CSCI 357: Algorithmic Game Theory

Lecture 4: Introduction to Auctions

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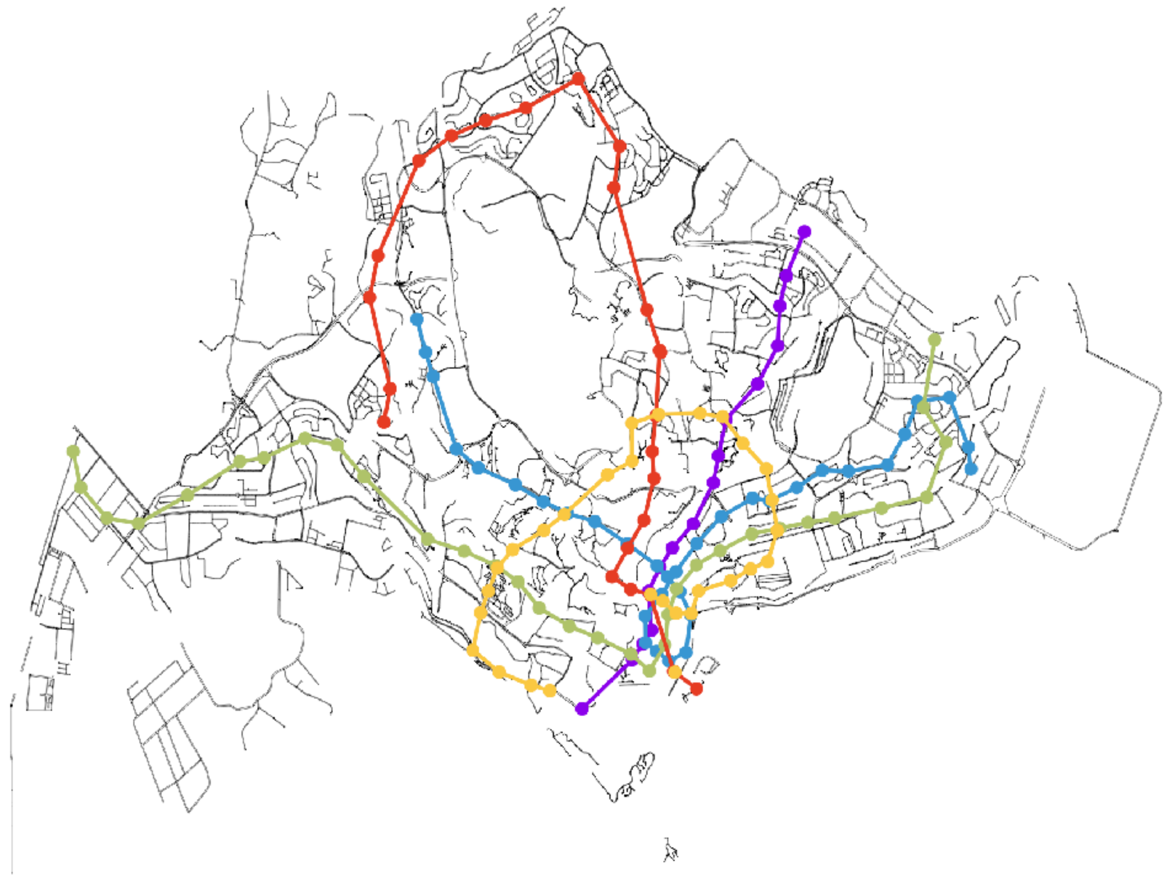


Announcements and Logistics

- **Assignment 2** out and due Thursday 10 pm
  - Office hours 2.30 - 4 on Tues & Wed in TCL 206
  - Thursday after lecture 4-5 pm in lecture room Schow 30A
  - Goal: simulate a lab/ problem solving environment
  - Encourage everyone to pick at least one that they can attend
- **Assignment 1:** goal to return feedback by Thursday

Questions?

