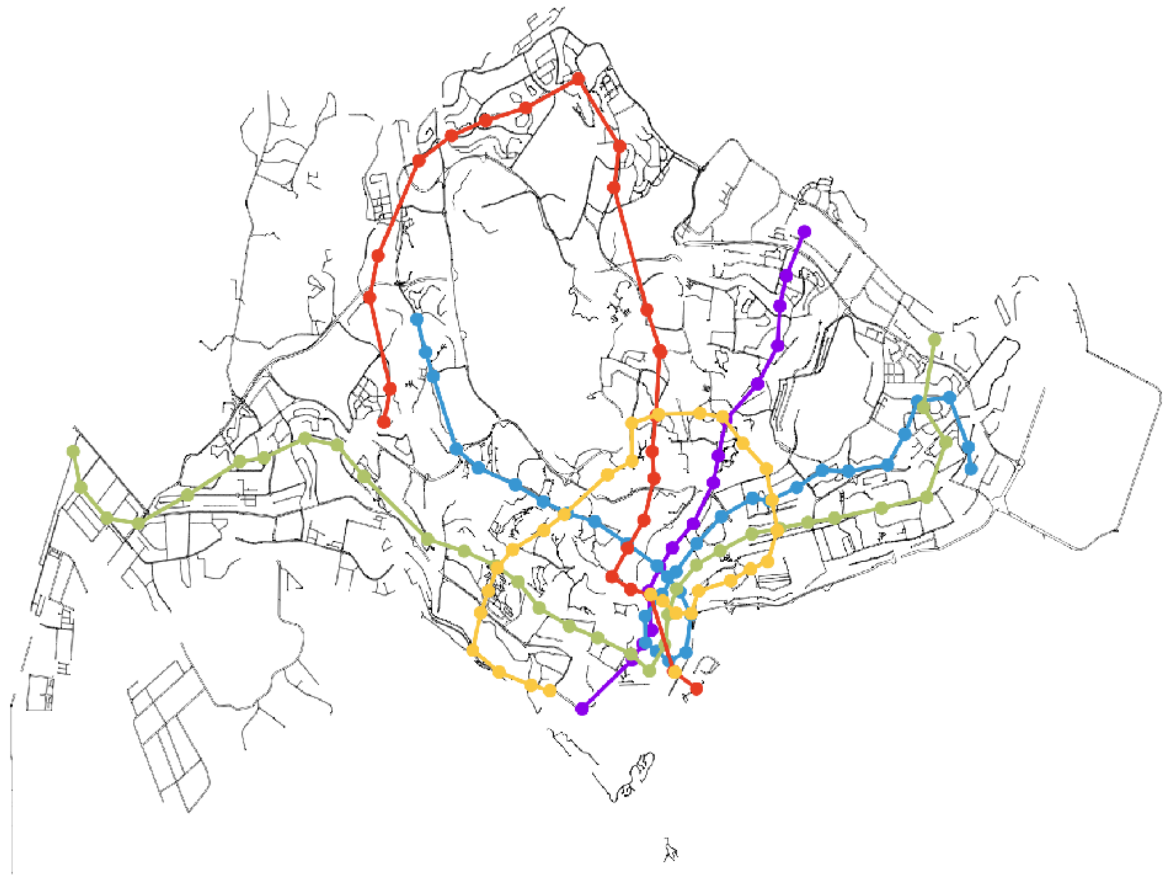
CSCI 357: Algorithmic Game Theory

Lecture 10: VCG and Simultaneous Auctions

Shikha Singh

Announcements and Logistics

- **Assignment 5** out and due Wed at 11 pm
  - Shorter single-person assignment
- Self-scheduled **midterm 1** on Saturday **March 12**
  - Open book, open notes
  - Pick up exam between 9am - 7pm (fill out google form beforehand)
  - TCL 202 reserved for students who want to take the exam there
  - Return completed exam within ~3 hours (more details on Thursday)
- **No homework due** the week before Spring break
- Assignment 6 will be due Thurs April 7 (week after break)

Questions?

Last Time

- How to use revenue equivalence to solve for BNE of single-parameter 0/1 mechanisms
- **Revenue maximization:** Vickrey (second price) auction with an appropriate reserve price optimal
  - This extends to all single-parameter mechanisms
- Power of one-additional bidder:
  - **[Bulow Klemperer]** Vickrey auction (with no reserve) with $n+1$ bidders generates just as much expected revenue as the revenue-optimal auction with n bidders!

Bulow Klemperer: Proof Idea

- Define a fictitious auction A that does the following:
    - Simulate the revenue-optimal auction on n bidders
    - If the item is not allocated, then give it to bidder $n+1$ for free
- This auction has two useful properties:
    - Its expected revenue with $n+1$ bidders is exactly that of the optimal with n bidders
    - It always allocates the item
- **Claim.** Vickrey auction obtains at least as much expected revenue as any auction that is guaranteed to allocate the item (and thus A)
    - We won't prove this
- Thus, $R(\text{Vickrey}\,.\,(n+1)) \geq R(A) \geq \text{OPT}(n)$

Today

- General mechanism design
  - VCG mechanism and its challenges
- Revelation principle in mechanism design
  - Why the field focuses on direct revelation mechanisms
- Application: Spectrum auctions

Week 6: Matching Markets w/o Money

Week 5: Matching Markets w Money

Week 4: Bayesian Analysis & General Mechanism Design

Week 3: Application : Sponsored Ad Markets

Week 2: DSIC Auctions

Week 1: Game Theory

VCG Mechanism for General Mechanism Design

General Mechanism Design

- So far we have focused on single-parameter mechanism design
- Bidders can have valuations for any subset of allocations
- Direction revelation is even more challenging:
    - Asking bidders for up to $2^{|S|}$ values in the worst case

n buyer with private valuations over all possible allocations

Multiple items S

General Mechanism Design

- Combinatorial (multi-parameter auctions): set S of items, and $2^{|S|}$ possible subsets that can be allocated (**outcomes**)
- Ingredients of a multi-parameter mechanism design problem
  - n strategic agents
  - A finite set A of feasible outcomes
  - Each agent i has a private valuation $\mathbf{v}_i(\mathbf{a})$ for each $\mathbf{a} \in A$
    - Each $\mathbf{v}_i$ is now a vector describing values for all possible outcomes
- **Goal:** Design a DSIC, surplus maximizing, and polynomial time mechanism
  - Polynomial time will be the biggest bottleneck here

Application: Unit Demand

- Matching markets to match buyers to items
  - n buyers and m items
  - Each buyer i has a valuation v_{ij} for item each j
  - Each buyer wants only one item **(unit demand)**
- Note that this is more general than the single-parameter examples:
  - Sponsored search, Knapsack auctions etc
- Many application domains: housing markets, matching renters to rooms etc
- Auctioning off government licenses or construction projects etc

Unit Demand Example

- Suppose you are organizing **a job fair** and each firm has a different preference of the booth assignment they receive
- There are three firms and three possible locations in the room front **(F),** middle **(M),** rear **(R)**
- They have the following private valuation of each option

	F	M	R
firm 1	\$10	\$2	\$1
firm 2	\$100	\$100	\$100
firm 3	\$50	\$45	\$40

- How should we allocate the booths and charge payments so that they are incentivized to report their true valuation?

