Spring 2022

CSCI 357: Algorithmic Game Theory

Lecture 1B: Game Theory I

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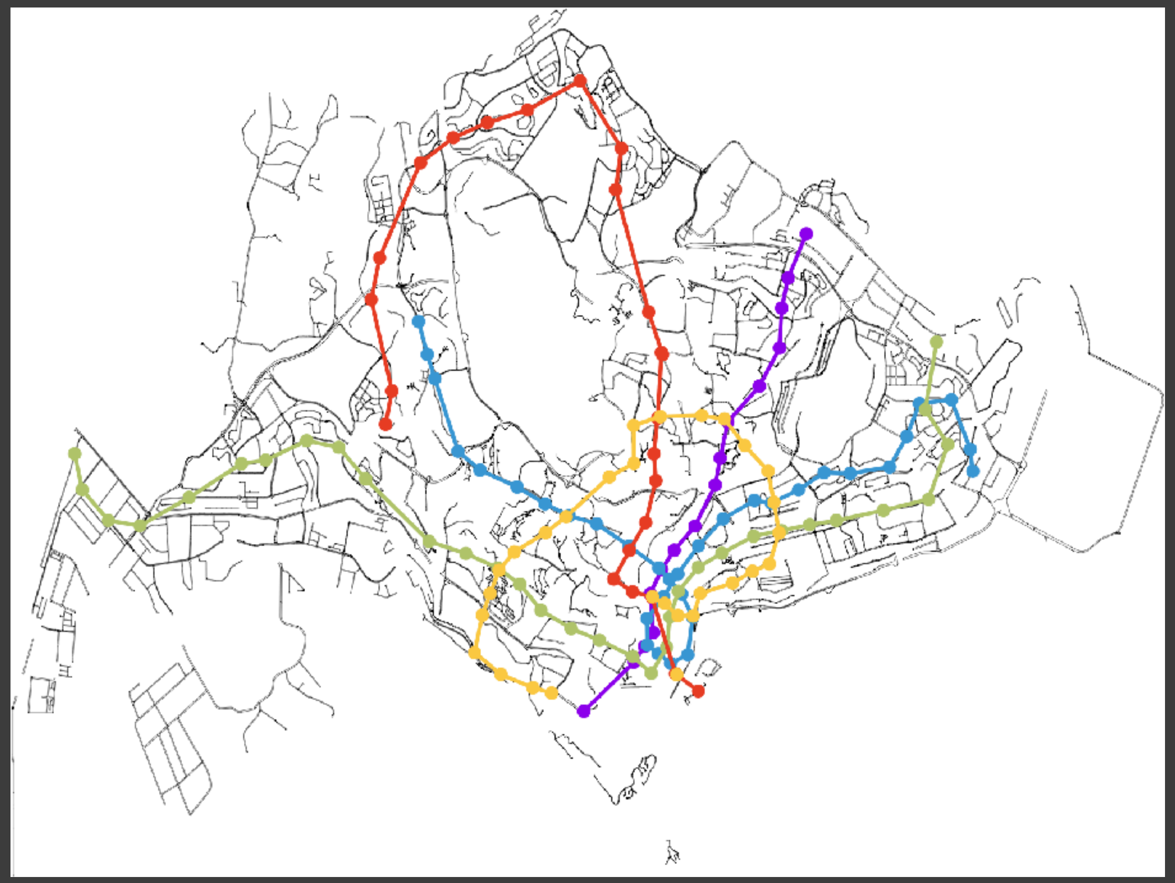
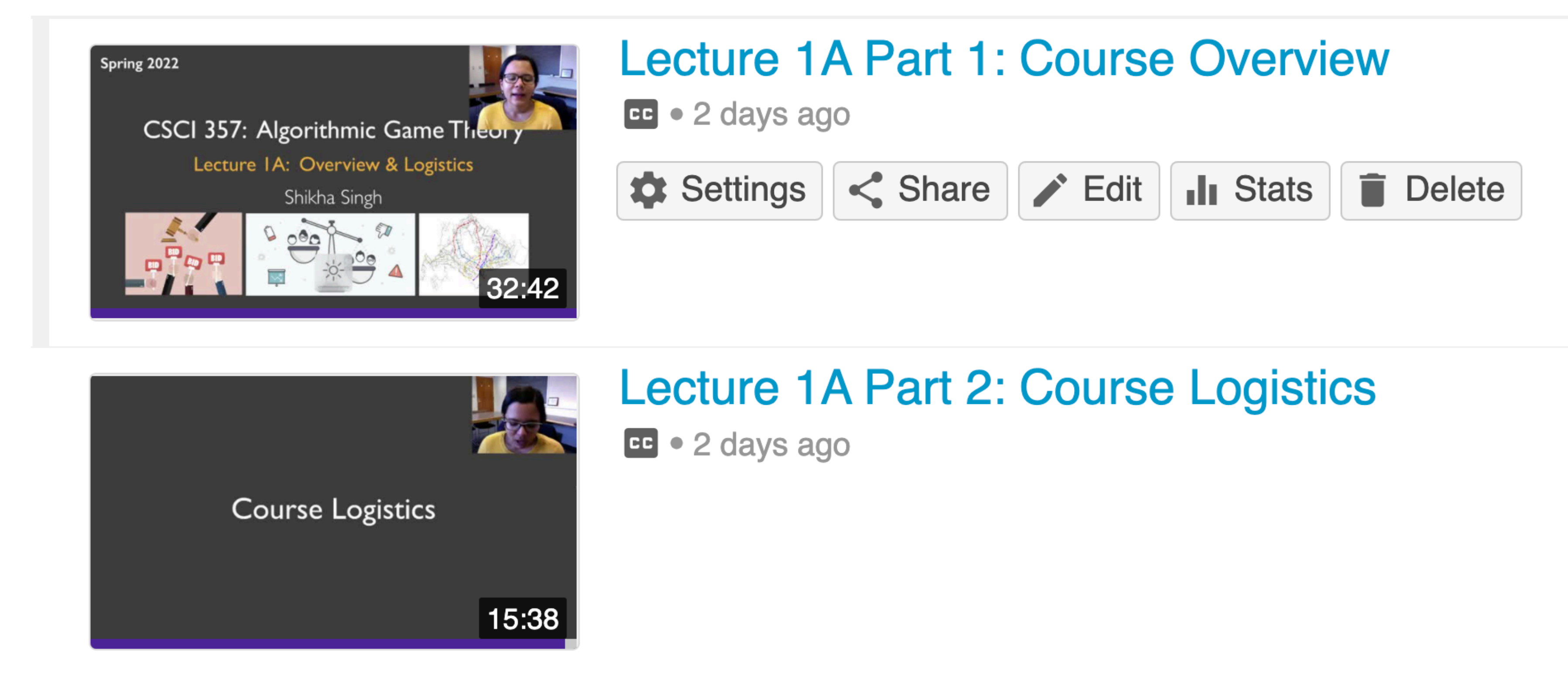