Last Time

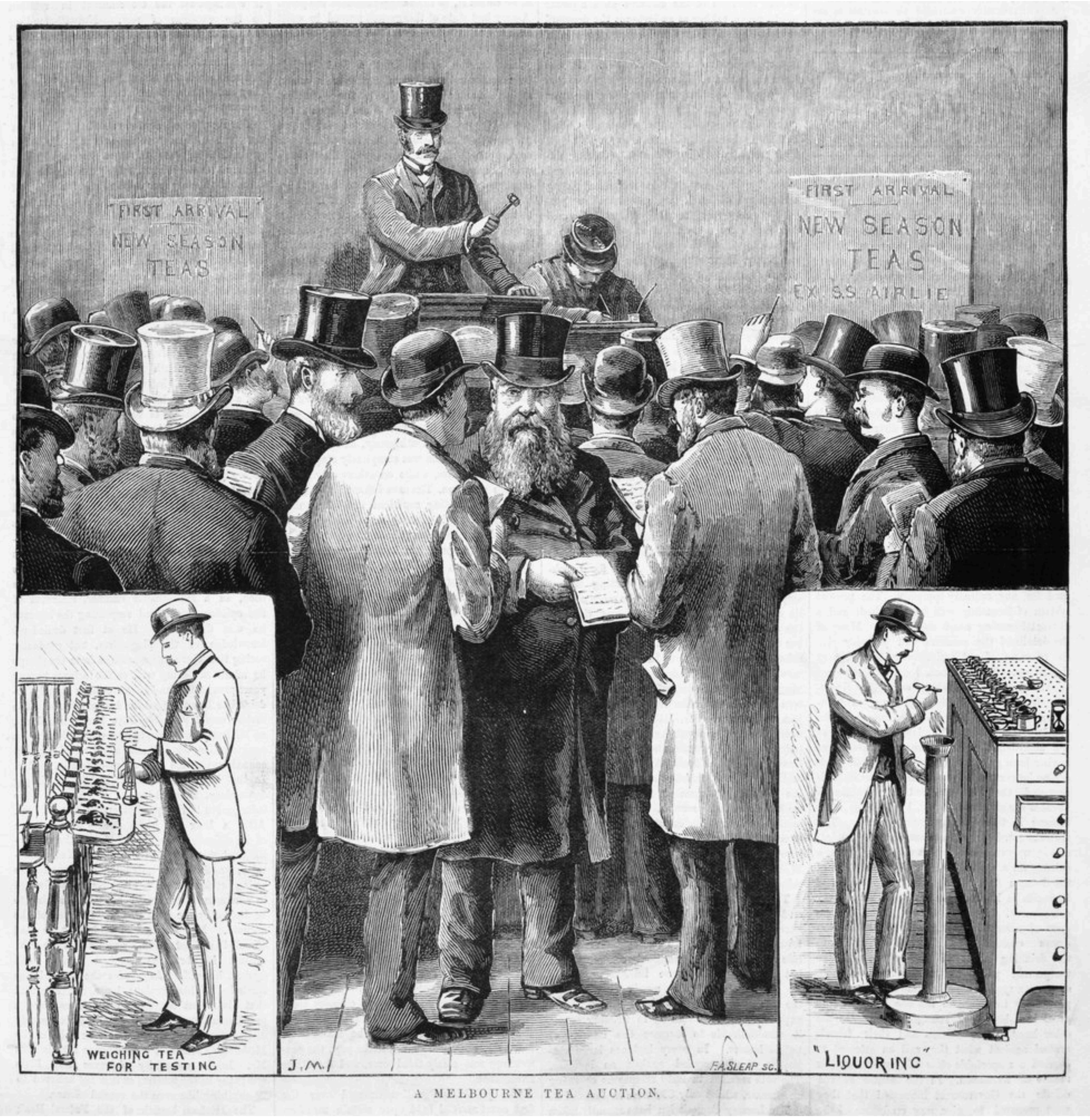
- Wrapped up foundation in game theory
- What we will build on
  - Strategic reasoning and best response
  - DSE and Nash equilibrium solution concepts
  - Important to be comfortable with these definitions!

This Week

- Start the study of mechanism design by looking at simple auctions
- Problems in mechanism design with money often reduce to an auction
- When we talk about auctions, what comes to mind?

Auctions: What Comes to Mind?

Traditional outcry style auctions



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Credit: Sotheby's

Question. Can you think of other other examples of auctions in real life?

Auctions Everywhere

Auction of perishable goods

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Bluefin goes for $3 million at 1st 2019 sale at Tokyo market

A 612- pound bluefin tuna sold for a record 333.6 million yen at the first auction of 2019, after Tokyo's famed Tsukiji market was moved to a new site.

https://www.nbcnews.com/news/world/bluefin-goes-3-million-1st-2019-sale-tokyo-market-n955101

Auctions Everywhere

Auction of sports memorabilia or sports players in premier league games

http://www.epathletichalloffame.org/2019-sports-memorabilia-auction-items.html

Credit: IPL

Auctions: Many Ways

- What we think when we think auctions:
  - Interactive and multi-round
  - **Dynamic prices:** prices are determined based on bids
- What are some examples of auctions that occur all the time but do not fit this picture?