General Demand: Challenges

- In principle, the valuation function for different outcomes can be more complex than the simple example
- An agent can have a different valuation for each possible winner of the auction (not just their own allocation)!
  - In a bidding war over a hot startup agent i's highest valuation may be for the outcome where they acquire the start up
  - But if they lose, they may prefer that the startup be bought by a company that is not a direct competitor
- To obtain tractable mechanisms often assumptions are made on the valuation function
  - But what we discuss holds for types of valuation

The VCG Mechanism

- Surprisingly, there exists a DSIC surplus-maximizing mechanism in general.
- **Theorem [Vickrey-Clarke-Groves (VCG) Mechanism]:** The following mechanism is DSIC for any general mechanism design problem:
    - Collect sealed bids
    - Allocated based on the surplus maximizing rule
    - Charge each bidder their "**externality**": the surplus loss inflicted on others by their presence
- Turns out the above allocation and payment imposes DSIC behavior

The VCG Mechanism

- **Allocation.** Given bids $\mathbf{b} = (\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n)$ where each $\mathbf{b}_i$ is now a vector indexed by $|A|$, the surplus maximizing allocation is (assuming bids as proxies for valuations)

$$\mathbf{a}^*(\mathbf{b}) = \text{argmax}_{\mathbf{a}\in A} \sum_{i=1}^{n} \mathbf{b_i}(\mathbf{a})$$

- **Payment**. Charge each bidder their externality:

$$p_i(\mathbf{b}) = \underbrace{\max_{a_{-i}\in A_{-i}} \sum_{j\neq i} \mathbf{b}_j(a_{-i})}_{\text{without } i} - \underbrace{\sum_{j\neq i} \mathbf{b}_j(a^*)}_{\text{with } i}$$

Where a^* is the surplus maximizing outcome in the presence of i

VCG Payments

- Payment alternate intuition:

$$p_i(\mathbf{b}) = \underbrace{\max_{a_{-i} \in A_{-i}} \sum_{j \neq i} \mathbf{b}_j(a_{-i})}_{\text{without } i} - \underbrace{\sum_{j \neq i} \mathbf{b}_j(a^*)}_{\text{with } i}$$

$$p_i(\mathbf{b}) = \mathbf{b}_i(a^*) - \left(\sum_{i=1}^{n} \mathbf{b}_i(a^*) - \max_{a_{-i} \in A_{-i}} \sum_{j \neq i} \mathbf{b}_j(a_{-i}) \right)$$

Rebate equal to the surplus generated by i's presence

The VCG Mechanism

- **Claim.** The VCG mechanism $(\mathbf{x}, \mathbf{p})$ is DSIC.
- **Proof.** Fix i and $\mathbf{b}_{-i}$. Suppose the chosen outcome is $\mathbf{x}(\mathbf{b}) = a^*$
  - Utility of i for outcome a^* is $\mathbf{v}_i(\mathbf{a}^*) - p_i(\mathbf{b})$
  - Term B is a constant (max surplus generated without i)
  - Maximizing i's utility $\iff$ maximizing term A

$$\underbrace{\mathbf{v}_i(a^*) + \sum_{j \neq i} \mathbf{b}_j(a^*)}_{A} - \underbrace{\max_{a_{-i} \in A_{-i}} \sum_{j \neq i} \mathbf{b}_j(a_{-i})}_{B}$$

The VCG Mechanism

- **Claim.** The VCG mechanism $(\mathbf{x}, \mathbf{p})$ is DSIC.
- **Proof.** Fix i and $\mathbf{b}_{-i}$. Suppose the chosen outcome is $\mathbf{x}(\mathbf{b}) = a^*$
  - Utility of i for outcome a^* is given by the expression:
  - Term B is a constant (max surplus generated without i)
  - Setting $\mathbf{b}_i = \mathbf{v}_i$ maximizes i 's utility under a surplus maximizing allocation.

Bidding truthfully maximizes i's utility

$$\underbrace{\mathbf{b}_i(a^*) + \sum_{j \neq i} \mathbf{b}_j(a^*)}_{A} - \underbrace{\max_{a_{-i} \in A_{-i}} \sum_{j \neq i} \mathbf{b}_j(a_{-i})}_{B}$$

The VCG Mechanism: Example

- Suppose you are organizing a job fair and each firm has a different preference of the booth assignment they receive
- There are three firms and three possible locations in the room front (F), middle (M), rear (R)
- They have the following private valuation of each option

	F	M	R
firm 1	\$10	\$2	\$1
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firm 3	\$50	\$45	\$40

- **Class exercise.** What outcome does the VCG mechanism select? What payments does it charge?

