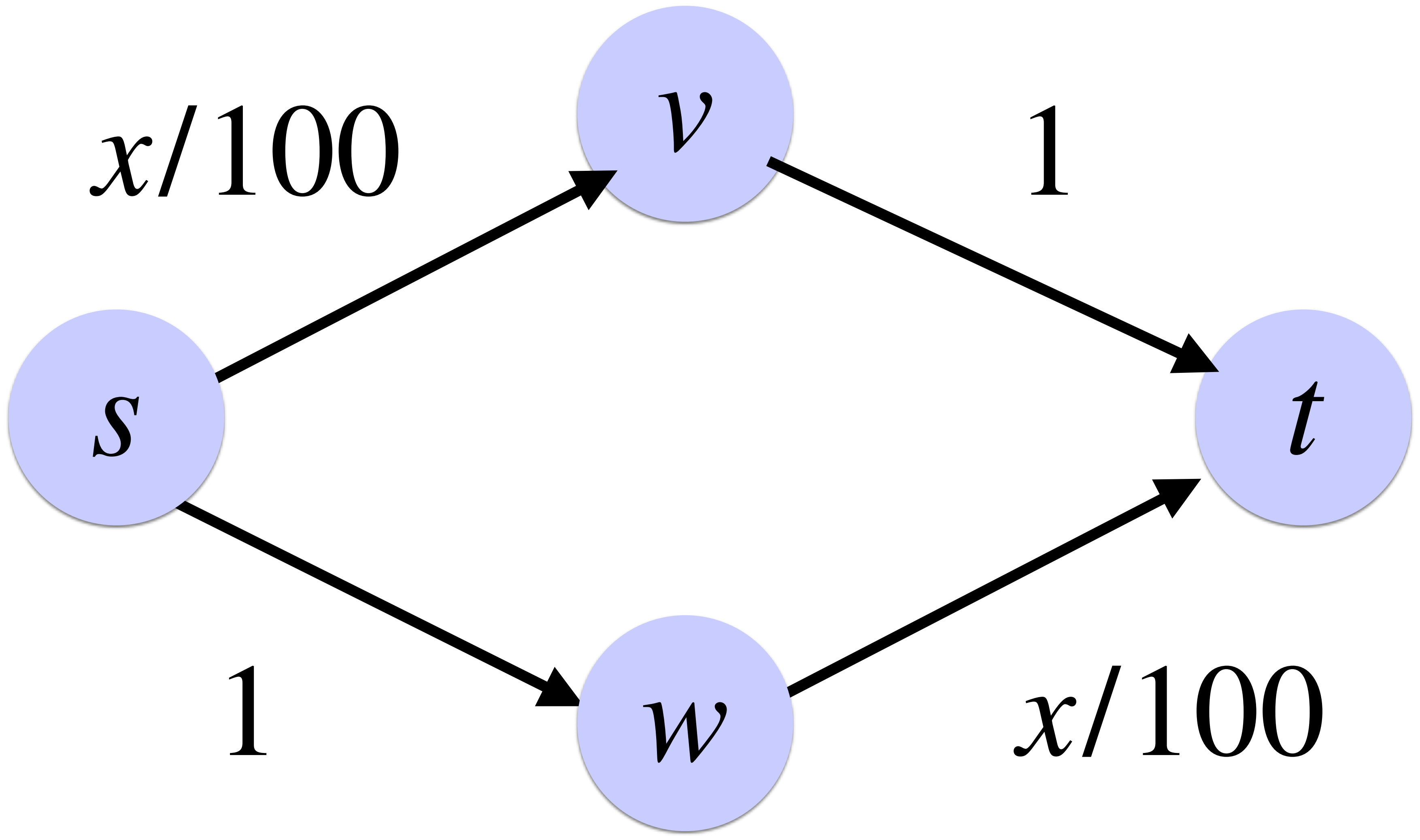
Example Network

- Suppose there are 100 drivers
- Cost function $c(x)$ on an edge which maps x (the number of players using it) to their commute cost on that edge
- Commute time on a given route (s to t): **sum of edge costs**



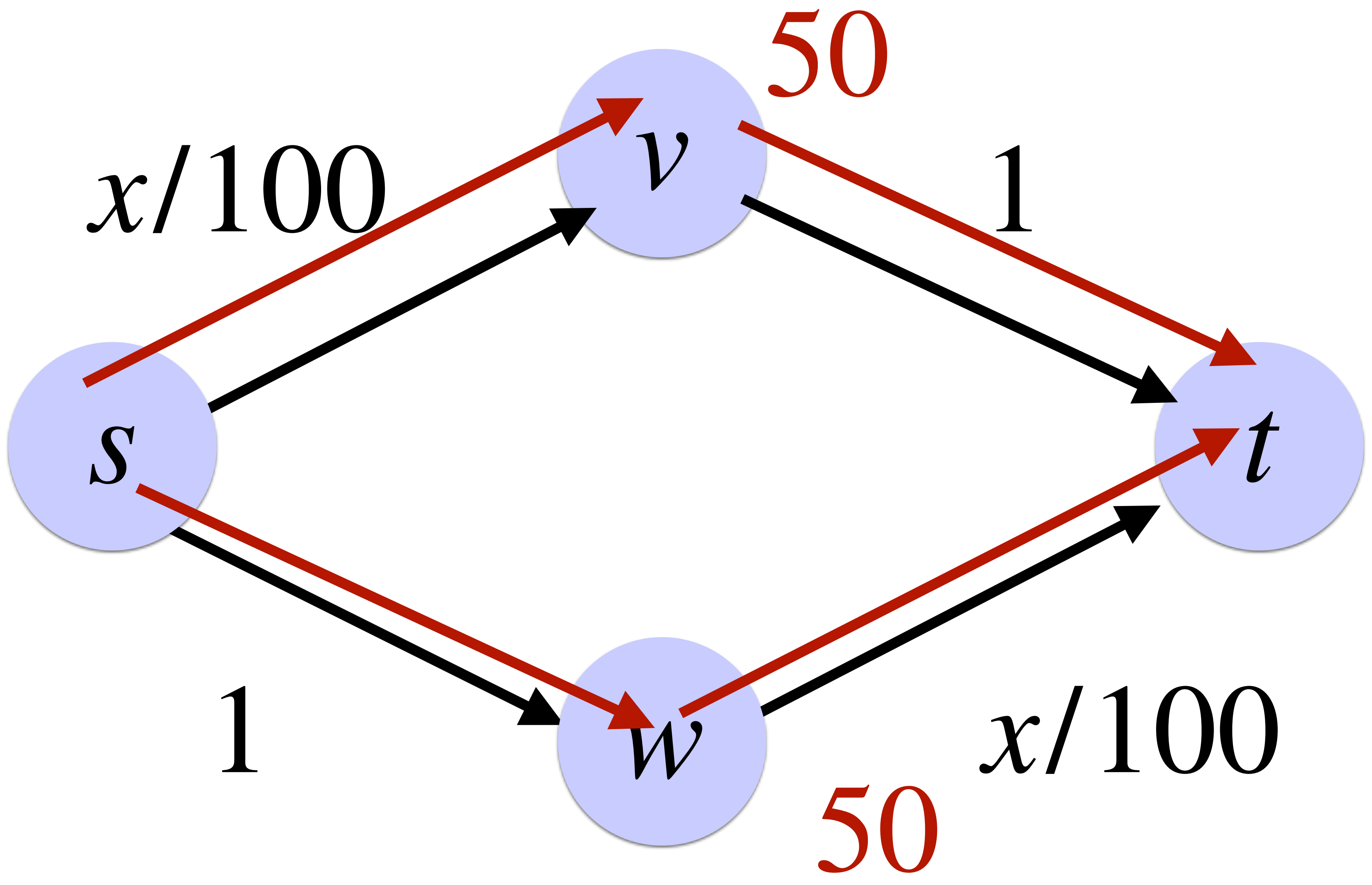
Nash Equilibrium

- At Nash equilibrium, what do we expect the state of traffic to be?
- **(Aside:** notice that in these types of graphical games, enumerating the entire payoff matrix is not reasonable: 100^2 action profiles)