Quick Recap

- Course website: https://williams-cs.github.io/cs357-s22-www/
- Course overview, syllabus, logistics and policies in **two recordings** on GLOW
- Problem sets: typeset in LaTeX, submit on Gradescope **(code: KYERN3)**
- **Assignment 0** on GLOW (due Friday):
  - Join Slack and post an introduction in #general
  - Fill out course survey
  - Sign up for a short Zoom chat with me
- Assignments will typically be due 10 pm Thursdays



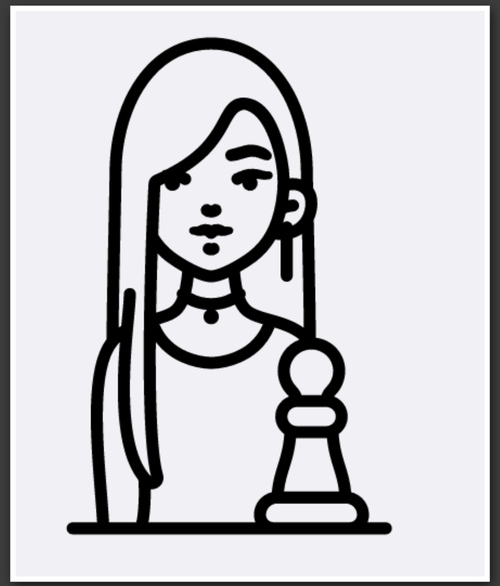
Any Questions about course overview and logistics?

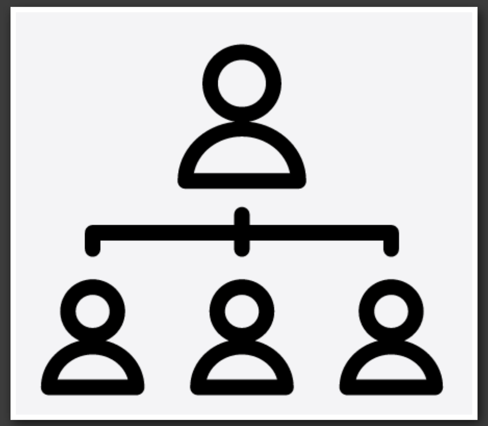
Classroom discussion:

Examples of procedures/algorithms in your life where the rules do not necessarily lead to desirable behavior or have unintended consequences? Or on the flip side: examples of well-incentivized algorithms?

Defining a Game

- **Players**: the decision makers
  - People, governments, companies
- **Actions**: what can the players do
  - Enter a bid in an auction
  - Decide when to sell stock
  - Decide who to vote for
- **Outcome**
- **Payoffs/Utility** of each outcome to players
  - Represented a number **(cardinal)**
  - Or ordering over outcomes **(ordinal)**






GAME THEORY

Towards a Game Represenation

- To start, consider the simplest games
- Simultaneous move, single-action games
    - Eg. Rock, paper, scissors
- How many players?
- What are the actions available to players?
- What are the outcomes?
- What are the payoffs to players of the outcomes?
    - How can we represent this?