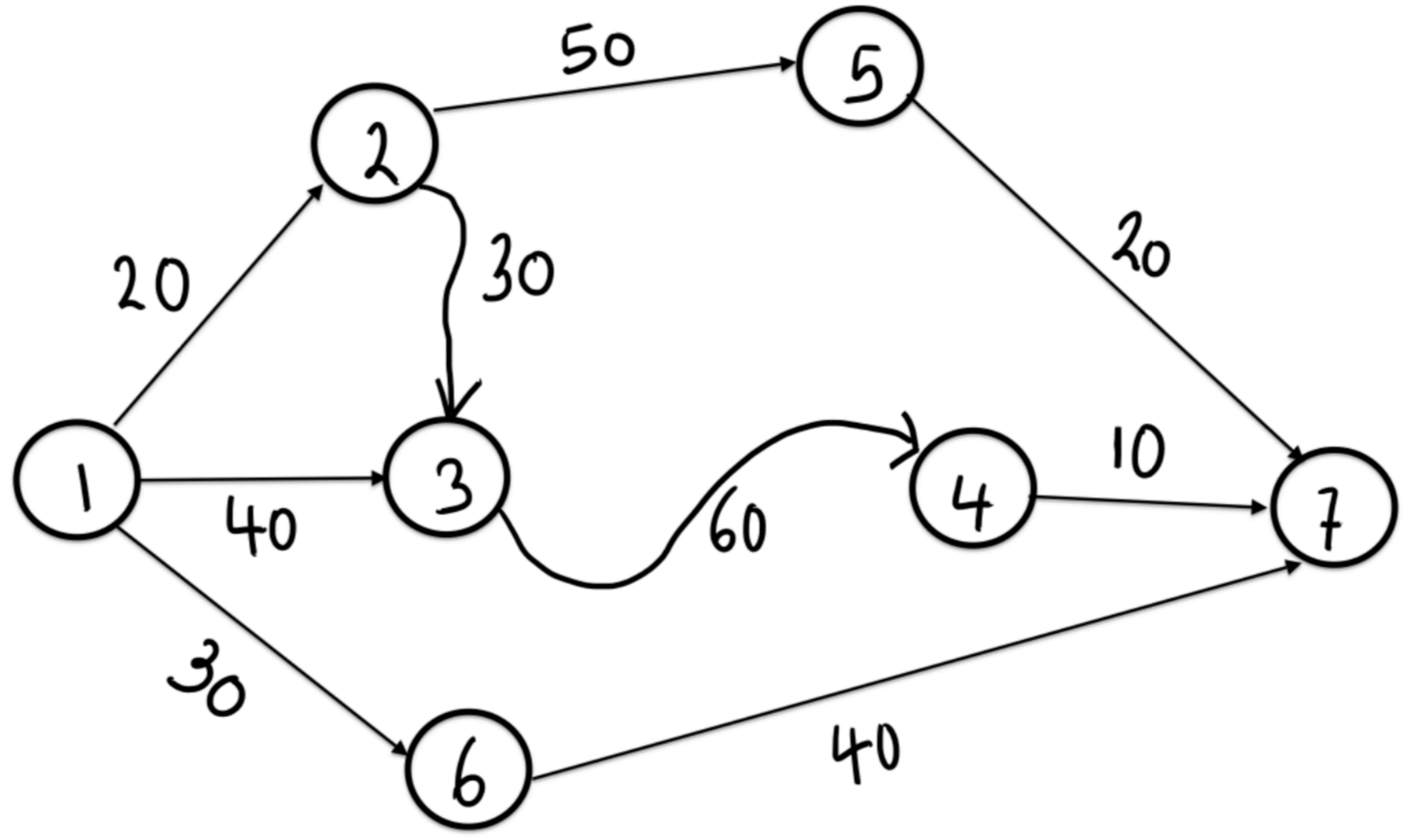
The VCG Mechanism: Example

- VCG does the following assignment:
  - $(1,F)$, $(2,R)$, $(3,M)$
  - Total surplus: $10 + 100 + 45 = 155$
- Without firm 1, the best outcome is $(2,M/R)$, $(3,F)$ with surplus $100 + 50 = 150$
- $p_1 = 150 - (155 - 10) = 5$
- Without firm 2, the best outcome is $(1,F)$, $(3,M)$ with surplus 55
- $p_2 = 55 - (155 - 100) = 0$
- Without firm 3, the best outcome is $(1,F)$, $(2,M/R)$ with surplus 1
- $p_3 = 110 - (155 - 45) = 0$
- Revenue generated: 5

	F	M	R
firm 1	\$10	\$2	\$1
firm 2	\$100	\$100	\$100
firm 3	\$50	\$45	\$40

Shortest Paths from s to t

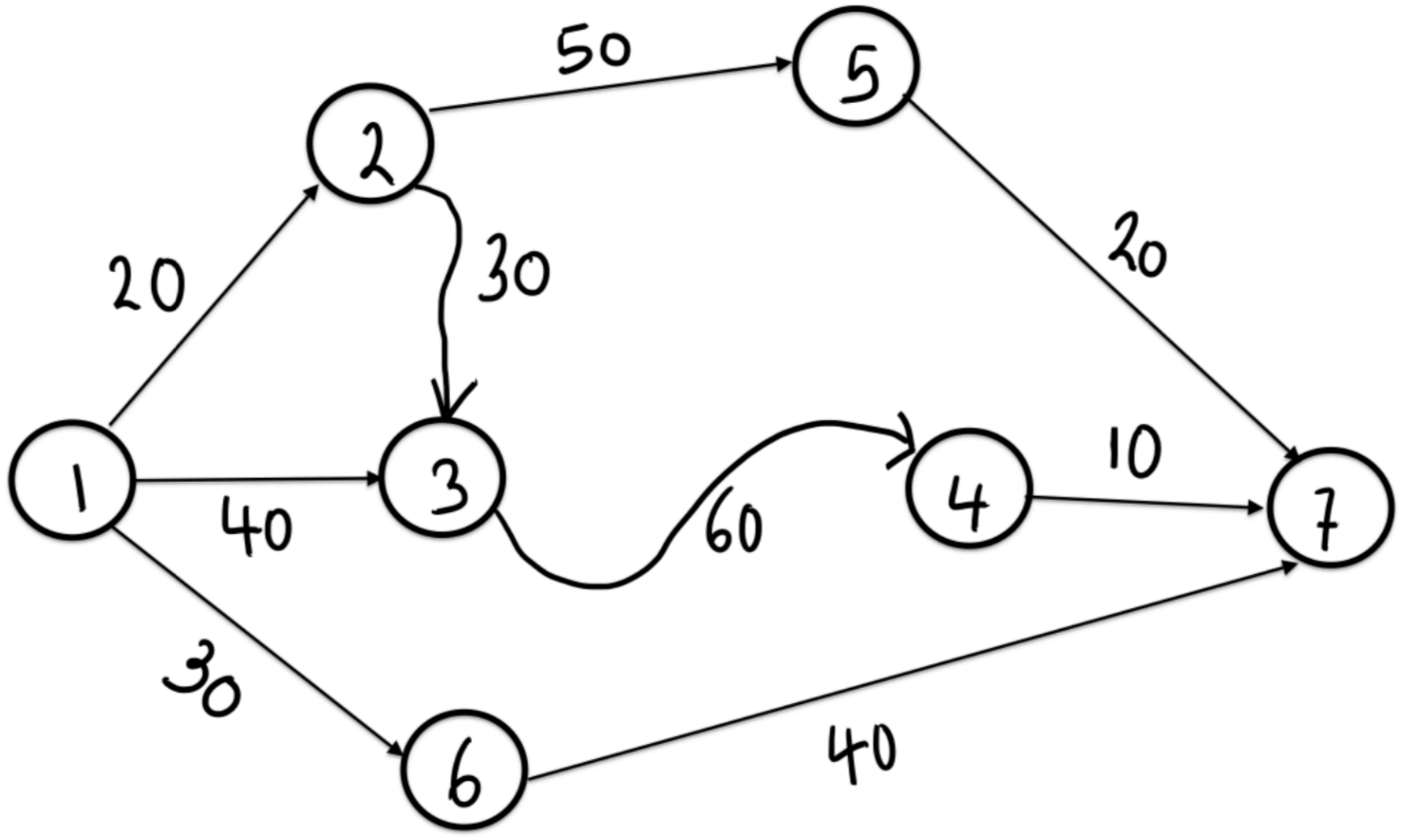
- Many mechanism design problems can be modeled as a combinatorial auction
- **Goal:** Select a lowest cost path from 1 to 7
- Each edge is an agent with cost $c_i > 0$ if their edge is used ($v_i = -c_i$)
  - Since agent's have costs when used, mechanism may pay them
- $A = \{$all s-t paths$\}$
- A_{-i} = $\{$paths that do not use edge $i\}$
- VCG mechanism selects path with maximum value:
  - Min cost path



Example: Shortest Paths from s to t

- Assuming truthful reports, the lowest-cost path is $1 \to 6 \to 7$
- What are the payments?
  - For all agents except (1,6) and (6,7): cost is zero
  - For agent (1,6)'s payment
    - What is the lowest cost path without that edge?
      - $1 \to 2 \to 5 \to 7$
    - $p(1\text{-}6) = -90 - (-40) = -50$
  - That is, 1-6 should receive a payment of 50
  - Similarly we can compute 6-7's payment:
    - $p(6 - 7) = -90 - (-30) = -60$

The agents receive as payment the **maximum cost** they *could have reported* and still been on the selected path!



