

Partial and Total Functions: Solutions

For each of the following function definitions, you were asked to give the graph of the function and say whether it was a partial function or a total function on the integers. If the function was partial, you were asked to say where the function was defined.

1. $f(x) = \text{if } x+2>3 \text{ then } x*5 \text{ else } x/0$

The graph of f is

$$\{\langle x, x \times 5 \rangle \mid x > 1 \wedge x \in \mathbb{Z}\}$$

This is a partial function. It is defined on all integers greater than 1 and undefined on integers less than or equal to 1.

2. $f(x) = \text{if } x < 0 \text{ then } 1 \text{ else } f(x-2)$

The graph of f is

$$\{\langle x, 1 \rangle \mid x \in \mathbb{Z}\}$$

This is a total function.

3. $f(x) = \text{if } x=0 \text{ then } 1 \text{ else } f(x-2)$

The graph of f is

$$\{\langle x, 1 \rangle \mid x \geq 0 \wedge x \text{ is even}\}$$

This is a partial function. It is defined on all positive even integers and undefined on all integers that are negative or odd.