Handout 10 February 27, 2024

In this example, I always choose the *normal order* reduction. Normal order is always "outermost leftmost" in the AST. The <a href="highlight">highlight</a> identifies the parts of the AST reduced in the next step.

expression	AST	derivation rule
$(\lambda f.\lambda x. f(fx))(\lambda z. (+xz))2$	@ /\ @ 2 /\ \lambda \l	given
	@ / \	
$(\lambda f.\lambda a.f(fa))(\lambda z.(+xz))2$	β 2  λ λ  f λ z +  Λ Λ Λ  a @ x z  f @  f a	$\alpha$ reduce a for x to avoid "capturing" the free x
	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
$(\lambda a.(\lambda z.(+xz))((\lambda z.(+xz))a))2$	X Z	$\beta$ reduce $(\lambda z.(+xz))$ for f

Handout 10 February 27, 2024

expression	AST	derivation rule
	$ \begin{array}{c cccc} \lambda & & & & & \\ \lambda & & & & & \\ \hline \lambda & & & & & \\ Z & + & & & & \\ X & Z & Z & + & \\ \end{array} $	
$((\lambda z.(+xz))((\lambda z.(+xz))2))$	/ \ x z	$\beta$ reduce 2 for a
$\frac{((\lambda z.(+xz))((\lambda z.(+xz))2))}{(\lambda z.(+xz))((\lambda z.(+xz))2)}$	Same tree.	eliminate parens
$((+x((\lambda z.(+xz))2)))$	+ /\ x	$\beta$ reduce $((\lambda z.(+xz))2)$ for z
$(+x((\lambda z.(+xz))2))$	Same tree.	Eliminate parens.
(+x(((+x2))))	+ /\ x + /\ x 2	$\beta$ reduce 2 for z
$\frac{(+x(((+x2))))}{(+x(+x2))}$	Same tree.	Eliminate parens. Done.
( ( – / /	1	r