Problems with VCG

- Suffers from **collusion**, same way as second-price auctions
- **Intractability** of surplus maximization
  - This is a challenge even when restricted to a single-parameter setting
- **Budget balance**: If an agent has a **negative value** (say a seller who has a **cost** involved with outcomes) then the mechanism may not generate enough revenue to compensate the seller
    - Positive payments may not equal negative payments
  - That is, the VCG mechanism may incur a **budget deficit**
- **Non- monotonity of revenue:** It may generate worse revenue when the competition increases!

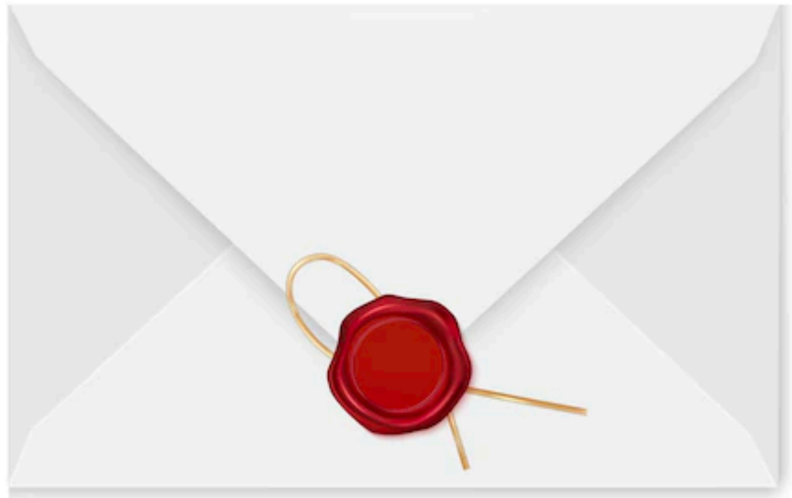
Mechanism Design Challenges

- So far, we have only looked at sealed bid (or direct revelation mechanisms)
- Challenges of these auctions, esp in a combinatorial setting?
- Communication complexity:
  - Asking bidders to report all their valuations upfront can be expensive
- Computational complexity: already a challenge with single-parameter mechanisms
  - Worse in general settings: no notion of "**monotonicity**" for **approximations**
- Challenges with general sealed-bid auctions:
  - Issues of privacy and transparency, synchronization of bidding
- **Question**. Why the focus on sealed-bid (direct revelation mechanisms)?

Revelation Principle

Direct vs Indirect Mechanisms

- So far we have focused on sealed bid mechanisms and truthfulness
    - Direct revelation: we ask bidders to upfront report their private value
- One can imagine many indirection mechanisms:
    - Place agents into a “priority order” and ask each agent in turn which item it wants from what is left
    - Do deferred acceptance or sequential "take-it or leave it" options
    - Ascending clock mechanisms
- These processes are more "interesting" than simultaneous sealed bidding
- **Question**. Are we restricting ourselves fundamentally when we focus on direct revelation mechanisms?



Cost of Truthfulness

- So far we have conflated two things when we say a mechanism is DSIC:
  - Each player in the mechanism has a **dominant strategy** no matter what their private value is
  - DSE strategy is *direct revelation:* agents truthfully report all their private value upfront
- We can define Bayesian-incentive compatibility the same way:
  - Each player in the mechanism **has a best response** (given others' strategies and the players beliefs about their private information)
  - DSE strategy is *direct revelation: agents truthfully report all their private value upfront*
- Rephrased question: *Are we missing out by insisting on direct revelation of true values?*
  - Or rather can indirect mechanisms that do not ask agents to upfront report their true values be inherently better?