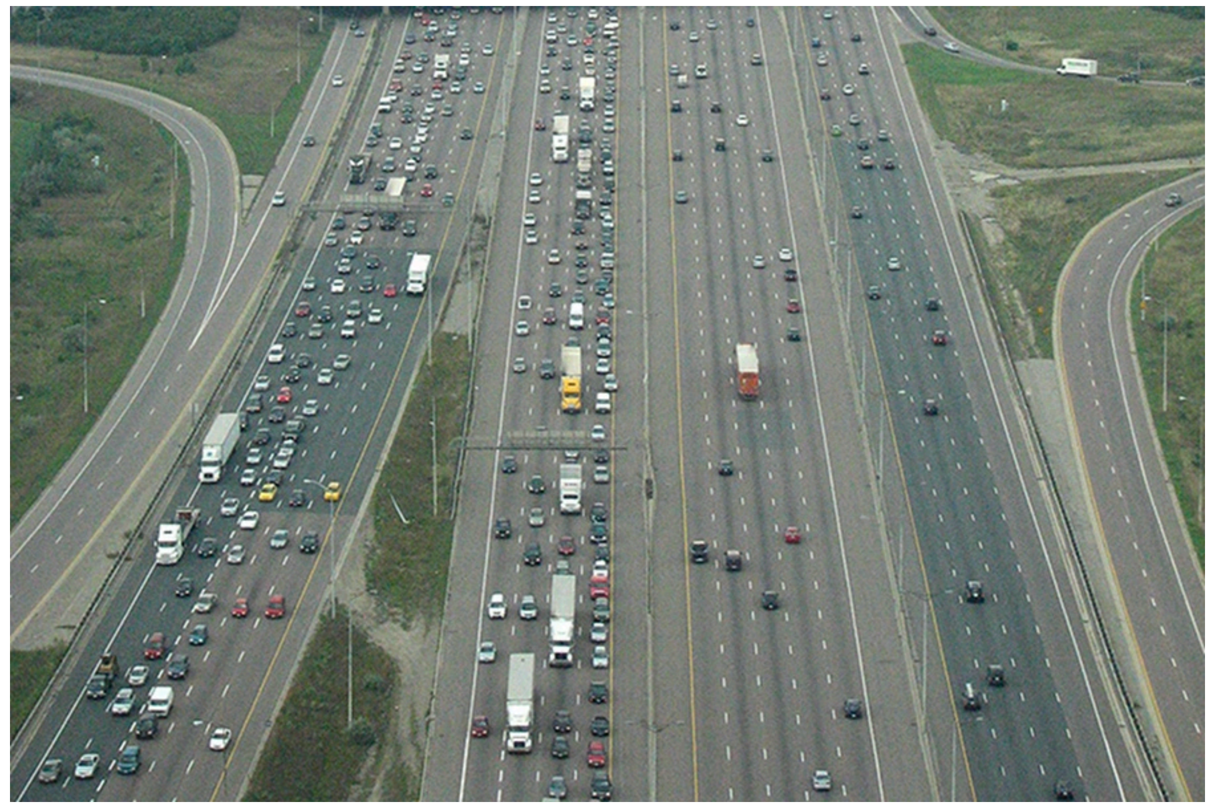
Nash Equilibrium

- At a Nash equilibrium, traffic splits $50 - 50$ across the routes
- What is the commute time of each agent?
  - $1 + 1/2 = 1.5$ (say hours)

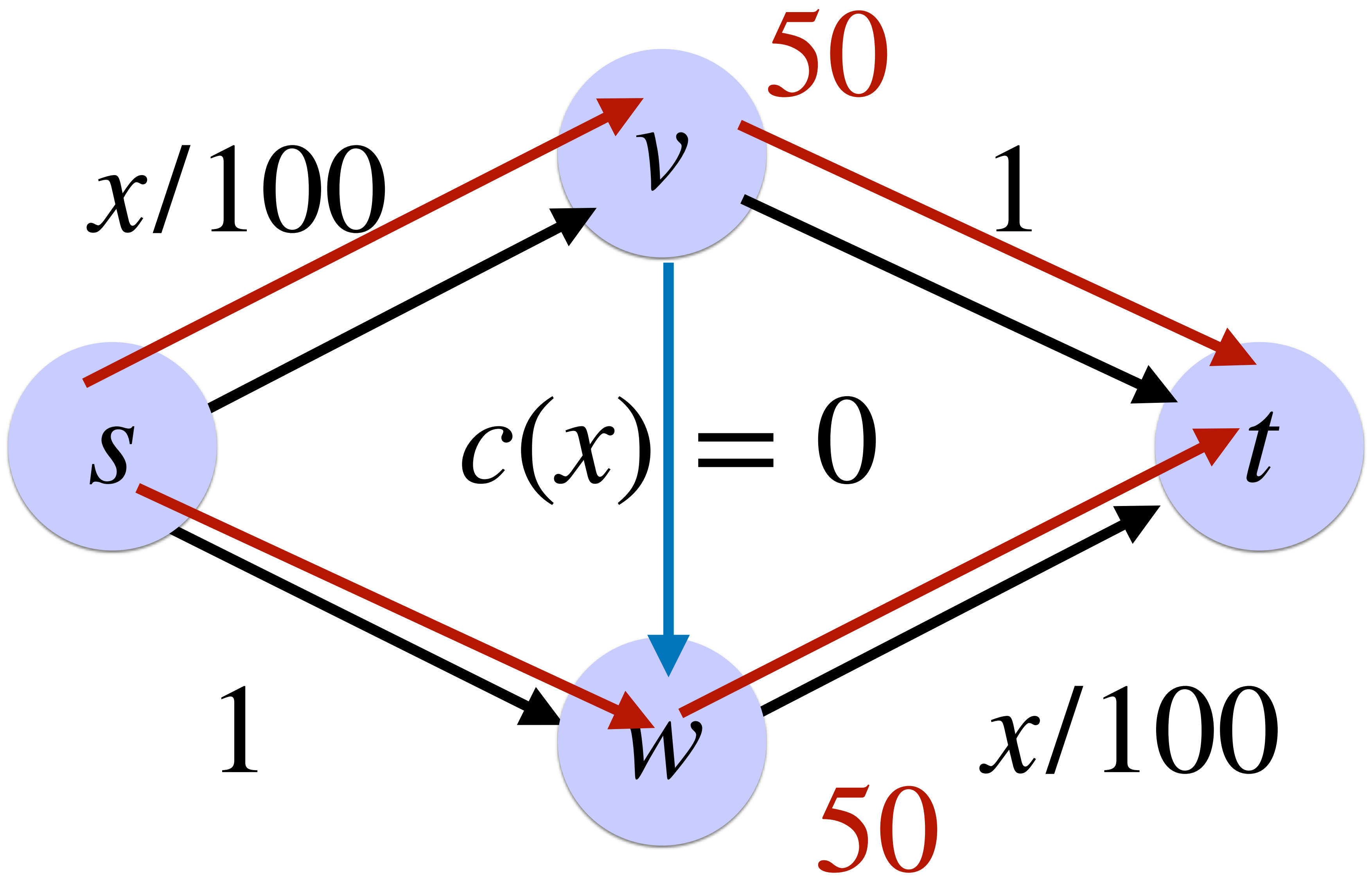


Braess Paradox

- Now suppose, to improve congestion, we introduce a "super highway" between v and w
  - Cost of this edge does not depend on traffic and is zero
  - Essentially "teleports everyone"
- How does this change effect the equilibrium flow?