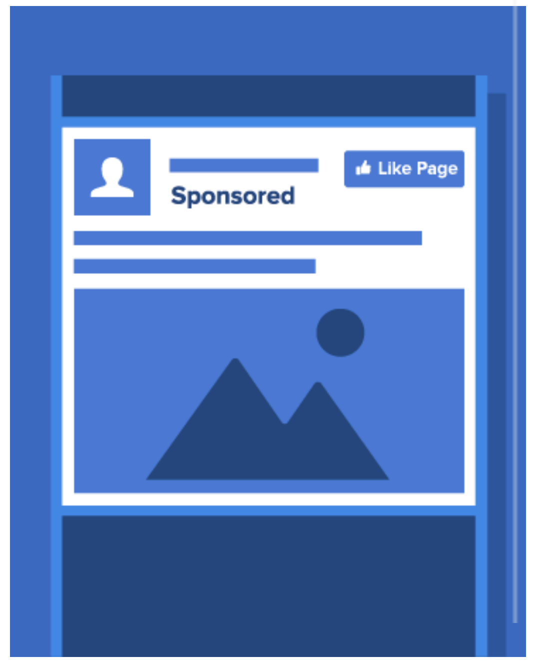
NYSE opening prices

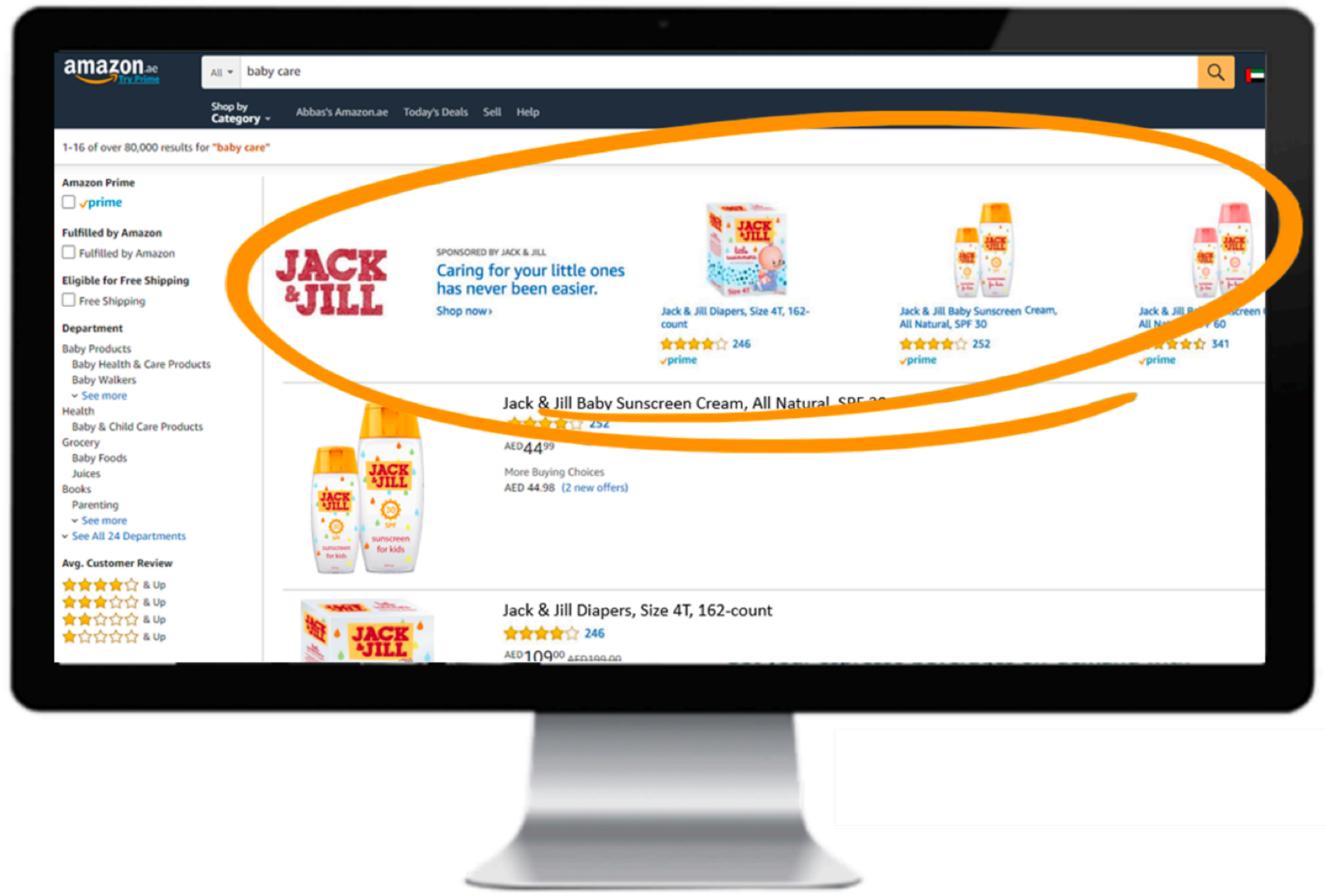
Auctions for used goods

ebay

Sponsored Ads on Amazon/ Google/ Facebook

Google Ads

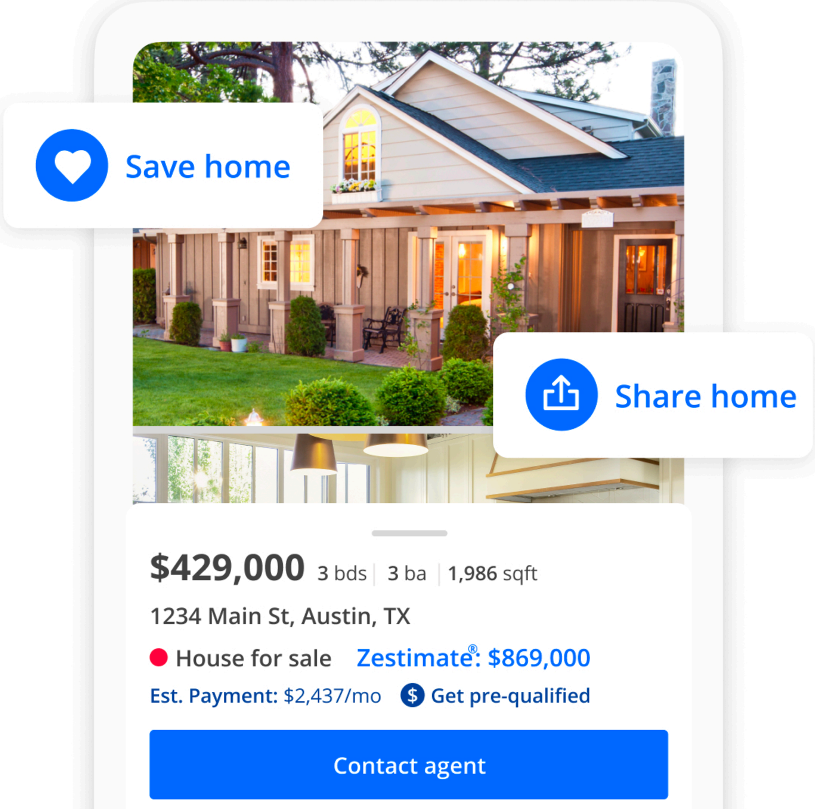




Silent auctions (sealed bid)