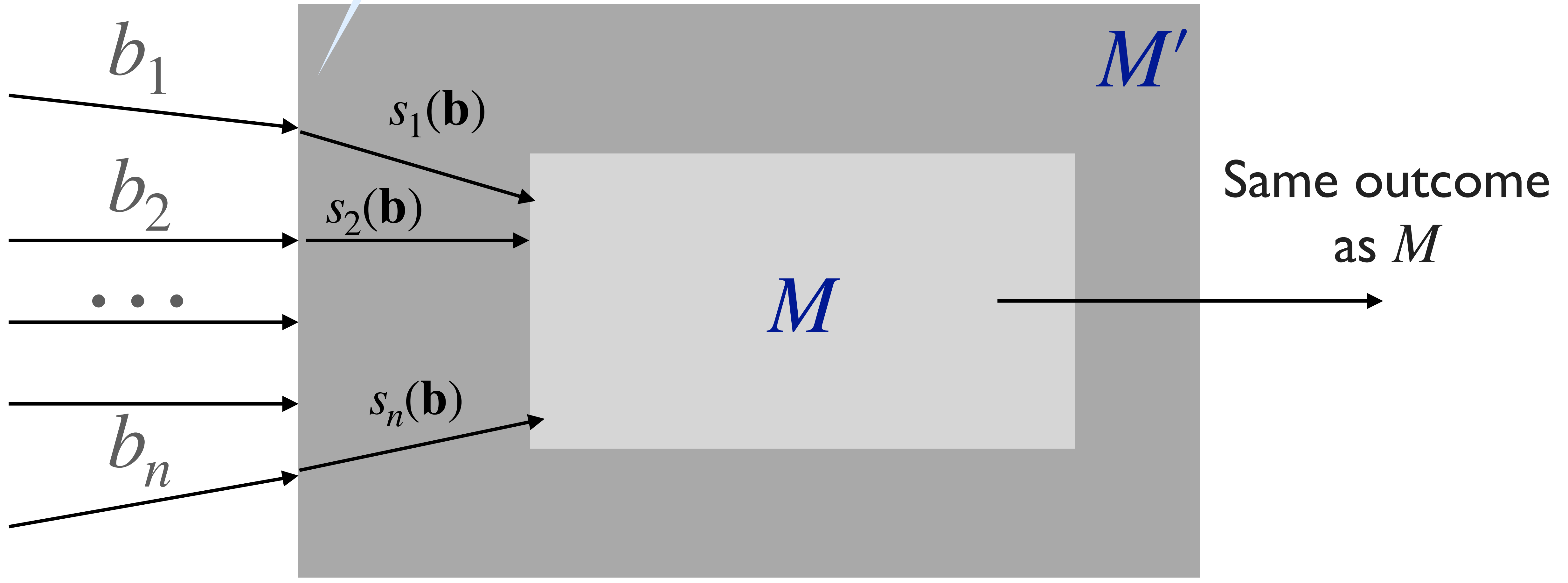
The Revelation Principle

- States that as long as participants have a dominant-strategy or Bayes' Nash strategy, direct revelation is without loss of generality
  - Indirect mechanisms cannot inherently do better than direct
- For every mechanism in which every participant has a dominant strategy equilibrium or BNE, we can design a direct revelation mechanism M' where truthfully reporting your value is a DSE/BNE achieves the same outcome.
- M' can simulate M and equilibrium strategies:
  - Suppose a strategy profile s was a BNE of M where each agent i with private valuation v_i has a BNE strategy $s_i(v_i, \mathbf{v}_{-i}) = b_i$
  - M' asks bidders to report their value and play s_i on their behalf

The Revelation Principle

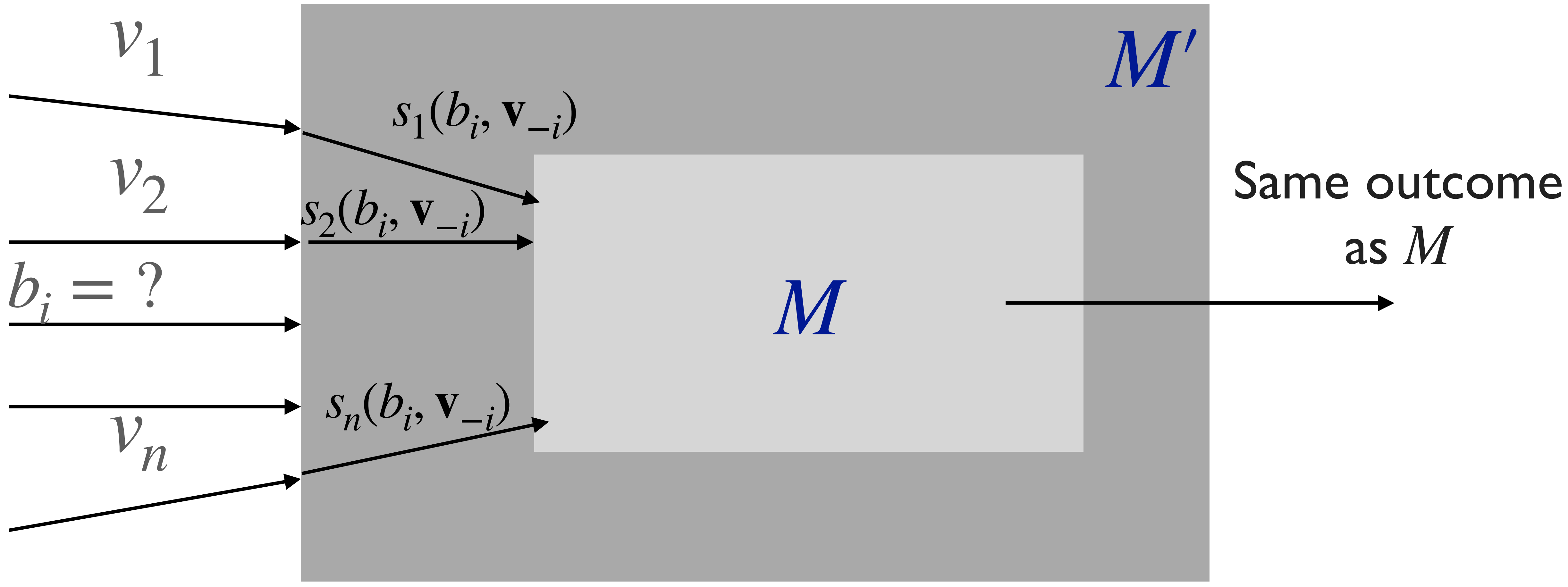
- Consider mechanism M where each agent i with private valuation v_i has a BNE strategy $s_i(v_i, \mathbf{v}_{-i}) = b_i$
- Mechanism M''s can simulate these strategies for agents:
  - Accept bids $\mathbf{b} = (b_1, \ldots, b_n)$
  - Submit bids $\mathbf{b}' = (s_1(\mathbf{b}), s_2(\mathbf{b}), \ldots, s_n(\mathbf{b}))$ to M
  - Output the same outcome as M

Agents do not need to strategize, the mechanism will do it for them



The Revelation Principle

- **Claim:** s is a BNE of M means that truth telling is a BNE of M' (for the same distribution F)
- **Proof.** Let s' be the truth telling strategy, to show it is BNE, fix s'_{-i} to be truthful:
  - $s'(v_j) = b_j = v_j \qquad \forall j \neq i$
- Then, in M all players $j \neq i$ are using $s(v_j)$
- What is i's best response?
  - To play $s(v_i)$ as s is a BNE
- This can be done by being truthful in M' ∎