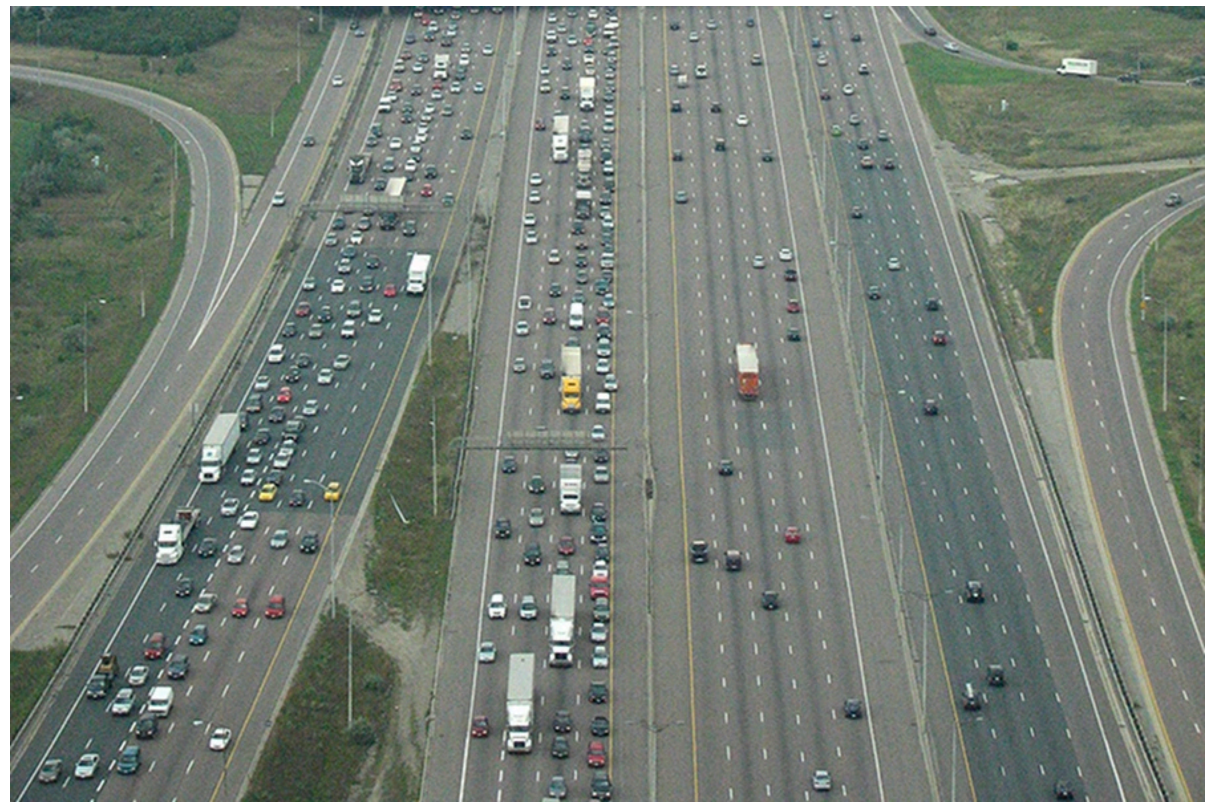


Katy free highway in Texas

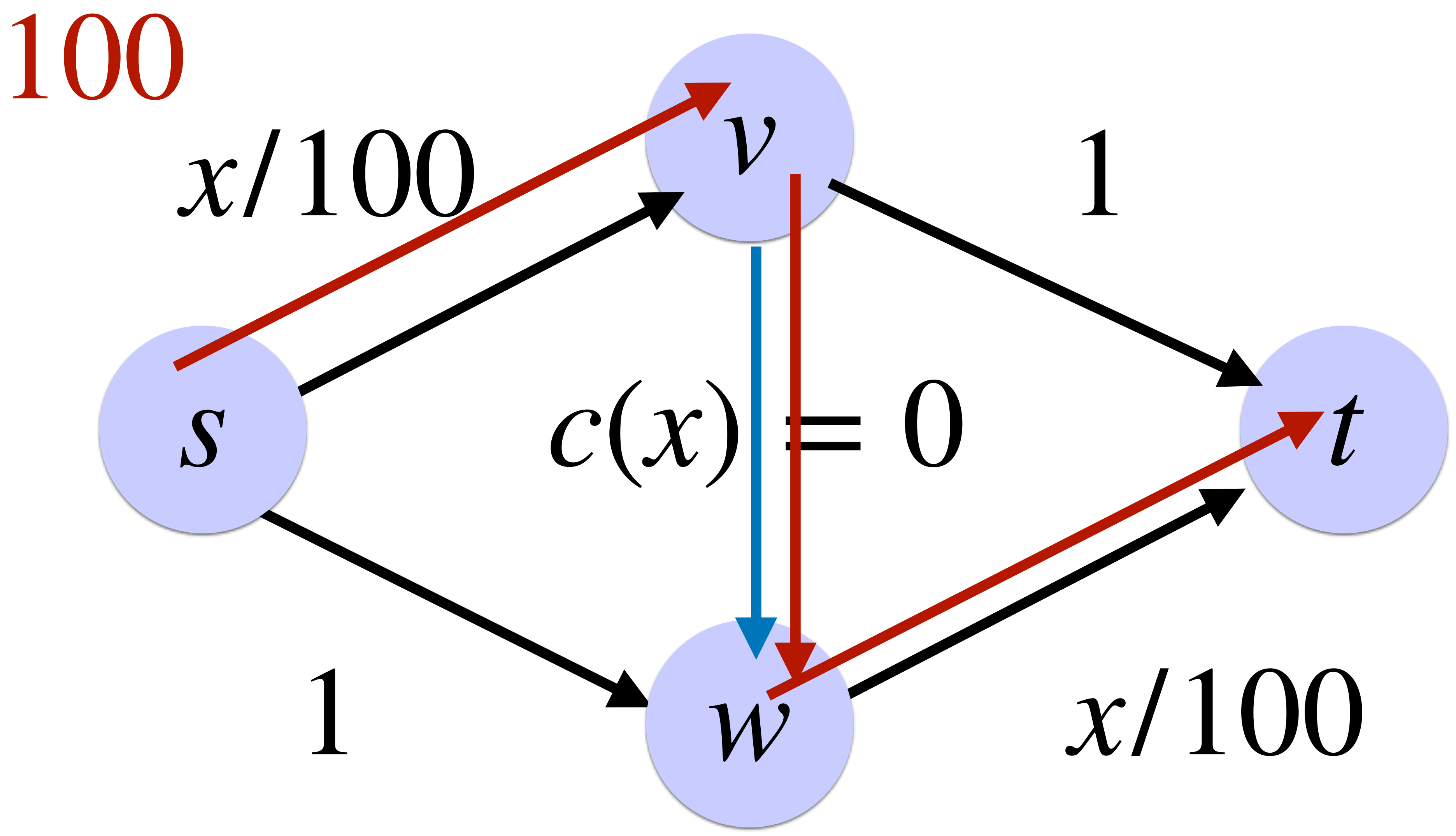


Braess Paradox

- Everyone taking s -> v -> w -> t is a Nash eq, why?
  - Can anyone gain by deviating unilaterally?
- What is the commute time now?
  - 2 hours (compared to 1.5 before)