http://online.dpsk12.org/2018/11/29/dohs-virtual-silent-

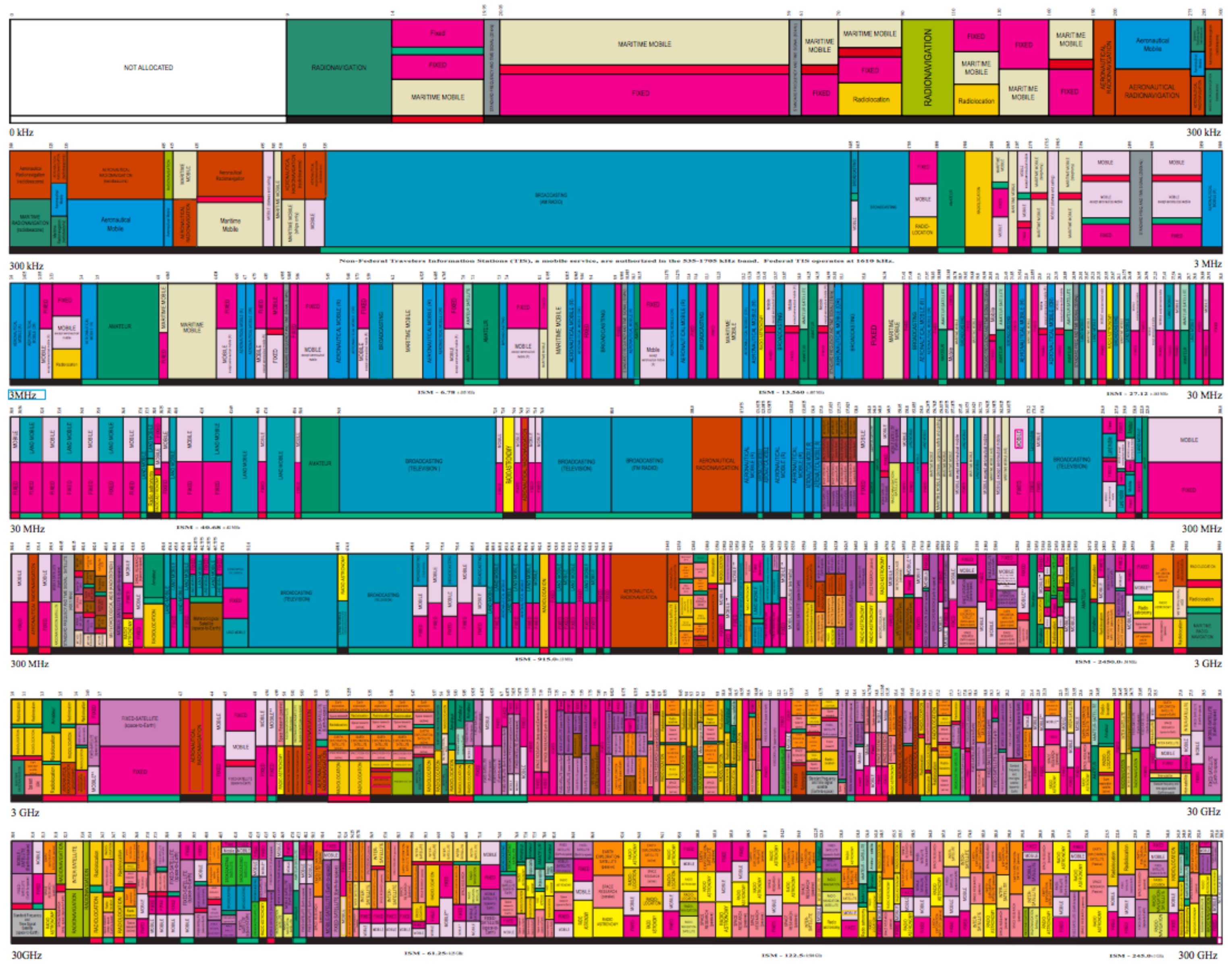
Housing market



Auction-based Marketplaces

- Google and Facebook make most of their revenue through **ad auctions**
    - **Case study.** Sponsored search auctions or keyword auctions in detail in this course
- Governments across the world use auctions to sell wireless spectrum rights
    - **Case study.** FCC wireless spectrum auctions
    - **Goal**: "Economic efficiency/social welfare": allocating resources to companies that need it most
- Auctions **set opening price on the NYSE**

https://www.rightmixmarketing.com/marketing/facebook-google-ad-auction-platforms-are-both-good-but-which-one-is-better/



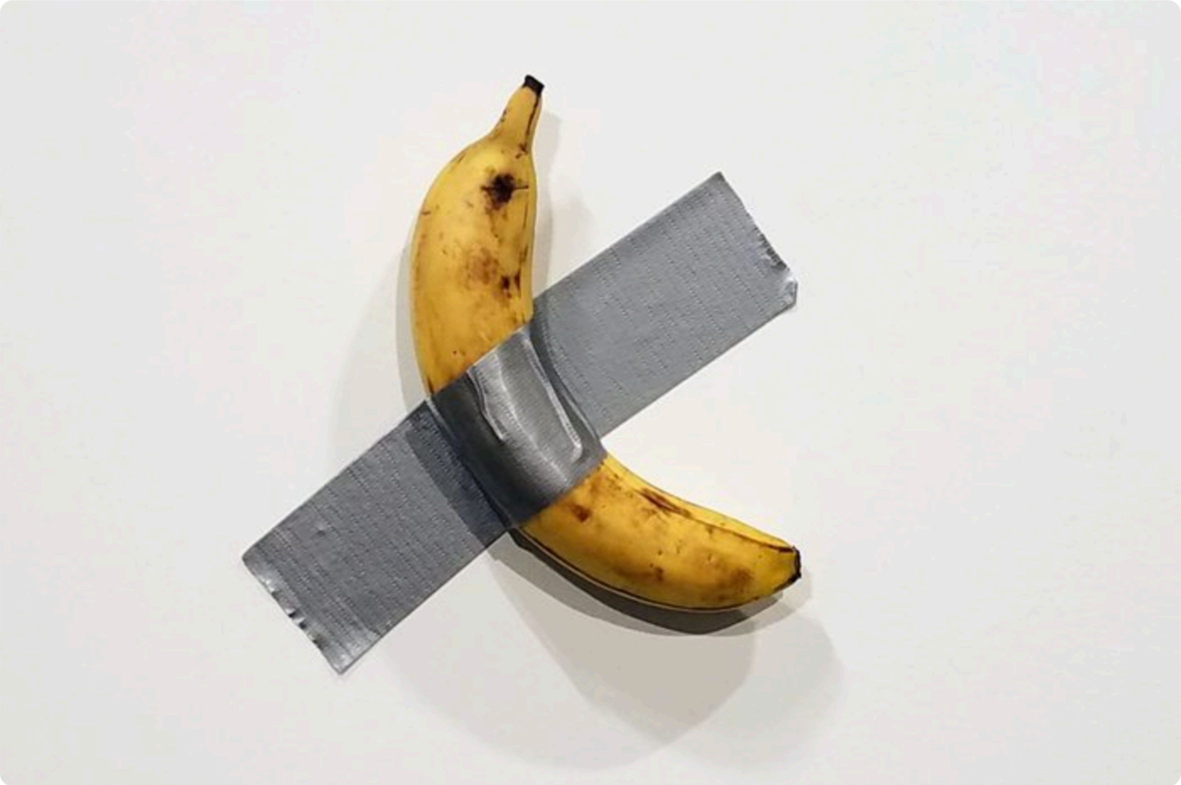
https://priceonomics.com/the-spectrum-auction-how-economists-saved-the-day/

