Reduction Proofs

Halt-no-Input

A <u>reduction proof</u> is a style of proof common in computer science. It is often used to show the equivalence (or non-equivalence) of two classes of algorithms. Do not confuse reduction-style proofs with the "reduction" steps in a lambda calculus proof; these are different ideas.

A form of a reduction proof is an algorithm: we construct a function called a <u>reducer</u>. So keep mind that when you are asked produce a reduction-style proof, you're being asked to write code. You write the code for the reducer function.

When employed to prove undecidability (i.e., that a problem has no solution), the "direction" of the reduction is counterintuitive. Let B be the problem you intend to prove something about. Let A be a problem you already know something about (e.g., the Halting Problem). We then attempt to construct a reducer that converts problems of type A into problems of type B and <u>assume</u> the existence of an algorithm that solves instances of type B.

If we are successful in constructing a reducer (using ordinary code), the existence of the reducer demonstrates a contradiction: we already know that solving problems of type A is impossible. We have therefore proved, by <u>reductio ad absurdum</u>, that we could solve problems of type B; that we could solve B was the only fact we assumed to be true.

Prove that the Halt-no-Input problem is undecidable.

Halt-no-Input problem: given a program P that requires no input, does P halt?

Form of Expected Answer. Your answer should be in the form of a reducer (an algorithm). Write your answer (code) in the language of your choice.

Hint. Be careful to pay attention to the direction of the reduction. Halt should make an appearance in your proof. On which side of the reduction does it appear?