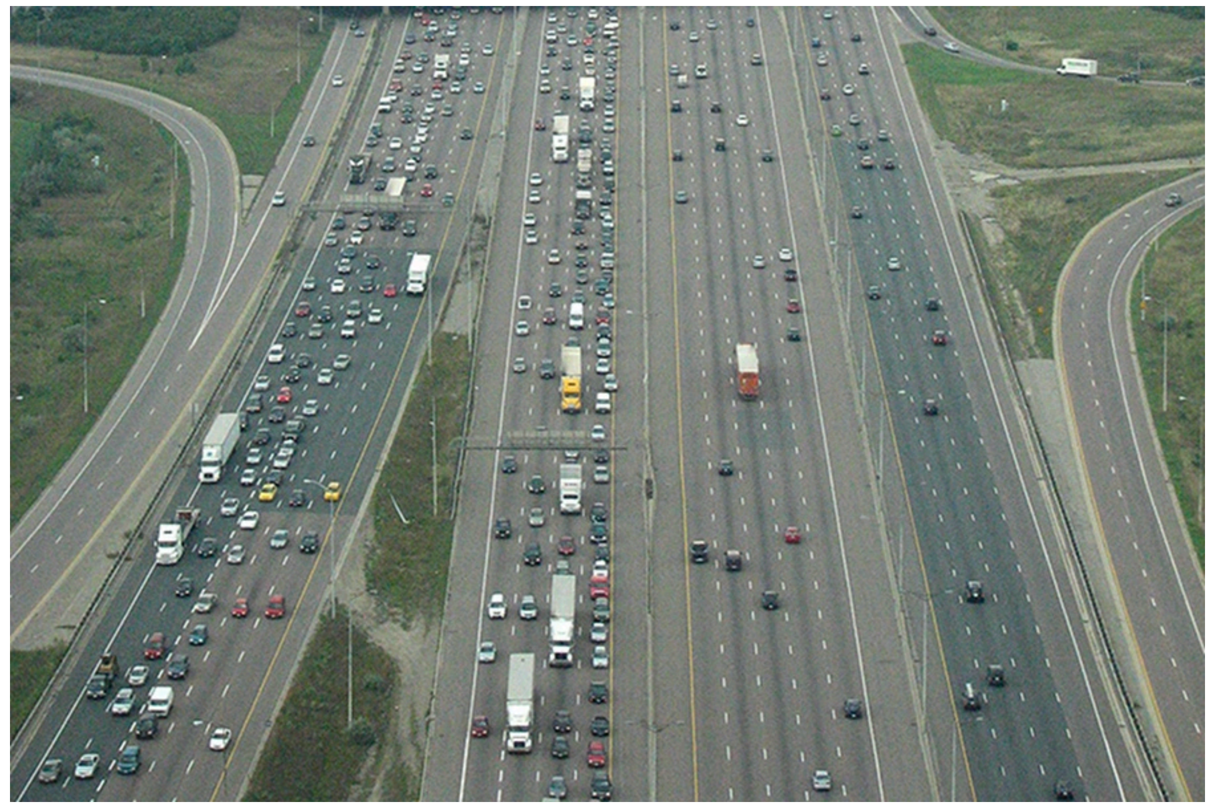


Katy free highway in Texas

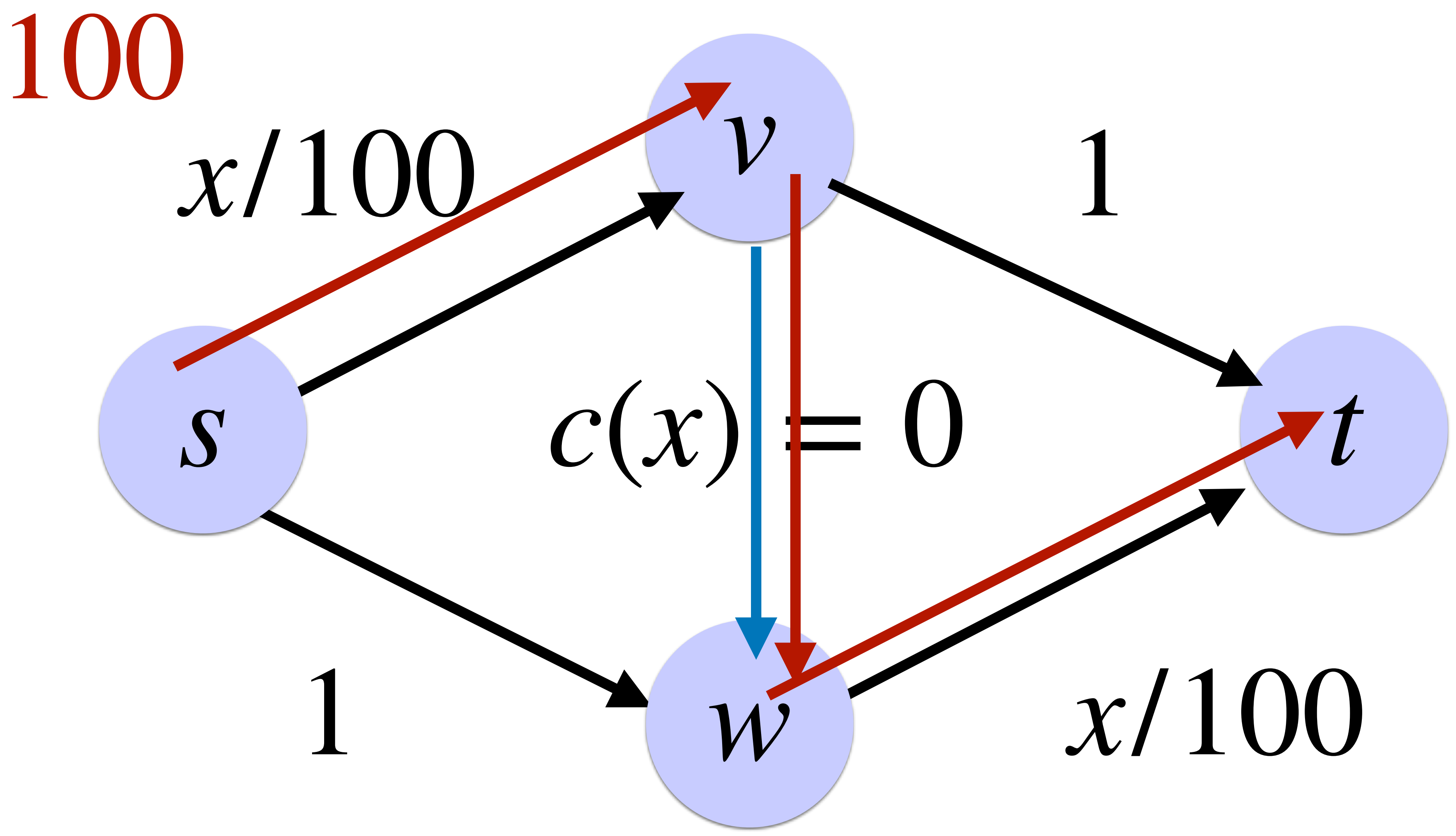


Braess Paradox

- Adding a super-highway made things much worse!
- Is this a phenomenon we experience in our lives?

