CS 357: Algorithmic Game Theory **Spring 2022**

Assignment 1 (due 02/10/2022)

Instructor: Shikha Singh *Solution Template*

Instructions. Use the Overleaf LaTeX template linked above to typeset the assignment.[1] If you go to the Overleaf dashboard and view the template, you will find an option to make a local copy that you can then edit.

You must submit this assignment on Gradescope (course code KYERN3). When submitting, you must assign a page to each question (each problem starts on a new page in the template to make this process easy). This step is important for anonymous grading.

Points will be awarded for clarity, correctness and completeness of the answers. Reasoning must be provided with every answer, i.e., please show your work. All assignments are due **at 10 pm EST** on the day of the deadline.

Readings. Before starting the assignment, review the definitions of normal-form games, dominant strategy equilibrium, and pure-strategy Nash equilibrium. Easley & Kleinberg Ch 6 is a great introduction to these concepts; Parkes & Seuken Ch 2 has more formal definitions (similar to those covered in lecture).

Normal-Form Games

Problem 1. (Forming a Game) Suppose you have to prepare a piece to perform at your music club. You only care about impressing a friend, who may or may not attend. You must decide whether to practice the piece (which is costly), or just "wing it" which is easy but may lead to embarrassment if your friend attends.

In particular, if you practice the piece and your friend attends the event, you are happy (utility 5), while if your friend does not attend, you feel like your effort was wasted (utility -5). If you do not practice, and your friend attends, you feel embarrassed (utility -10). If you do not practice and your friend skips, you don't care (utility 0).

On your friend's side, if they show up and you perform a well-practiced piece, they enjoy it (utility 10), otherwise, they feel like they wasted their time (utility -5). If your friend does not attend the event, their utility is 0 regardless of how you perform.

(a) Formalize the normal-form game by drawing the payoff matrix.

(b) Do either player have a dominant strategy? Explain.

(c) Find all pure-strategy Nash equilibria of the game.

[1] If you are new to LaTeX, see the information and links on the Resources page of the CS357 website.

(d) Which of the pure-strategy Nash equilibria from part (c) are Pareto-optimal among pure strategy profiles?

Problem 2. (Machine Scheduling: Parkes & Seuken) In a load balancing games, we have jobs that need to get executed and machines that can process jobs. The jobs here are players and can choose which machine to run on. Consider such a game with two identical machines and four jobs. Each job i has size $s_i > 0$, representing the length of the task. The sizes are $s_1 = s_2 = 2$ and $s_3 = s_4 = 1$, so that there are two large tasks and two small tasks. Each machine's speed is 1 unit per second. The completion time of a job i assigned to machine j is equal to the time when machine j completes all jobs assigned to it (that is, all jobs assigned to a particular machine finish at the same time)

Each job selfishly selects a machine to minimize its completion time.

(a) Identify two pure-strategy Nash equilibrium of this game.

(b) What is the socially optimal assignment; that is, the assignment of jobs to machine that minimizes the maximum completion time across both machines (this is termed the "makespan" in scheduling algorithms).

Iterated Elimination of Dominated Strategies

Problem 3. In class we considered one way of solving a game: by iterated elimination of *weakly* dominated strategies. We can also consider iterated elimination of *strictly* dominated strategies. An action $a_i \in A_i$ is **strictly dominated** if $u_i(a_i, a_{-i}) < u_i(a'_i, a_{-i})$ for some $a'_i \in A$ and for all $a_{-i} \in A_{-i}$.

Algorithm 1 describes the method where the input is a set of n action sets $A_1, \ldots, A_n$ and a set of a n utility functions $u_1, \ldots, u_n$, where each $u_i : A_1 \times \cdots A_n \to \mathbb{R}$

Algorithm 1: Iterated Elimination of Strictly Dominated Strategies

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Input: $A_1, \ldots, A_n, u_1, \ldots, u_n$

Let S_i denote the set of undominated actions for agents i

For each player i, initialize $S_i = A_i$

Initialize DominatedActionFound = True

while DominatedActionFound **do**

    if *there exists some agent i, some action $a_i \in S_i$ and some action $a'_i \in S_i$ such that $u_i(a_i, a_{-i}) < u_i(a'_i, a_{-i})$ for all $a_{-i} \in S_{-i}$* **then**

        Remove action a_i from S_i

    else

        DominatedActionFound = False

    end

end

return $S_1, \ldots, S_n$

(a) Which strategy profiles survive the iterated elimination of strictly dominated strategies in the following normal-form game?

	L	C	M
T	$0,6$	$3,4$	$1,5$
M	$2,1$	$3,3$	$2,4$
B	$2,3$	$5,1$	$3,4$

(b) Give an example of a game where no action can be eliminated.

(c) Let m be the maximum number of actions available to any player, and n, the number of players. What is the worst-case run time of iterated elimination of strictly dominated strategies? Give the best possible Big Oh bound in terms of n and m. Assume that the utility comparison of two distinct action profiles (a_i, a_{-i}) and (a'_i, a_{-i}) can be done in constant time.

Problem 4. (a) Prove that if only a single strategy profile s survives iterated elimination of *weakly* dominated strategies, then s is a pure strategy Nash equilibrium of the game.

(b) Prove that if only a single strategy profile s survives iterated elimination of *strictly* dominated strategies, then s is the unique pure strategy Nash equilibrium of the game.

Problem 5. Read Maskin's (brief) commentary on *Nash equilibrium and mechanism design* (paper linked here and available in `GLOW->Files->papers`). Provide very brief (couple of sentences) answers to the following questions.

(a) Why does Maskin think that Nash equilibrium has been a successful solution concept in game theory?

(b) What are some of the shortcomings of Nash equilibrium highlighted by Maskin?

(c) Why are these shortcomings less problematic in the context of mechanism design?

Extra Credit

Problem 6 (Osborne and Rubinstein)**.** Two players are involved in a dispute over an object. The value of the object to player i is $v_i > 0$. Time is modeled as a continuous variable that starts at 0 and runs indefinitely. Each player chooses when to concede the object to the other player; if the first player to concede does so at time t, the other player obtains the object at that time. If both players concede simultaneously, the object is split equally between them, player i receiving a payoff of $v_i/2$. Time is valuable: until the first concession each player loses one unit of payoff per unit of time.

Formulate this situation as a strategic game and show that in all Nash equilibria one of the players concedes immediately.

Acknowledgment. Remember to cite your collaborators and resources in the acknowledgment section of the solution template.