Caution: Revelation Principle

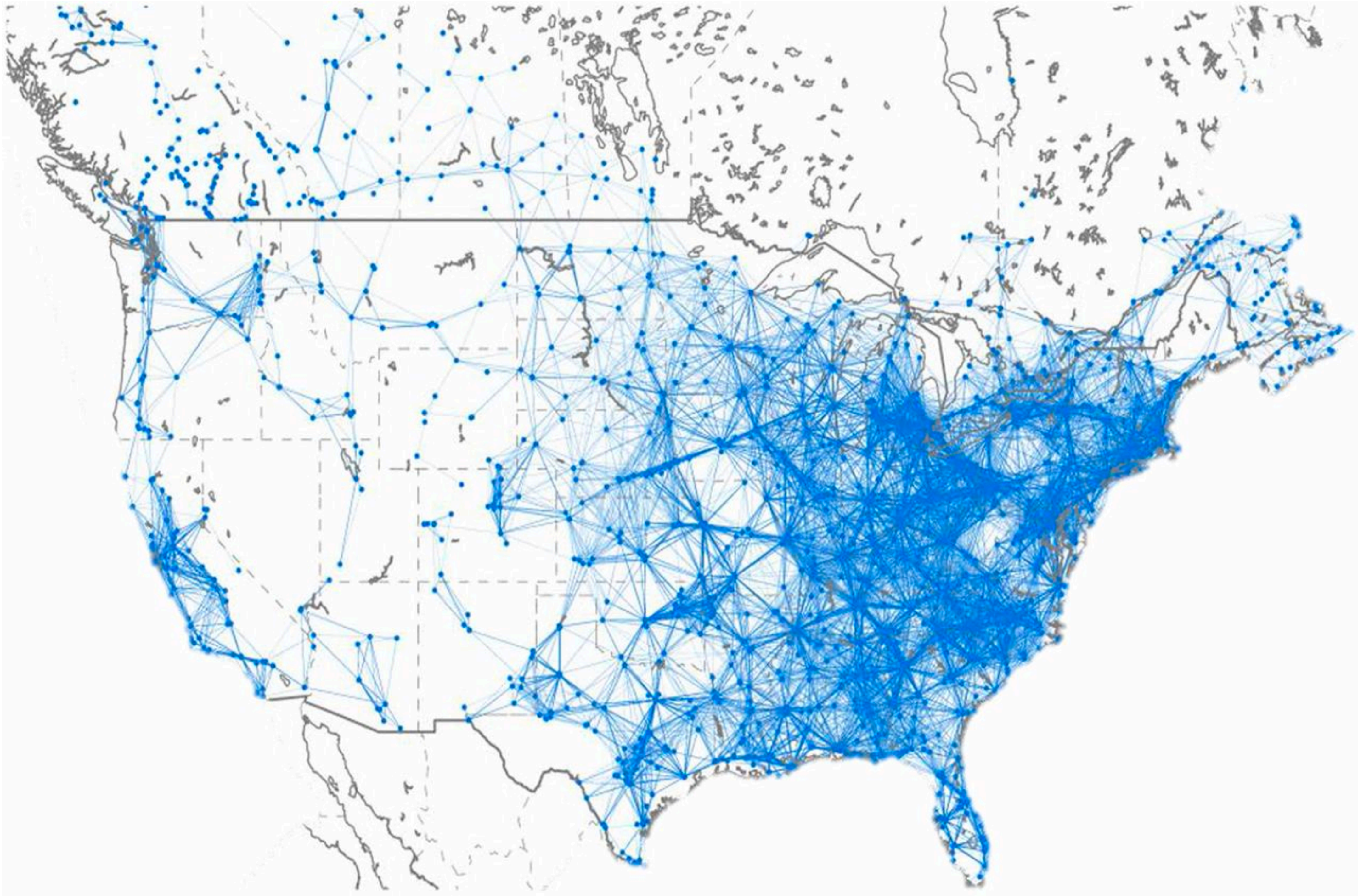
- The **set of equilibria** is not always the same in the original mechanism and revelation mechanism
  - What we proved is that that the revelation mechanism does have the original equilibrium of interest
  - However, it could be that the indirect mechanisms had a unique equilibrium and the transformation introduces new, bad equilibria
  - Multiple equilibria are highly undesirable
- Direct revelation leads to **communication blowup**
- Revelation principle **fails to hold** when:
  - Agents learn their value over time
  - Mechanism designer does not know the common prior F

Beyond Direct Revelation

- So what is the revelation principle good for?
  - Recognizing that truthfulness is not a restrictive assumption
  - Recognizing that **indirect mechanisms can't do (inherently) better than direct mechanisms**
- Thus, truthfulness per se is not important, what makes mechanism design hard is the requirement that a **desired outcome is accomplished under an equilibrium** of some type
- Criticisms of the revelation principle?
  - Ignores practical challenges involved with direct revelation mechanisms
  - Communication, syntonization, trust and privacy issues
- **Question**. What is an alternative indirect mechanism for combinatorial auctions?

Application: Spectrum Auctions

Spectrum Auctions



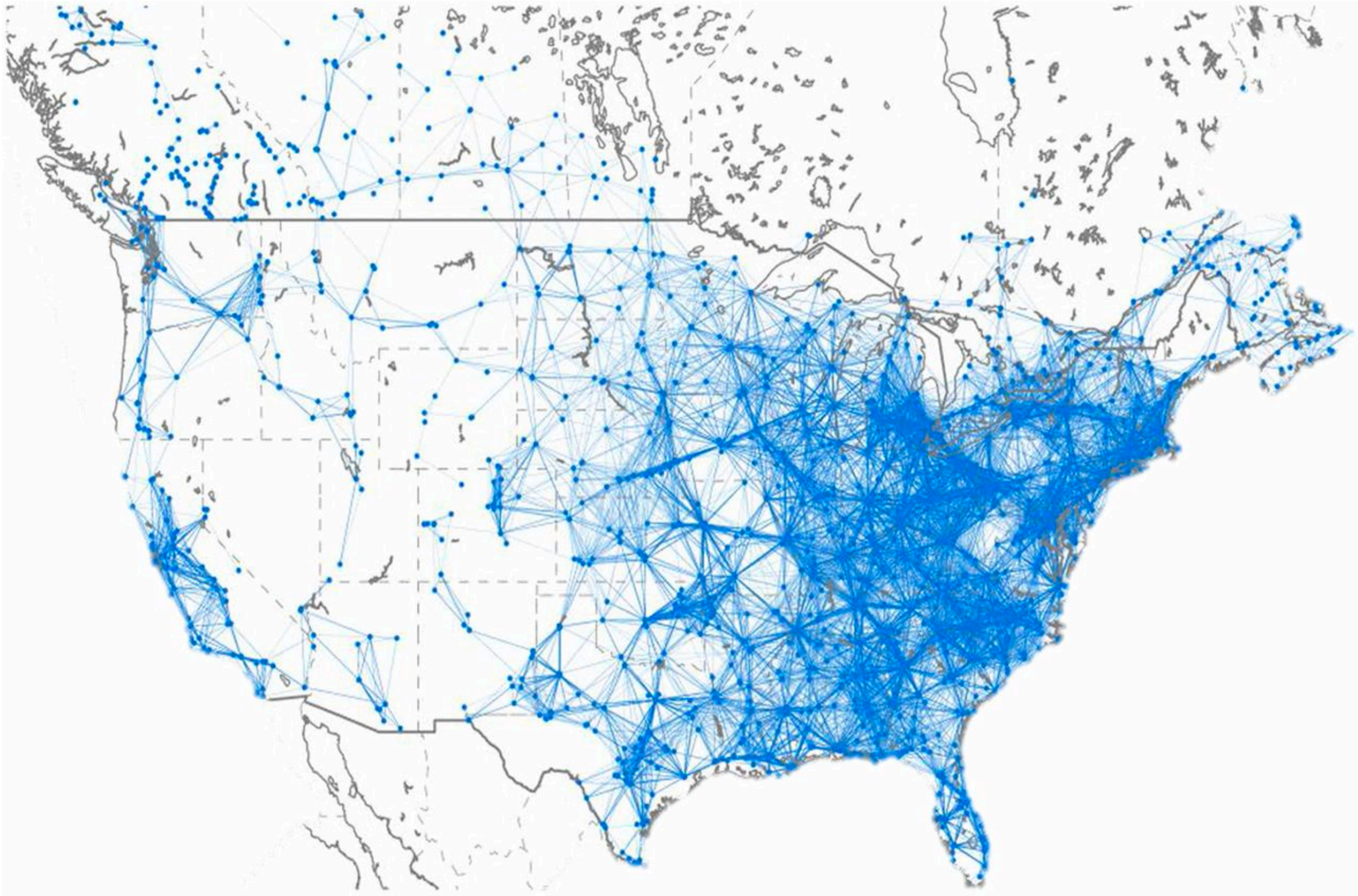
What is Spectrum?