GAME THEORY

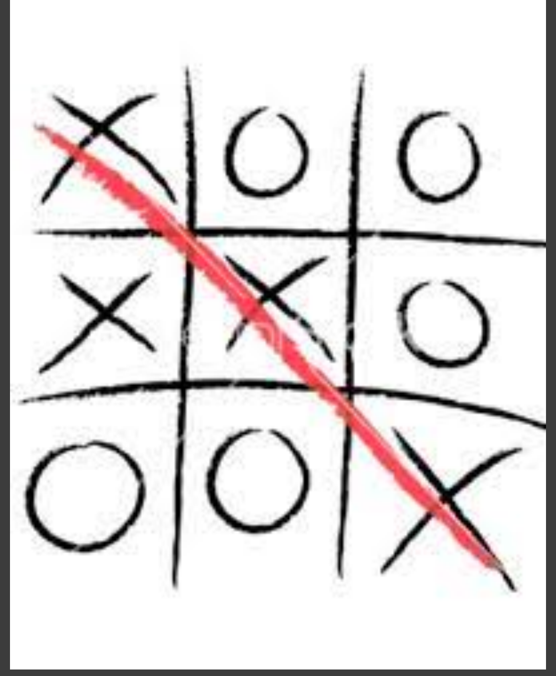
Normal-Form Representation

- **Normal form**/ Matrix Form/ Strategic Form:
  - List payoffs of players as a function of their actions
  - Assume players move simultaneously
- **Conventions**:
  - Row player is usually player 1
  - Column player is player 2
  - Payoffs for each outcome are written in each cell as a tuple, where first is player 1's payoff, then player 2

	Rock	Paper	Scissors
Rock	$0, 0$	$-1, 1$	$1, -1$
Paper	$1, -1$	$0, 0$	$-1, 1$
Scissors	$-1, 1$	$1, -1$	$0, 0$

Extensive-Form Games

- **Extensive-form (Sequential form)**: later in course
  - Encodes round-by-round actions/ timing of moves
  - Captures the information players learn during the game
  - Players keep track of history and act accordingly
    - Tic-tac-toe
    - Chess
    - Poker
    - Repeated games
  - Analyzing such games is more involved



Normal-Form Representation

- Finite, n-person normal form game (N, A, u)
  - **Players:** $N = \{1,\ldots,n\}$
  - **Action set**: for player i, set of actions A_i available
  - **Action profile**: $a = (a_1, \ldots, a_n) \in A = A_1 \times \cdots A_n$
  - **Outcome** of the game is action profile played
  - **Utility function** or Payoff function for player i is $u_i : A \to \mathbb{R}$
    - $u = (u_1, \ldots, u_n)$ is a profile of utility functions
- **Rationality assumption**. Players will always act to maximize their utility
- **Common knowledge assumption:** Player rationality is common knowledge
  - Each players knows that everyone else knows that everyone else is rational.....

Normal-Form: Formalize

- Example: Rock, paper, scissors
  - **Players:** $N = \{1,2\}$
  - **Action set**: $A_i = \{$Rock, Paper, Scissors$\}$ for all i
  - Action profile/outcome example: (Rock, Paper)
  - **Utility function**
    - Symmetric, and maps to $\{-1,0,1\}$

	Rock	Paper	Scissors
Rock	$0, 0$	$-1, 1$	$1, -1$
Paper	$1, -1$	$0, 0$	$-1, 1$
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Strategies

- For simultaneous move games, we will use the term **strategies** and **actions** interchangeably
- A strategy, in general, is a sequence of actions that a player makes
  - e.g., in chess you need to "act" several times over the play

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Rock	$0, 0$	$-1, 1$	$1, -1$
Paper	$1, -1$	$0, 0$	$-1, 1$
Scissors	$-1, 1$	$1, -1$	$0, 0$

Complete Information

- In a game of **complete information**, every player knows the everything about the game:
  - Actions available to other players, and their utilities
  - Know that every player knows this as well
  - Know that every player is rational and is going to play to maximize their utility
- Let's players reason about "equilibrium" behavior: simplifies analysis
- This is not always true!
  - We will study incomplete information games as well

Prisoner's Dilemma

- Two alleged criminals questioned in separate rooms
- Each player has two actions:
  - **Cooperate (C):** stay silent and not admit to anything
  - **Defect (D)**: testify agains the other person
- If both stay silent **(C, C)**, each serves 1 year in prison for minor offense
- If one confesses against the other **(C, D)** or **(D, C)**, confessor goes free while other person gets a long prison sentence
- If both confess **(D, D)**, they each serve 3 years in prison
- We can write their preferences as an ordering

	C	D
C	a, a	b, c
D	c, b	d, d

$$c > a > d > b$$

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- If both confess **(D, D)**, they each serve 3 years in prison
- But more commonly, we use numbers to denote their utility

	C	*D*
C	3, 3	0, 5
D	5, 0	1, 1