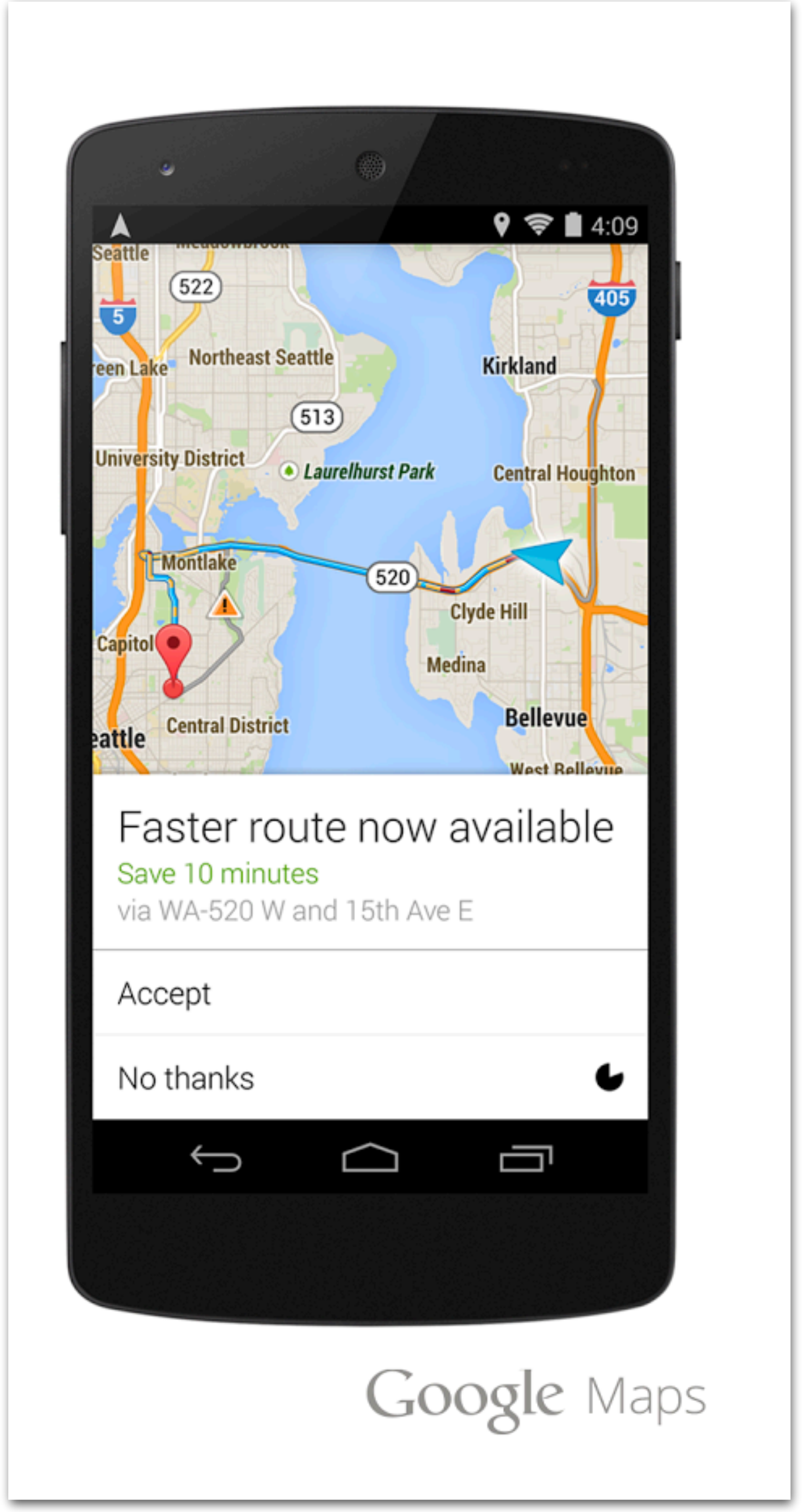


Katy free highway in Texas



Braess Paradox in Practice

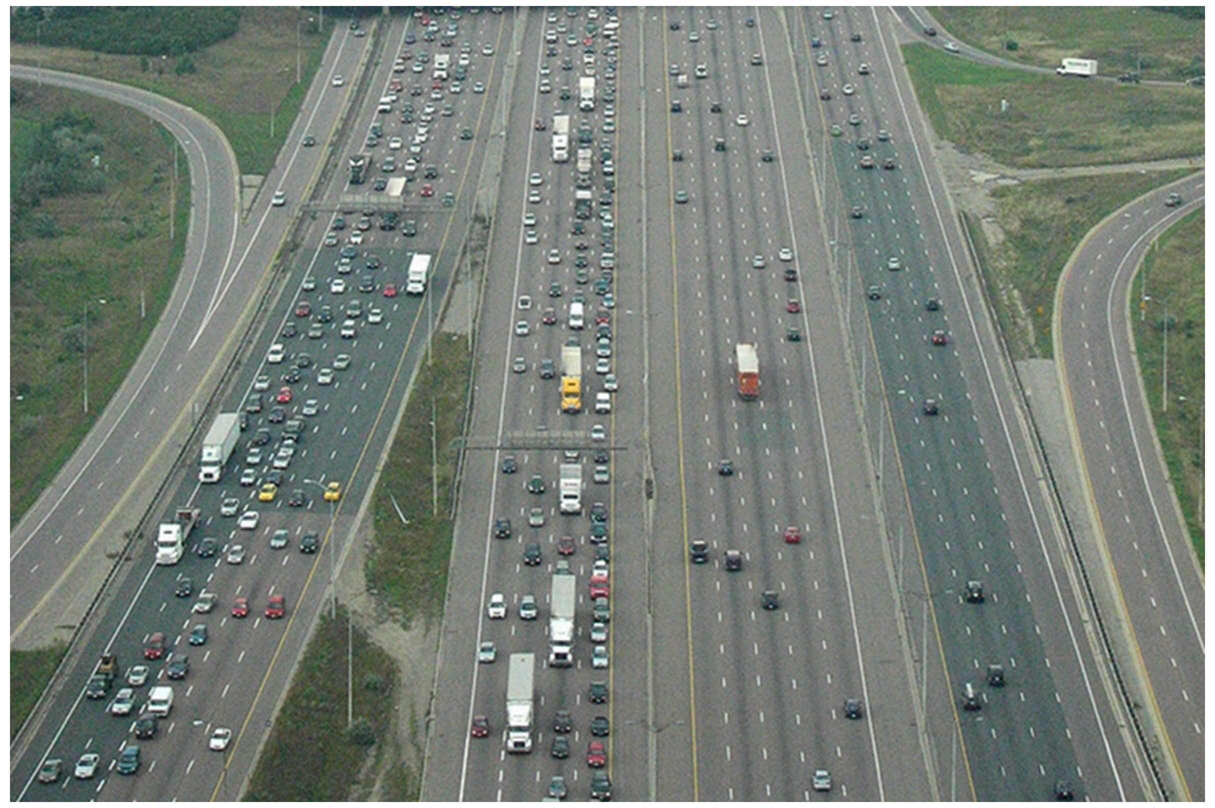
- Adding a super-highway made things much worse!
- Is this a phenomenon we experience in our lives?
- Google updates best route due to congestion
    - What if all drivers change that switch?