What and Why: Auctions

- A way for sellers to sell goods to a group of buyers
- A way for the market to determine price
  - **Resource allocation** or matching problem (with an additional term we need to compute: prices)
- Mechanisms are auctions even though we do not think of them as such
  - When a seller posts a price on a good, that is a form of auction: called **posted-price** or **"take it or leave it"** auction
- How to determine a good price a priori is not necessarily clear
- Great way to generate revenue for "weird" items

Generate Revenue from Surprising Items

https://www.rd.com/list/quirkiest-auction-items-that-sold-for-millions/



RHONA WISE/EPA-EFE/SHUTTERSTOCK

A banana taped to a wall

It's hard to say what is art anymore. One may think of the *Mona Lisa*, while another might value, say, a banana duct-taped to a wall. We're not being cute. That is literally what someone bought at the Art Basel art fair in Miami recently. Italian artist Maurizio Cattelan's controversial piece (said banana duct-taped to a wall), titled *Comedian*, sold for a whopping $120,000. The

Jack Dorsey's first ever tweet sells for $2.9m

jack
@jack

just setting up my twttr

8:50 PM · Mar 21, 2006

161.2K 129.6K Copy link to Tweet

Owned by @sinaEstavi

https://www.bbc.com/news/business-56492358

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Last Updated: Tuesday, 23 November, 2004, 11:54 GMT

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'Virgin Mary' toast fetches $28,000

A decade-old toasted cheese sandwich said to bear an image of the Virgin Mary has sold on the eBay auction website for $28,000.

An internet casino confirmed it had purchased the sandwich, saying it had become a "part of pop culture".

Goldenpalace.com says it will take the sandwich on world tour before selling it and donating the money to charity.

The toast is not intended for consumption

eBay Auctions

THE TOP 20 WEIRDEST THINGS EVER TO BE SOLD ON EBAY

https://pumpkeen.com/entertainment/comics/top-20-weirdest-things-ever-sold-ebay.html

Mechanism Design

- Auctions are the simplest and most-well studied type of mechanism
- Let us start to think of them as games
- Suppose I have a single item and n buyers

Game Parameters

- To start thinking of them as games, we need to identify our parameters
    - Players? Actions? Rules? Utilities? Outcomes?

What do Buyers Want?

- Suppose each player i has a value v_i: the amount that captures how much they value it/ most they are willing to pay for it
- **Utility:** $u_i = v_i - p$ (if they get the item for price p) and 0 otherwise
- Individual rationality assumption: Buyers want non-negative utility

Rules and Outcome

- How do we think of the rules and outcome of a single-item auction?
    - Who gets the item
    - What do players "pay" for it (if anything)

Quality of Outcome

- From a mechanism designer's point of view, how good is an outcome?
  - Generate/ maximize revenue
  - Generate "social welfare/ surplus" : maximize value generated

Game Information

- The games we studied so far were complete information games: the players knew the utilities other players derived from each outcome
- We need to think about whether this will still be the case....

Getting Started

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Maximize their utility (value minus price)

n buyers, each have a **public** value v_i for the item

Single item

No action from players, give item to buyer with the highest value, no need for prices.

Sealed Bid Auctions

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Maximize their utility (value minus price)

n buyers, each have a **private** value v_i for the item

Single item

Actions: Report "values" (sealed bids). Is this easier or more challenging to analyze?

Sealed-Bid Auctions

- **Step 1.** Collect sealed bids from buyers.
- **Step 2.** Decide who wins **(allocation rule)**
- **Step 3.** Decide what they pay **(payment rule)**

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Maximize their utility.

n buyers, each have a **private** value v_i for the item

Single item

Allocation rule. Who should win: what is a reasonable allocation rule?

Auction: First Attempt

- Collect sealed bids
- Give item to highest bidder for **free**
- Is this good?

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Maximize their utility.

n buyers, each have a **private** value v_i for the item

Single item

Need Prices. Even if goal is not to generate revenue, just social welfare.

Auction: Allocation and Prices

- Suppose we give item to highest bidder
- What should we charge them?

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Maximize their utility.

n buyers, each have a **private** value v_i for the item

Single item

Auction 2. Collect sealed bids, give item to highest bidder and charge them their bid.

