Midterm Study Guide

━━━━━ Reading ━━━━━

1. "Proof by Reduction"

2. "How to Fix a Motorcycle"

3. "Parsing"

4. "Evaluation"

━━━━━ Problems ━━━━━

Q1. (0 points) .... Terminology

Review the required readings for this assignment, specifically looking for definitions for new terminology. Write down the term and a brief definition.

Hint: Look for italicized words. Italics are sometimes used for emphasis, but it is also often used to draw your attention to new technical terms. For example, in "A Slightly Longer Introduction to F#", one of the first italicized phrases is "architecture independent." So write something like:

Architecture independence: the compiled version of a program can be run on any computer.

Find as many technical terms as you can and define them.

Q2. (10 points) .... Decision Problem

The `bothHalt` function has the following form,

```
bothHalt(f: int -> int)(g: int -> int)(i: int)
```

and it has the following behavior,

$$\texttt{bothHalt} = \begin{cases} \texttt{true} & \text{if both } \texttt{f(i)} \text{ and } \texttt{g(i)} \text{ halt.} \\ \texttt{false} & \text{if either } \texttt{f(i)} \text{ or } \texttt{g(i)} \text{ do not halt.} \end{cases}$$

Prove that `bothHalt` is not computable.

Q3. (10 points) .... Parsing and Evaluation

The EPHSPRESSO language is a sequence of operations on a tape having the following grammar.

```
<expr> ::= <op>*
  <op> ::= + | - | < | > | p
```

A *tape* is an array of length 10, with all elements initially set to zero. A *tape head* is the current index of the tape, also initially zero. The + and - operations increment and decrement the number stored under the tape head, respectively. The < and > operations move the tape head left (decrement the index) or right (increment the index), respectively. The p operation prints a character representation of the number written on the tape under the tape head according to the following scheme:

$$0 \to \mathtt{a}$$

$$1 \to \mathtt{b}$$

$$2 \to \mathtt{c}$$

$$\dots$$

$$25 \to \mathtt{z}$$

Moving the tape head off the tape causes the machine to terminate abnormally (i.e., with an error). Otherwise, the machine terminates normally when the program finishes executing.

Implement EPHSPRESSO in F#. You should be able to write a program that prints `ephs` to the console.

Q4. (10 points) .... Partial and Total Functions

Find the function graphs for the following functions and state whether the function is total or partial. Use the domain $\mathbb{Z}$ as input. Assume that an `int` can represent any $z \in \mathbb{Z}$.

(a)
```
let rec fibonacci n =
    match n with
    | 0 -> 0
    | 1 -> 1
    | _ -> fibonacci (n - 1) + fibonacci (n - 2)
```

(b)
```
let rec gcd a b =
    if b = 0 then
      a
    else
      gcd b (a % b)
```

(c)
```
let abs x =
    if x < 0 then
      -x
    else
      x
```