Faster route now available
Save 10 minutes
via WA-520 W and 15th Ave E

Accept

No thanks



Katy free highway in Texas

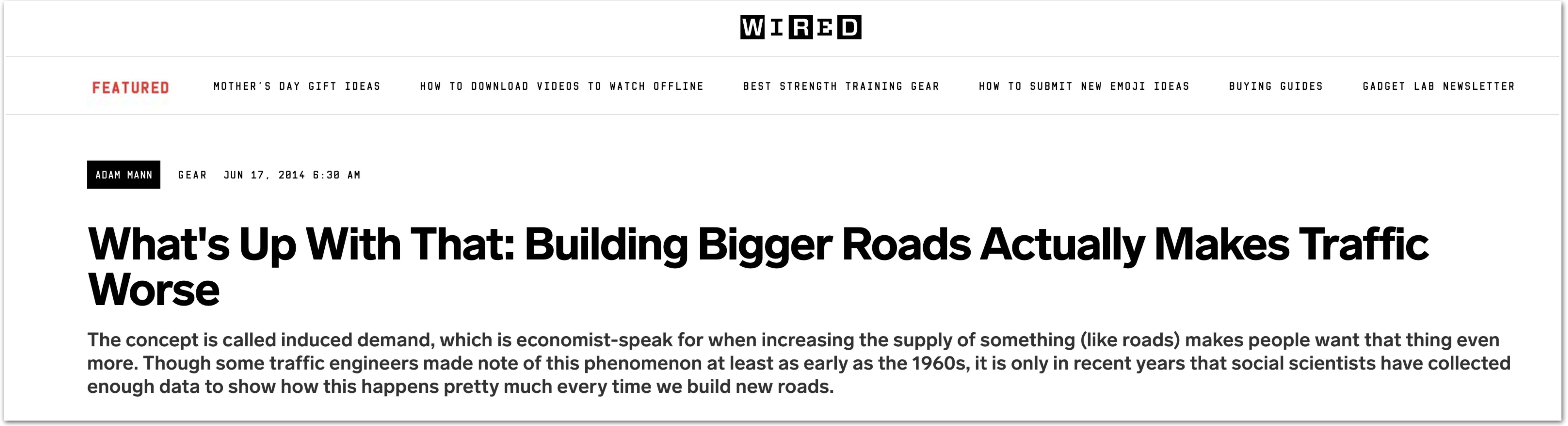
Braess Paradox in Practice

- In Seoul, the mayor undertook a massive revitalization project
  - Demolished a six-lane highway over the Cheonggyecheon river
  - Turned it into a recreation space
- Initially unpopular decision
- Since then has significantly improved traffic congestion

https://wwf.panda.org/wwf_news/?204454/Seoul-Cheonggyecheon-river

Braess Paradox in Practice

- In 2009, NYC experimented with road closures in 2009 to reduce congestion
- Closed off Broadway/Times Sq and Herald Sq
- Overall congestion improved
- Experiment considered to be a success and the road closures were made permanent



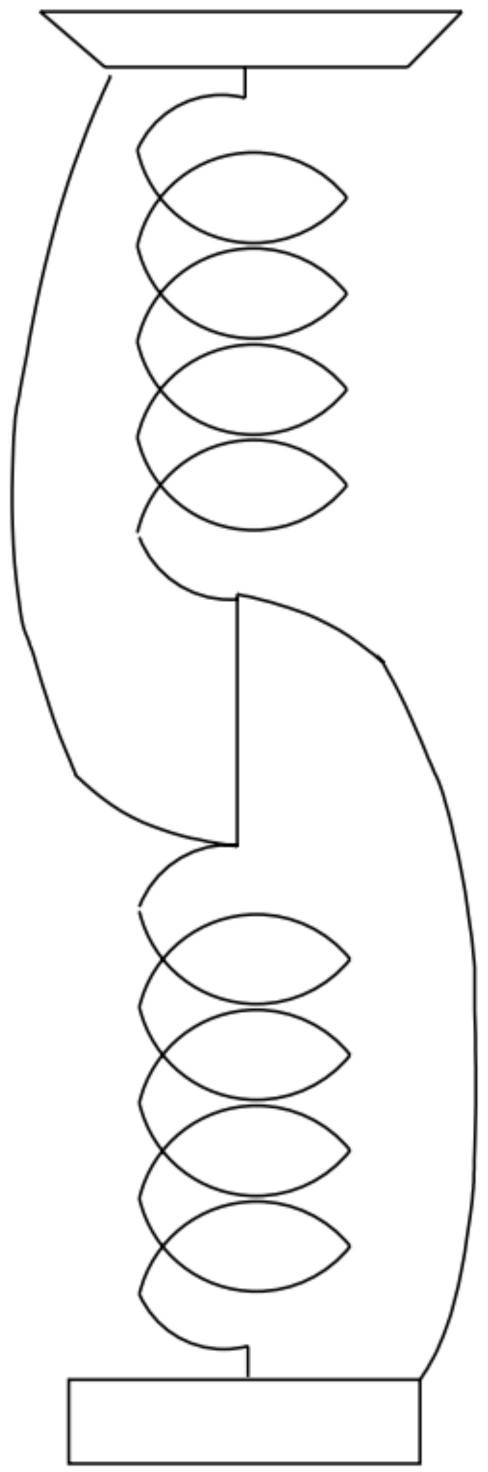
ADAM MANN GEAR JUN 17, 2014 6:30 AM

What's Up With That: Building Bigger Roads Actually Makes Traffic Worse

The concept is called induced demand, which is economist-speak for when increasing the supply of something (like roads) makes people want that thing even more. Though some traffic engineers made note of this phenomenon at least as early as the 1960s, it is only in recent years that social scientists have collected enough data to show how this happens pretty much every time we build new roads.