First-Price Auction

- **Natural scheme**
  - Collected bids
  - Give item to highest bidder
  - Charge winner their bid, others zero.
- This auction is difficult to reason about... why?
- To drive this point home, we will conduct a **first-price auction**

Class Auction Setup

- Your valuation: sum of the **last four digits** of your Williams student ID **times 0.2 cents**
    - E.g. 3124578 leads to value 4+5+7+8 * 0.20 cents = $4.80
- There will be two auctions:
    - Two-person: you will be paired with a random person
    - Three-person: you will be paired with a random pair
- What I need from you
    - your name and your Williams ID
    - your valuation, and your two bids (one for each auction)
- If you win (highest bid), you get your utility (value - your bid)
- Send your bids at https://tinyurl.com/357auction by **4 pm today**

Second Price Auction

- Suppose we give item to highest bidder
- Charge the winner the **second-highest bid**
- Called second-price or **Vickrey auction**

Designer's Goal: Allocate the item to the buyer **who values it the most.**

Buyer's Goal: Maximize their utility.

n buyers, each have a **private** value v_i for the item

Single item

Vickrey Auction. How good is this auction? How should bidders bid?

Second-Price Auction

- Second-price sealed bid auction
  - Collected sealed bids
  - Sort bids and relabel bidders s.t. $b_1 \geq b_2 \geq \ldots \geq b_1$
  - Allocate item to bidder 1 and charge payment b_2
- Do bidders have an incentive to under/ over bid?

Single-Item Sealed Bid Auction

- Single item, $N = \{1,2\ldots,n\}$ bidders
- Each bidder i has private value $v_i \in \mathbb{R}$ for the item, and submits a bid $b_i \geq 0$
- **Strategy** $s_i : \mathbb{R} \rightarrow \mathbb{R}$ of bidder i defines a bid for **every possible value** v_i the bidder can have (mapping from values to bids)
    - In general, strategy maps information available during play to the action
- Given bid profile $\mathbf{b} = (b_1, \ldots b_n)$:
    - An allocation rule $\mathbf{x}(\mathbf{b}) \in \{0,1\}^n$, indicates whether bidder i receives the item or not, i.e. $x_i(b) = 1$ or 0
    - A payment rule $\mathbf{p}(\mathbf{b}) \in \mathbb{R}^n$, specifies the payment $p_i(b)$ bidder i must make

Quasi-linear Utility

- We already defined this intuitively, here is some notation for it
- Let $u_i(\mathbf{b})$ denote the utility of bidder i given bid profile $\mathbf{b} = (b_1, \ldots, b_n)$
  - Note that a bidders utility depends on its valuation v_i, the allocation rule and the payment rule, we write $u_i(\mathbf{b})$ for simplicity
- **Quasi-linear utility.** Given a bid profile $\mathbf{b}$, the utility of bidder i for the allocation rule $x_i(\mathbf{b}) \in \{0,1\}$ and payment $t_i(\mathbf{b})$ is $u_i(\mathbf{b}) = x_i(\mathbf{b}) \cdot v_i - p_i(\mathbf{b})$
  - If a bidder wins item and pays p then its utility is $v_i - p$
  - If a bidder loses item and pays nothing, its utility is 0
- The goal of the bidders is to maximize their utility

Strategyproofness of SBSP Auction

- A mechanism is **dominant-strategy incentive compatible (DSIC)** or **strategyproof** if truth telling is the dominant strategy for every player.
- **Lemma 1.** In a second-price auction, each bidder has a (weakly) dominant strategy: set its bid b_i equal to its private valuation v_i, that is, this strategy maximizes the utility of bidder i, no matter what other bidders do.
- Proof.
  - (On board; also in book)
- In fact, truth telling is the **unique dominant strategy** in a SBSP auction
  - **Exercise**. Think about how you would prove this!

Individual Rationality

- When designing mechanisms you want to make sure that buyers are willing to participate by ensuring they always get non-negative utility
- We can show that no truth-telling bidder will regret participating in a second-price auction
- **Lemma 2.** In a second-price auction, every truth-telling bidder is guaranteed non-negative utility.
- Proof. Fix an arbitrary bidder i,
  - If i loses: utility is zero
  - If i winds, utility is $v_i - p$: since $b_i = v_i$ and $p \leq b_i$ (p is second-highest bid), thus $v_i - p \geq 0$

Surplus Max & Linear Time

- Strategyproofness/DSIC alone is not always great
- Can you give an example of a stupid auction that is DSIC?
    - Giving the item away for free to a random bidder is DSIC
- Vickrey auction maximizes surplus:
    - Gives the item to the bidder with the highest valuation (at the unique DSE)
- Solves the surplus-maximization optimization problem as well as if **the valuations where known in advance**!
- **Linear time.** All the auction needs to do is compute maximum and second maximum from a list of bids, and thus is linear-time

Auction Design Goals

When designing auctions, ideally, we want the following properties

- **Strong incentive guarantees**
    - Truthful reporting is a dominant strategy equilibrium (strategyproof)
    - Truth-telling guarantees non-negative utility (individually rational)
- **Strong performance guarantees**
    - Maximizes social surplus $\sum_{i=1}^{n} v_i x_i$, where $x_i = 1$ if i wins and 0 otherwise; and $\sum_{i=1}^{n} x_i = 1$ (single item case)
- **Computational efficiency**:
    - the auction can be implemented in polynomial time

We will also talk about revenue maximization later

Questions: Design Choices

- We have established that sealed bid second-price auctions are awesome
- But what about the other design choices?
    - Does it ever make sense to give the item to "not" the highest bidder?
- How good/bad are other payment rules?
- Are multi round auctions **inherently "richer"** than sealed bid ones?

