Lab 4

Due Monday, March 4 by 11:59pm

Handout 8
CSCI 334: Spring 2024

Coding Guidelines

Each question in this assignment must be written using LaTeX. I provide a LaTeX template in your repository for you to use to get started.

Keep in mind that LaTeX is a programming language. The template I provide compiles without error as-is. Treat this homework as you would with any other programming language: make small changes and compile frequently using `pdflatex`. For full credit, you must submit both your `.tex` source file as well as the rendered `.pdf` file. (5 points) Your source file should be called `lab-4.tex` and your PDF should be called `lab-4.pdf`. (5 points) Your source file should compile without error. (5 points)

Turn-In Instructions

Turn in your work using the `git` repository assigned to you. The name of the `git` repository will have the form `https://aslan.barowy.net/cs334-s24/cs334-lab04-<USERNAME>.git`. For example, if your CS username is `abc1`, the repository would be `https://aslan.barowy.net/cs334-s24/cs334-lab04-abc1.git`.

You should have received an invite to commit to the repository via email. If you did not receive an email, please contact me right away!

Single-Author Programming Assignment

This is a solo lab. You may work with another classmate to understand what the problems ask, but you are not permitted to develop solutions together. Submitted solutions must be exclusively your own. Please refer to the section "single author programming assignments" in the honor code handout for additional information. You do not need to submit a `collaborators.txt` file for this assignment. You are always welcome to ask me for clarification if the above is unclear in some circumstance.

This assignment is due on Monday, March 4 by 11:59pm.

Reading

1. **(Required)** "Introduction to the Lambda Calculus, Part 2"

Problems

Q1. (30 points) ..... Lambda Calculus Reduction

(a) Reduce the following lambda expression.

$$(\lambda x.\lambda y.xy)(\lambda x.xy)$$

Begin by performing alpha reduction. Do all possible reductions to find the normal form. Your reduction should be in the two-column format shown in the course packet.

(b) How do you know that your final expression is a normal form?

(c) What goes wrong if you do not rename bound variables? Perform a second reduction (again in two-column format) that shows a mistake that can happen when you fail to perform alpha reduction.

Q2. (30 points) ..... Lambda Calculus Reduction

Reduce the following lambda expression.

$$(\lambda x.mx)((\lambda x.\lambda y.\lambda z.x)ohn)o$$

Q3. (25 points) ..... Church Numerals

Church encoding is a means of representing data and operators in the lambda calculus. The data and operators form a mathematical structure which is embedded in the lambda calculus. The Church numerals are a representation of the natural numbers using lambda notation. The method is named for Alonzo Church, who first encoded data in the lambda calculus this way.

The natural numbers are written using Church numerals as follows.

Number	Lambda Expression
0	$\lambda f.\lambda x.x$
1	$\lambda f.\lambda x.fx$
2	$\lambda f.\lambda x.f(fx)$
3	$\lambda f.\lambda x.f(f(fx))$
...	...
n	$\lambda f.\lambda x.f^n x$

Addition by one can be achieved using the successor function, defined as

$$\text{succ} \equiv \lambda n.\lambda f.\lambda x.f(nfx)$$

Prove that $0 + 1 = 1$.

Q4. ($\frac{1}{10}^{\text{th}}$ bonus point) ..... Optional: Feedback

I always appreciate hearing back about how easy or difficult an assignment is.

For $\frac{1}{10}^{\text{th}}$ of a bonus to your final grade, please fill out the following Google Form.