Braess Paradox: Strings & Springs

- Not only a traffic phenomenon: strings and springs
- https://youtu.be/cALezV_Fwi0?t=415



(a) Before

Takeaways

- Braess's Paradox is observed in any system that can be modeled as a network
  - Water systems, electric systems, any flow network
- Recurring theme: selfish behavior does not always lead to globally efficient outcomes
  - Seen this in Prisoner’s dilemma
- **Question:** “how bad is selfish behavior?”
- Quantify the loss in welfare caused by letting the game play out in the wild, rather than centrally controlling it

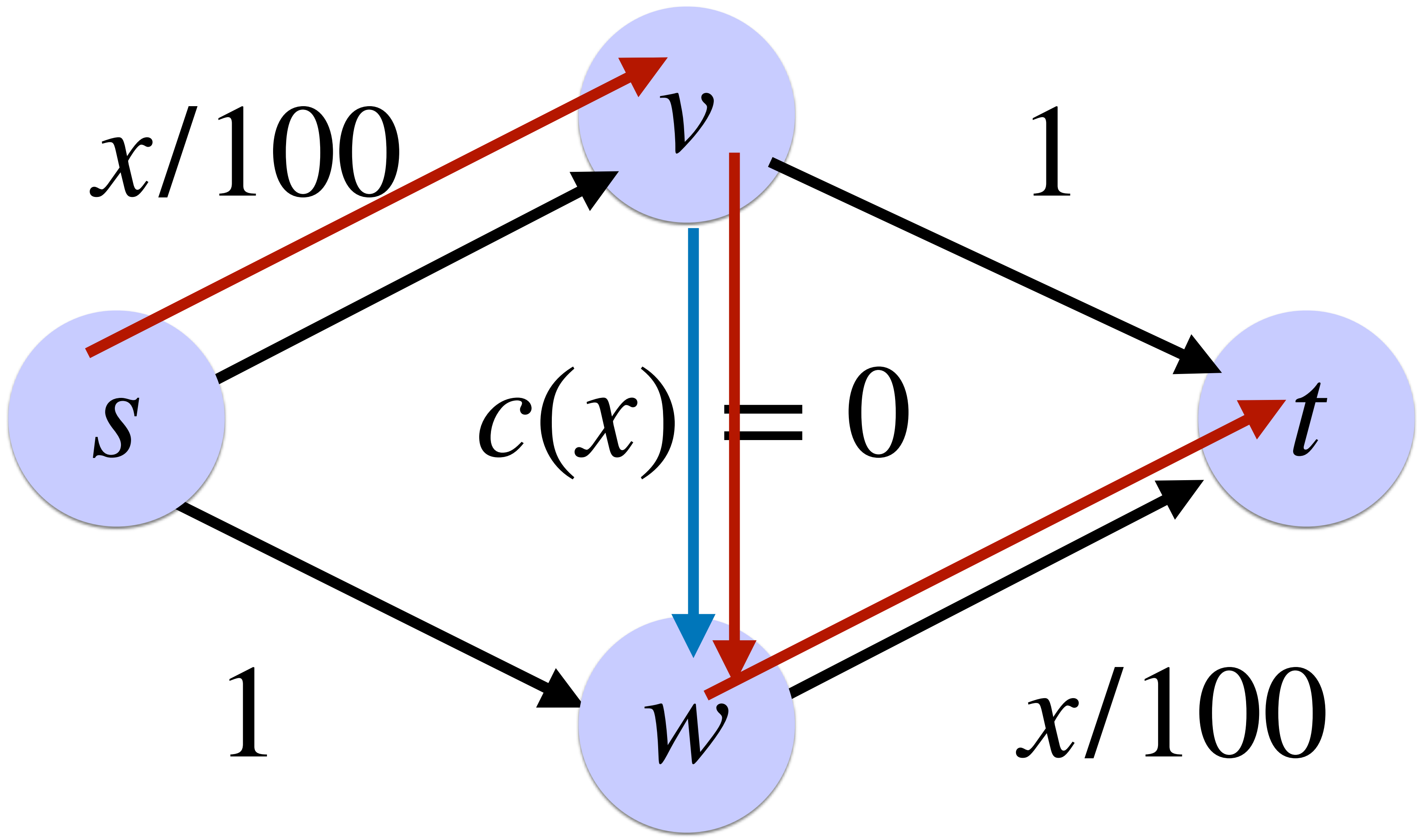
Price of Anarchy

- Concept that measures how the social welfare of a system degrades due to selfish behavior of its agents
- Captures how well equilibria approximates social welfare
- CS driven area in AGT: Introduced and studied primarily by computer scientists
- Does the PoA definition remind you of something from 256?

$$\text{PoA} = \frac{\text{Opt SW}}{\text{SW at (Worst) Eqm}} \qquad \text{PoA} = \frac{\text{SC at (Worst) Eqm}}{\text{Opt SC}}$$

PoA is not too Bad

- Turns out, pure Nash eq always exists in routing networks
- In Braess Paradox, equilibrium commute time is 2
  - Optimal commute time is at least as good as splitting traffic 50-50: $\geq 3/2$
- PoA $\leq 4/3$
- **Theorem**. (Roughgarden & Tardos) PoA of any selfish routing network with linear costs $c(x) = ax + b$) is at most $4/3$.
  - Regardless of the network topology!
  - Linear cost function:
- We will show a weaker bound of 2 today



Best Response Dynamics

- We will show that a pure Nash equilibrium always exists through a "best response dynamics" process which eventually reaches equilibrium
  - Start with a state: if it is not an equilibrium then there exists a player who is not playing their best response
  - Keep updating actions until equilibrium is reached (if ever)
- Best-response dynamics ends in an equilibrium, how do we know it halts?
  - Potential function argument: system starts with some potential energy
  - If at every step this energy moronically reduces: the process must halt when it "runs out"

Potential Function

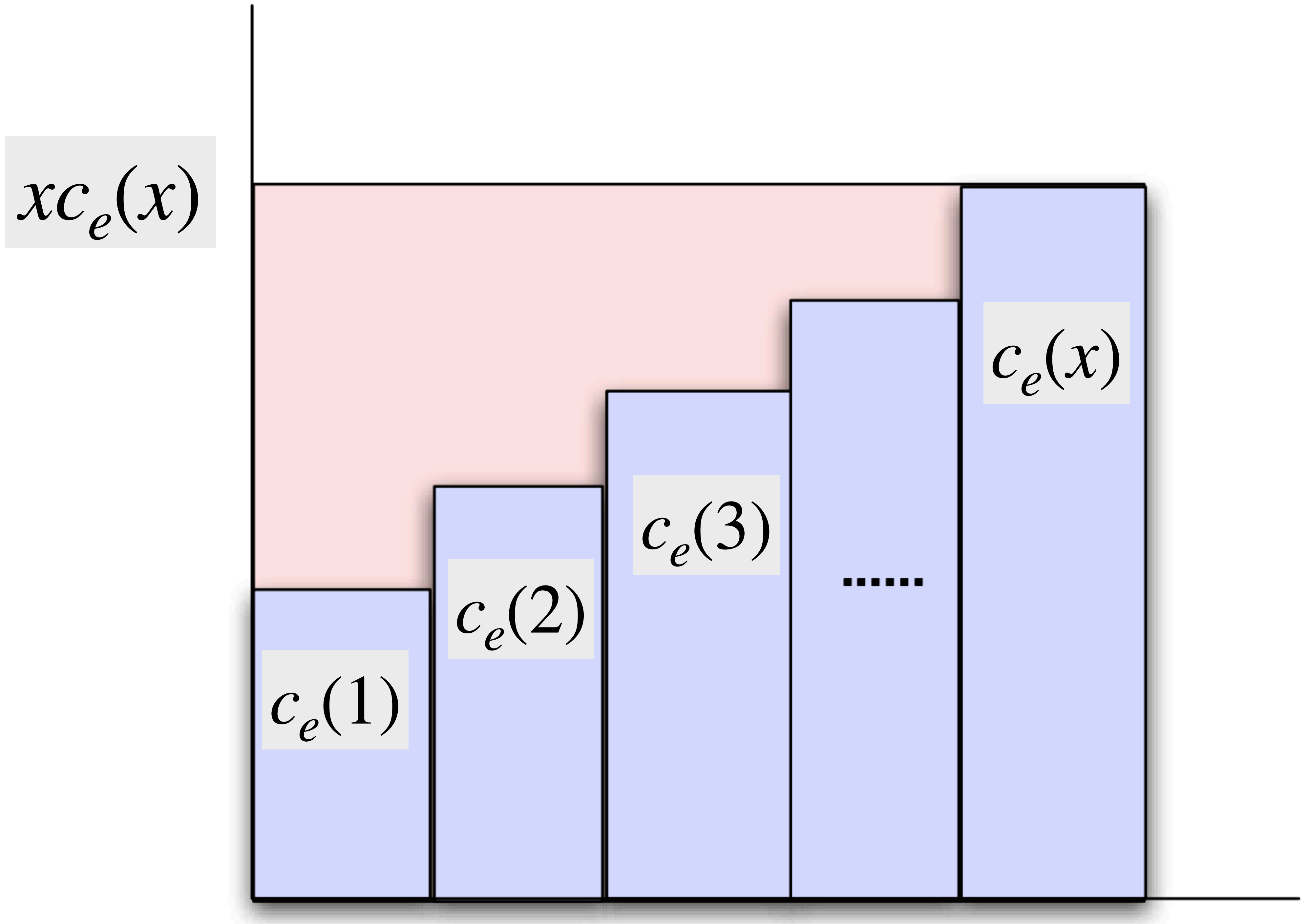
- Let edge e have x units of traffic on it and cost function $c_e(x)$
- Define energy of an edge as: $P(e) = c_e(1) + c_e(2) + \cdots + c_e(x)$
  - Notice this is always positive
- Total potential energy $P = \sum_e P(e)$
- If the current traffic pattern is not an equilibrium
  - Someone can improve their utility by unilaterally changing their path
  - Show that this causes the potential energy to decrease
  - Since $P \geq 0$ this process must eventually come to an end exactly when the system is at equilibrium