Spectrum refers to the invisible radio frequencies that wireless signals travel over. Those signals are what enable us to make calls from our mobile devices, tag our friends on Instagram, call an Uber, pull up directions to a destination, and do everything on our mobile devices.

The frequencies we use for wireless are only a portion of what is called the electromagnetic spectrum.

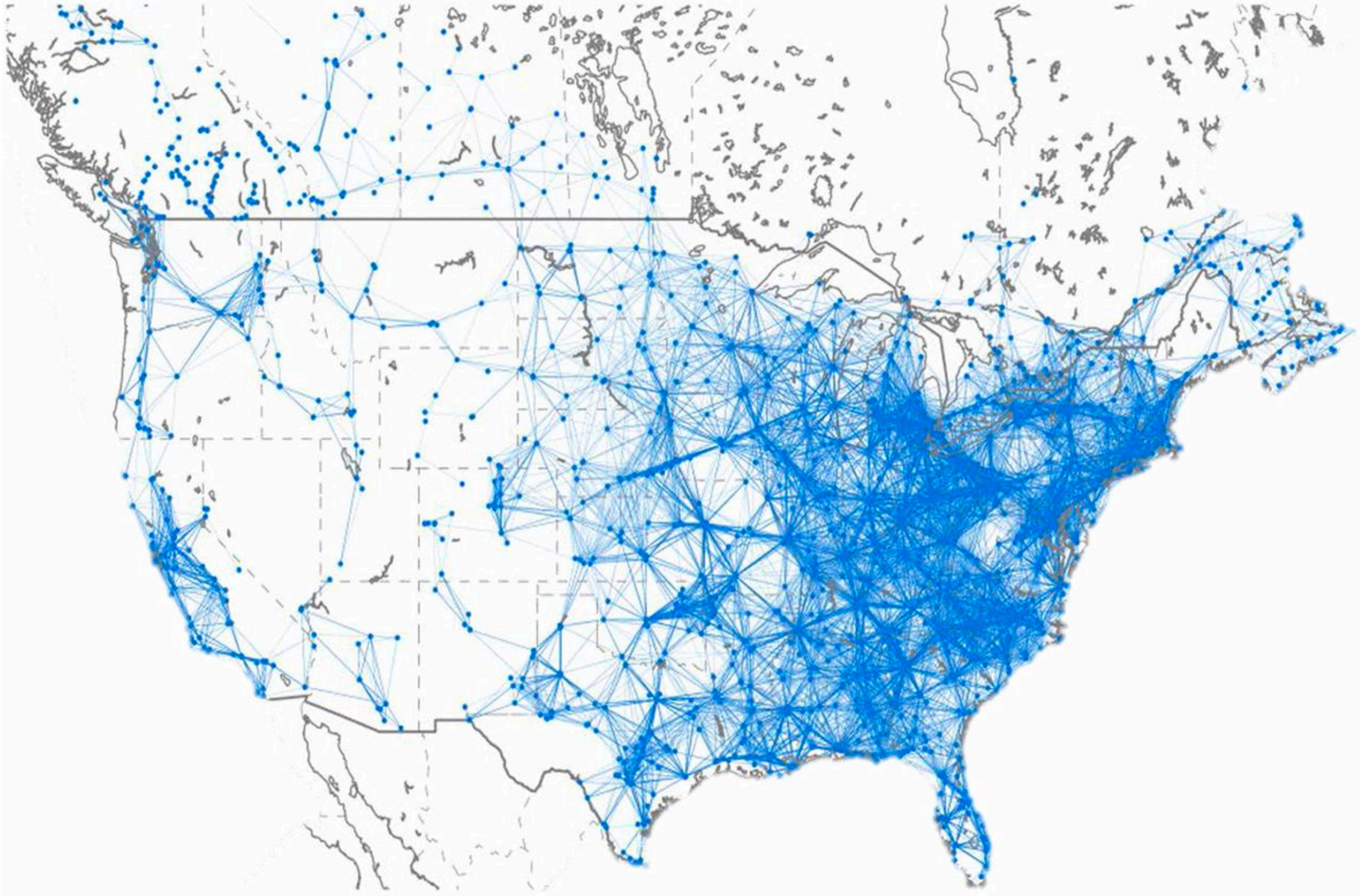
https://www.ctia.org/news/what-is-spectrum-a-brief-explainer

Wireless Spectrum



"The easiest way to understand what spectrum really is and how it provides services is to look at your radio. When you tune your radio to 93.9 FM, you are tuning into a station that is broadcasting at 93.9 megahertz. If you want to a listen to a different station, like one that only plays country music or jazz, you turn the dial to another frequency like 104.7 FM. And a different radio station will be transmitting over that particular frequency on a different setting on your radio dial. No two stations transmit over the same spectrum at the same time in the same area, because if they did, they'd cause interference with one another." -- https://www.cnet.com/news/wireless-spectrum-what-it-is-and-why-you-should-care/

Wireless Spectrum



The New York Times

F.C.C. Backs Proposal to Realign Airwaves

By Edward Wyatt

Sept. 28, 2012

WASHINGTON — The government took a big step on Friday to aid the creation of new high-speed wireless Internet networks that could fuel the development of the next generation of smartphones and tablets, and devices that haven't even been thought of yet.

The five-member Federal Communications Commission unanimously approved a sweeping, though preliminary, proposal to reclaim public airwaves now used for broadcast television and auction them off for use in wireless broadband networks, with a portion of the proceeds paid to the broadcasters.

The initiative, which the F.C.C. said would be the first in which any government would pay to reclaim public airwaves with the intention of selling them, would help satisfy what many industry experts say is booming demand for wireless Internet capacity.

Mobile broadband traffic will increase more than thirtyfold by 2015, the commission estimates. Without additional airwaves to handle the traffic, officials say, consumers will face more dropped calls, connection delays and slower downloads of data.

The F.C.C. will issue proposed rules for what it calls incentive auctions — the sale of airwaves that are voluntarily given up by broadcasters in exchange for a portion of the auction proceeds.