HW Questions

- **Question 2.** Show that charging the highest bidder the third-highest bid is not DSIC.
- **Question 3**. Show that sealed-bid second price auctions are susceptible to collusion: give **necessary** and **sufficient** conditions
  - Even though for a single player truth telling is dominant
  - For a group, they can cheat and get better **total utility**

HW 2 Question 4

Auctions

Sealed-bid (Simultaneous-move auctions)

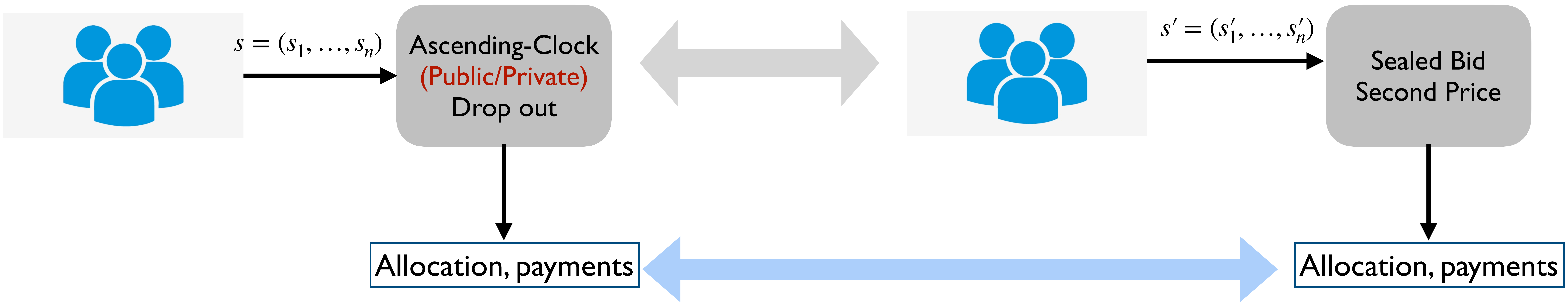
https://www.rightmixmarketing.com/marketing/facebook-google-ad-auction-platforms-are-both-good-but-which-one-is-better/

Multi-round open-outcry style auctions where bidders respond to other bids

- Ascending
- Descending, etc

HW 2 Question 4

- **Problem 2.** Strategic equivalence definition
- Not about what bidders "should do" (rationality), but rather what they "can do" in each auction
  - Important distinction in AGT



Generalizing Second-Price Auctions

Beyond Single Item

- What are some challenges of generalizing to multiple items?
- What do we need from the bidders?

Multiple items

n buyer with private valuations

Single Parameter Settings

- If we consider a set of items S and agents having a different valuation for any subset $A \subseteq S$, then it is called a **combinatorial auction**
    - More challenging setting, will discuss later
- First, we study a generalization to the single-item setting, which nonetheless covers many applications, e.g. sponsored search auctions
- **Single parameter setting**: Valuation for whatever allocation a bidder receives can be captured by a single number
    - E.g., buyer i has value v_i for a certain subset $S_i \subseteq S$, 0 other others
    - E.g., buyer i has a value v_i for every click and we have ad slots with different click-through-rates

Single Parameter Settings

- Single parameter settings are more general than auctions
- For example, deciding whether or not to build a public project that can be used by everyone can be modeled by the allocation $X = \{(0,\ldots,0),(1,\ldots,1)\}$
- Auctions are a special case of general mechanisms
- Auctions involve transfer of goods and money but this is not necessary for the results we will study

auction	mechanism
bidder	agent
bid	report
valuation	valuation

Table 3.1: Correspondence of terms in auctions and mechanisms. An auction is the special case of a mechanism that is designed for the exchange of goods and money.

Example: k identical goods

- Simple example of single-parameter setting: we have k copies on an item
- Suppose we want a DSIC auction to maximize **social surplus**:

$$\sum_{i=1}^{n} v_i x_i, \text{ where } x_i = 1 \text{ if } i \text{ gets an item and } 0 \text{ otherwise; and } \sum_{i=1}^{n} x_i \leq k$$

n buyers, each has private value v_i for a single copy of the item

k identical items

Our Design Approach

- Challenge of mechanism design (with money): jointly design two pieces: who gets what, and how much do they pay
    - Not enough to figure out who wins, if don't charge them the right amount, strategic agents will game the mechanism
- Usually, the recipe we will follow:
- **Step 1.** Assume truthful bids, and decide how to allocate so as to maximize surplus (in polynomial time)
- **Step 2.** Using the allocation in step 1, decide how to charge payments so as that the mechanism is strategyproof (DSIC)