Best Response Dynamics

- Suppose a player changes its path: stops using some edges e and starts using edges e' (all else fixed)
- How does it change the energy of edges it is no longer using
  - Difference: $c_e(x)$
- How does it change the energy of edges it is now using
  - Similarly, the difference is $c_{e'}(x+1)$
- Overall change: $\sum_{e'} c_{e'}(x+1) - \sum_{e} c_e(x)$, is this negative?
- Exactly the change in travel time of player: must go down
- Thus, eventually this process terminates in a pure Nash equilibrium

Price of Anarchy

- Compare the social cost of a pure Nash equilibrium to the optimal cost
  - Social cost = total travel time for all drivers
- If x drivers are traveling on edge with cost $c_e(x)$, their total travel time?
  - $T(e) = xc_e(x)$
- Energy of an edge versus total travel time?
  - $\frac{1}{2}T(e) \le P(e) \le T(e)$
- SocialCost of any traffic pattern is $\sum_e T(e)$
- SocialCost$(Z)/2 \le P \le$ SocialCost(Z)



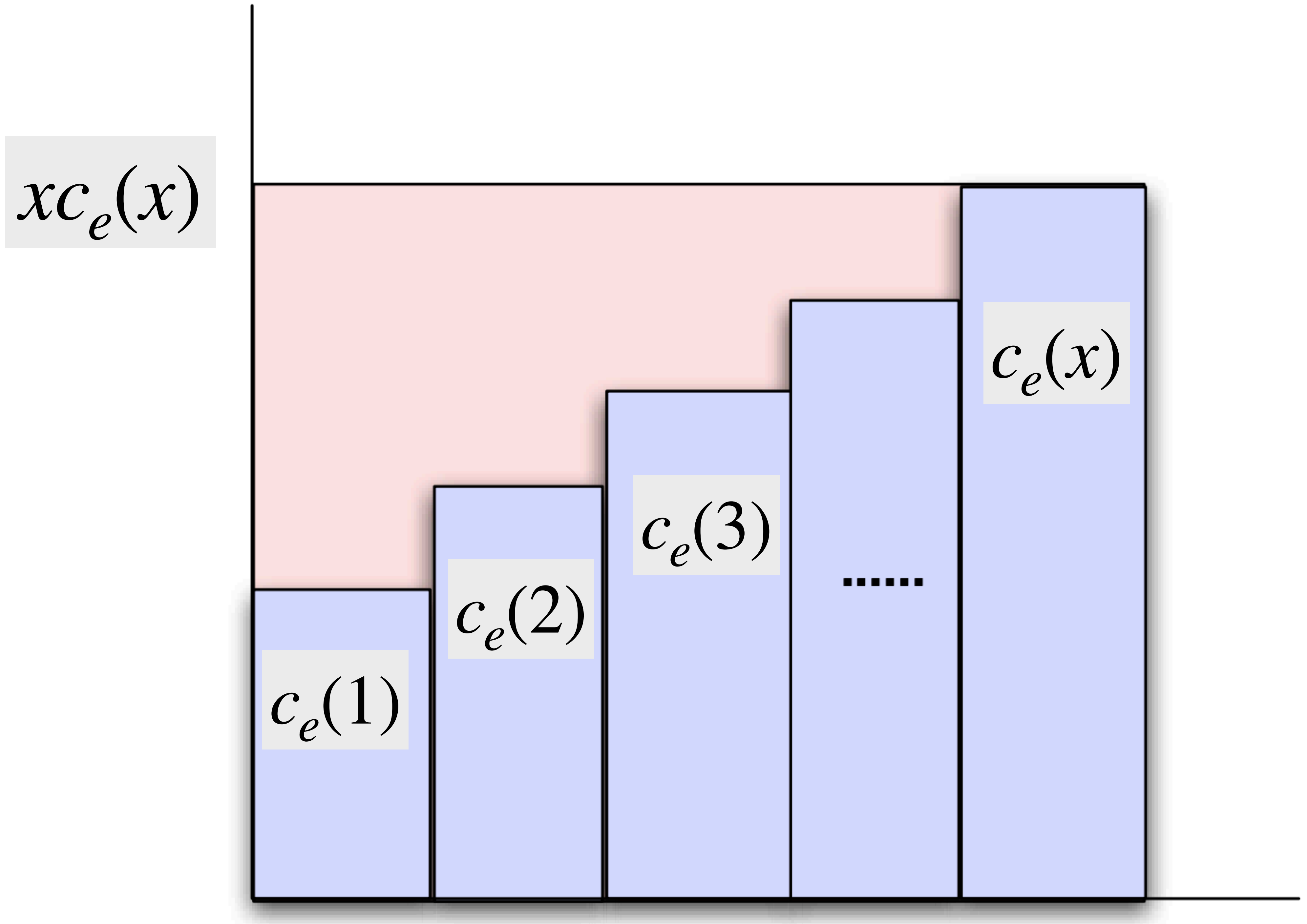
Price of Anarchy

- Let Z^* be the optimal traffic pattern that we start with
- Since potential energy only goes down when reaching an equilibrium Z
- $P(Z) \leq P(Z^*)$
- SocialCost(Z)

$$\leq 2P(Z) \;\; \leq 2P(Z^*)$$

≤ 2 SocialCost(Z^*)

- PoA is at most 2



Applications of Selfish Routing

- Understanding network over-provisioning:
    - Relatively easy and cheap to overprovision computer networks: provide additional capacity than what is needed
    - This means the network will not be fully utilized
    - Empirically observation: networks perform better (fewer delays and packet drops) when they have extra capacity
    - Theory of PoA and selfish routing has been used to corroborate and explain why overprovisioned networks tend to perform better

Liquid Democracy