FCC Incentive Auction

- For over 20 years, US and other countries have used spectrum auctions to sell licenses for wireless spectrum
- What's new and different that's happened this decade:
  - Decision to "reassign" airwaves: most popular parts of the spectrum are already owned by TV broadcasters
  - FCC decided to design a new auction (**FCC Incentive Auction**) to buy these back so they can be sold to companies that will put it to better use, e.g. wireless broadband services
- **Double auction:** reverse auction to buy back licenses, forward auction to sell
- We will focus on the **forward auction** to sell licenses today

Selling Items Separately

- Spectrum auctions are combinatorial in nature because corporations often want a subset of frequencies
  - Having a particular license may make others redundant/ more desirable
- Direct mechanisms are impractical to run in this setting
- What is a reasonable indirect/ asynchronous way to sell multiple items?
- A simple idea: instead of selling bundles, we can try to **sell items separately**
  - Already know truthful mechanisms for single-item auction
  - Can we just use it to sell multiple items?
- **Question**. Could selling items separately work?
- **Question**. How do we organize these single-item auctions?

Sequential Auctions

- **Question.** Is it a good idea to sell hold single-item auctions sequentially?
- Consider two nearly identical items, sold back to back using a second-price auction
- Suppose you are high-valuation bidder (likely to win any auction)
- What is your best strategy?
    - Suppose everyone is bidding truthfully
    - If you skip the first auction, second-highest bidder wins and is out of the auction
    - Now you can pay third-highest price in the second auction
- Not a dominant strategy to bid in a straightforward way

What Does not Work

- **Mistake 1.** Holding single-item auctions sequentially
- In March 2000, Switzerland auctioned off 3 blocks of spectrum via a sequence of Vickrey auctions
- Resulted in some unexpected variation in prices:
    - Two identical 28 MHz blocks sold for quite different prices: 121 million and 134 million
    - In a third auction, a larger 56 MHz block sold for 55 million!
- It was clear the bids were far from equilibrium
- Reasonable to speculate that the revenue generated was far from optimal
- **Takeaway**: items should be auctioned simultaneously

What Does not Work

- **Question 2.** What we use a (private) sealed-bid format?
- Difficult for bidders to figure out how to bid in such auctions
- Suppose you want 1 out of 10 licenses
- What are some good bidding strategies?
    - Pick one at random and go for it
    - Bid less aggressively in a few different auctions in the hopes of getting lucky in one of them and getting a deal
- **Challenge**: how to trade off risk of winning too many licenses with the risk of winning too few

Sealed Bid Format

- **Mistake 2.** Use sealed-bid single-item auctions
- In 1990, New Zealand auctioned off almost identical licenses for TV broadcasting using simultaneous (sealed-bid) Vickrey auctions
- Revenue of auction was **far below projected revenue**
  - Actual revenue: $36 million; projected: $250 million
  - In contrast, most spectrum auctions exceed projected revenues
- In one auction, the high bid was **$100,000 million** and second highest was **$6**
- In another, highest bid was $7 million, and second highest $5000
- **Takeaway**: Bidding behavior should be more public (public drop outs)

Simultaneous Ascending Auction

- So how are spectrum auctions run these days?
- Simultaneous ascending auctions (SAAs) **form the basis of the** state-of-the-art spectrum auction format
  - Essentially a bunch of English auctions (ascending clock) run in parallel
  - In reality, many other details
- Main component is that **bids are visible** to all
  - Even though this may lead to strategic behavior
  - Overall, leads to more-informed decisions

Simultaneous Ascending Auction

- Why do SAAs work better?
  - Main reason is *price discovery*
  - Bidders have more information about likely selling prices and can change their strategy midway: abandon highly-competitive licenses, finding unexpected bargains, etc
- What is another benefit of this format?
  - bidders only need to determine valuations on a need basis
- **General wisdom**: SAAs perform well and achieve good welfare and incentive properties

Next Topic

- **Decentralized matching markets** that can be viewed as a simultaneous ascending auction

[Aside] Reverse Auction

Descending-clock auction (Milgrom and Segal)

- Each round each broadcaster offered a buyout price
    - Buyout prices decrease over time
- Broadcasters can decline or accept in each round
    - **Decline**: exits auctions, retains license
    - **Accept**: moves to next round
- Different prices allowed for different broadcasters
- **Intuition**: stop auction when prices are as low as possible, with the constraint of clearing enough spectrum

[Aside] Reverse Auction

- To reclaim spectrum, FCC ran a reverse auction to buy back licenses
- E.g., suppose they want to clear at least 10 channels from 38-51 nationwide
- Because this is nationwide, there is a problem:
  - Bidders drop out in an uncoordinated way (channel 40 in MA may go first, followed by 51 in OH, etc)
- How to clear out a channel nationwide without buying out everyone?
  - **Solution**: repack TV stations into a smaller subset of channels example (38-41)
  - Stations who drop out retain their license but may get assigned a different channel

Reverse Auction

- Descending clock auction maintains the invariant that stations that have dropped out of the auction can be assigned channels so that at most a target most of channels are used
- Called the **repacking problem**
- **Challenge:** two stations with overlapping broadcasting regions cannot be assigned the same channel
- Turns out to be essentially a graph coloring problem
    - Overlapping regions (adjacent) cannot be same color
    - NP-complete problem

Algorithms to the Rescue

- A team led by Kevin-Leyton Brown was given the responsibility to quickly solve repacking problems
- FCC gave them the budget of **one minute per problem**, ideally with most instances solved within a second
- Leyton-Brown's team improved upon the state-of-the-art SAT solvers and optimized for the specific problem
- Were able to solve 99% of the repacking problems in under a minute!

Cutting edge techniques to solve NP complete problems was the reason FCC was able to use this auction!

Final Report Card

- Broadcast Television Incentive Auction (3/16-3/17)
- Reverse auction: led to **$10 billion cost**
- Forward auction: generated **$20 billion revenue**
- Revenue will cover auction costs, funds a new first responder network and go towards reducing deficit
    - "Middle Class Tax Relief and Job Creation Act"

"While being unique in a number of ways, we believe that the auction offers a good example of how recent advances in economic theory and computer science can be combined to design radically new marketplaces, unlocking substantial economic value and benefiting all market participants as well as the US public as a whole." : *Leyton-Brown, Milgrom, Segal*

Economics and computer science of a radio spectrum reallocation

Kevin Leyton-Brown[a,b,1], **Paul Milgrom**[b,c,1,2], **and Ilya Segal**[b,c,1]

Edited by Jose A. Scheinkman, Columbia University, New York, NY, and approved May 25, 2017 (received for review February 4, 2017)

The recent "incentive auction" of the US Federal Communications Commission was the first auction to reallocate radio frequencies between two different kinds of uses: from broadcast television to wireless Internet access. The design challenge was not just to choose market rules to govern a fixed set of potential trades but also, to determine the broadcasters' property rights, the goods to be exchanged, the quantities to be traded, the computational procedures, and even some of the performance objectives. An essential and unusual challenge was to make the auction simple enough for human participants while still ensuring that the computations would be tractable and capable of delivering nearly efficient outcomes.

algorithmic mechanism design | auction theory | incentive auction | market design | dominant strategies

Next:
Matching Markets (w Money)