k identical goods: Allocation

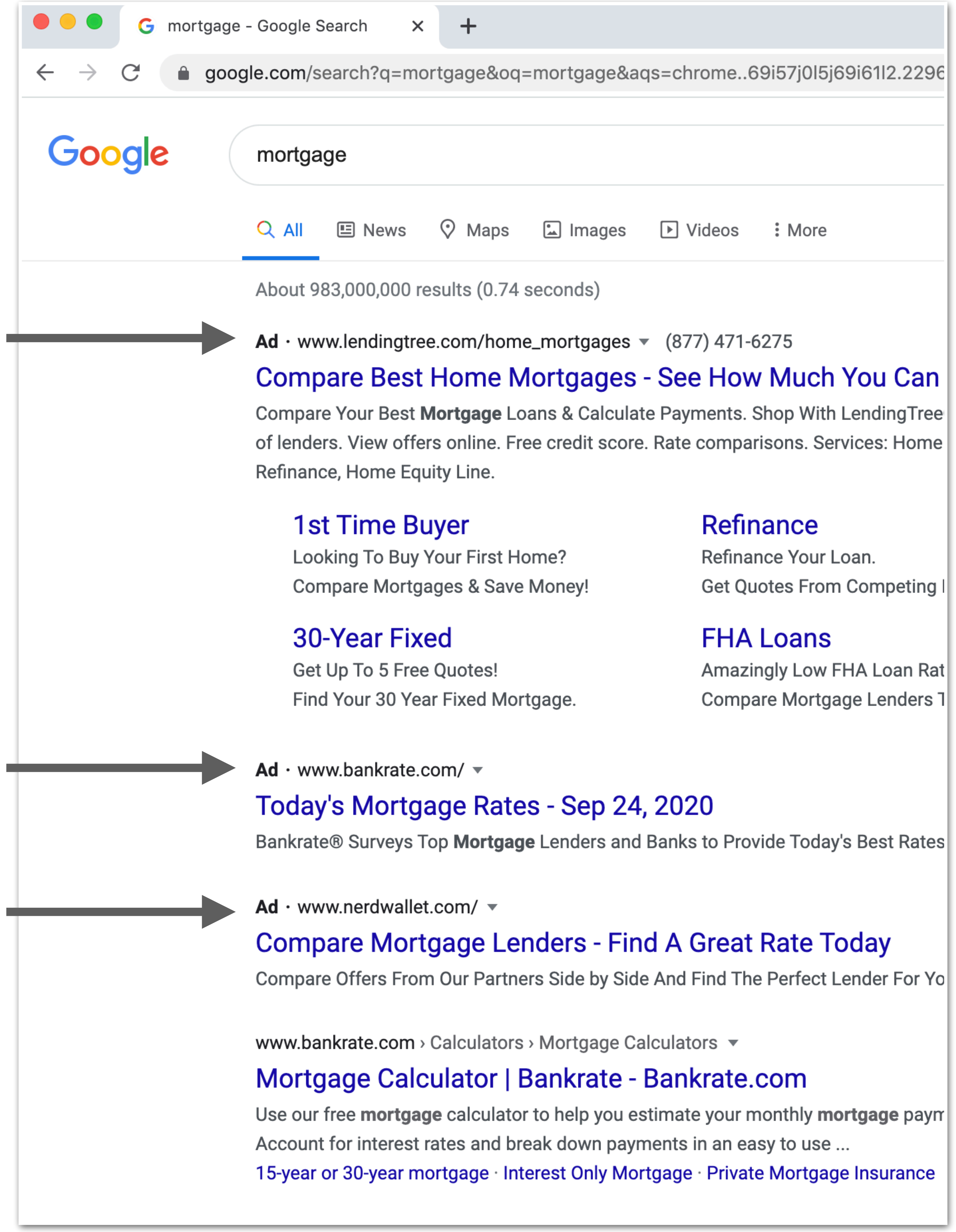
- Collect sealed bids
- Who should we give the k items to?
  - Top k bidders
- **Question**. What should we change them so that truth telling is dominant strategy?

n buyers, each has private value v_i for a single copy of the item

k identical items

Sponsored Search

- Sponsored ads appear on almost all web platforms
  - Facebook, Google, Amazon, etc
- Every time someone searches a query, an auction **is run in real time** to decide: which advertisers links are shown, in what order, and how they are charged
- Extremely impactful to the internet economy: **around 80% of Google's revenue** is through sponsored ads
- We look at a simplified but effective model to study sponsored search auction



Sponsored Search Model [Edelman & Varian]

- Items for sale are k slots for sponsored links on a page
- Bidders (advertisers) have a standing bid on a keyword that was searched on
- Slots higher up on the page are more valuable than lower
  - Quantified through **click-through-rates (CTRs)**
  - CTR α_j of a slot j is the probability of clicks that slot is expected to receive
  - Reasonable to assume $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$
- **Simplifying assumption.** CTR of a slot is independent of its occupant, that is, doesn't depend on the quality of the ad
- We assume advertisers have a private valuation v_i for each click on its link: value derived from slot j by advertiser i is $v_i \cdot \alpha_j$

Sponsored Search: Model

- Given an assignment of bidders to slots, such that each slot is assigned to at most one bidder and each bidder is assigned at most one slot, a feasible allocation is $X = (x_1, x_2, \ldots, x_n)$
  - where $x_i = \alpha_j$, the click through of slot j if bidder i is assigned to it; otherwise $x_i = 0$ if bidder is unassigned
- **Question.** Is there an awesome auction for sponsored search?
- What we want:
  - Dominant-strategy incentive compatible
  - Surplus maximization: allocation maximizes $\sum_{i=1}^{n} v_i x_i$
  - Polynomial time (tons of these auctions need to run every day!)

Sponsored Search: Allocation

- **Question.** How do we do we assign slots to maximize surplus?
    - Greedy allocation is optimal (can be showed by an exchange argument)
    - Recall that CTR rates $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$
    - Sort and relabel bids $b_1 \geq b_2 \geq \cdots \geq b_n$
    - Assign jth highest bidder to jth highest slot
- Can we create a payment rule (an analog of second-price rule) that makes the greedy allocation incentive compatible?
    - What is the analog of the second-price auction here?

Towards a General Characterization

- **Question.** Can any allocation rule be paired with a payment rule such that the mechanism is strategyproof (truthtelling is a dominant strategy)?
    - When is this possible and how should we design the payment rule?
- Myerson's lemma gives a **general characterization** of allocation rules that can be turned into a truthful (DSIC) mechanism
    - We can use it to create payment rules for both k item and sponsored search auctions!

Myerson's Lemma: Informal

- In a fixed-parameter setting,
    - an allocation rule $\mathbf{x}$ can be made dominant-strategy incentive compatible if and only if $\mathbf{x}$ is monotone (non decreasing), and
    - if $\mathbf{x}$ is monotone, there is a unique payment rule $\mathbf{p}$ such that $(\mathbf{x}, \mathbf{p})$ is DSIC.
- Question of whether there exists a payment that makes an allocation DSIC (a difficult to answer question) reduced to a question of whether a rule is "monotone" : a computation/ operational question