CSCI 334:
Principles of Programming Languages

Lecture 10: Computability, part 2

Instructor: Dan Barowy

Williams

Topics

Garbage collection

Halting problem

Reduction proofs

Your to-dos

1. Lab 5, **due Sunday 10/15** (partner lab)
2. As a **part of lab 5**, read *How to Fix a Motorcycle.*

Announcements

- **Midterm exam**, in class, Thursday, Oct 19.
- **Field trip to WCMA**, Thursday, Nov 2.
- Colloquium: **What I Did Last Summer (Research Edition)**, 2:35pm in Wege Auditorium.

Announcements

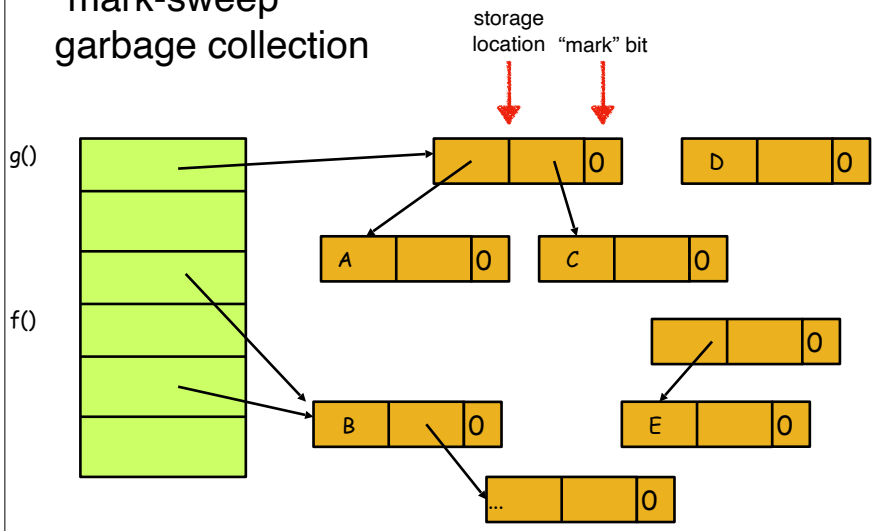
- **TA Applications** due Friday, Oct 27.
- **TA Evaluation** forms due Friday, Oct 27.
-

Garbage collection

A **garbage collection algorithm** is an algorithm that determines whether the storage, occupied by a value used in a program, **can be reclaimed for future use**. Garbage collection algorithms are often tightly integrated into a programming language **runtime**.

John McCarthy

"mark-sweep" garbage collection



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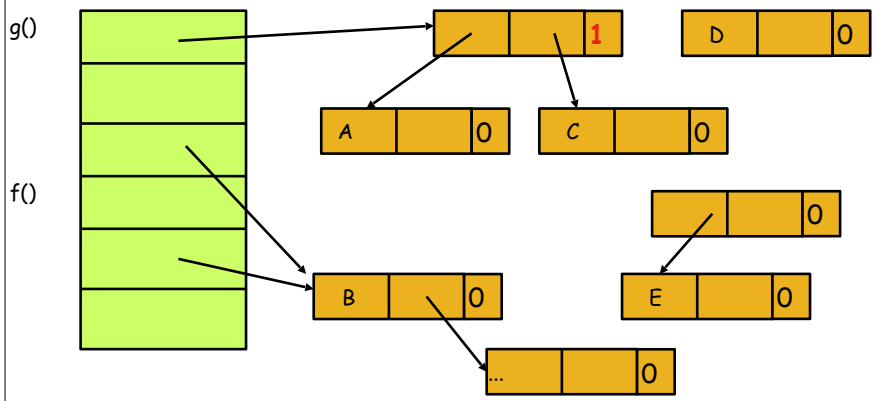

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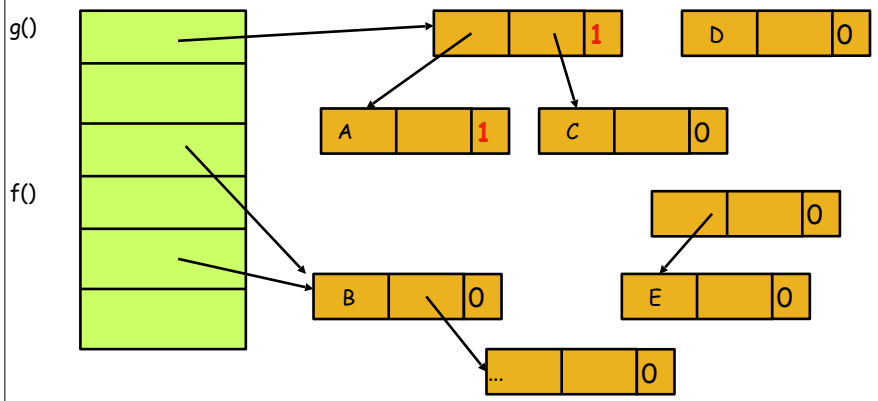

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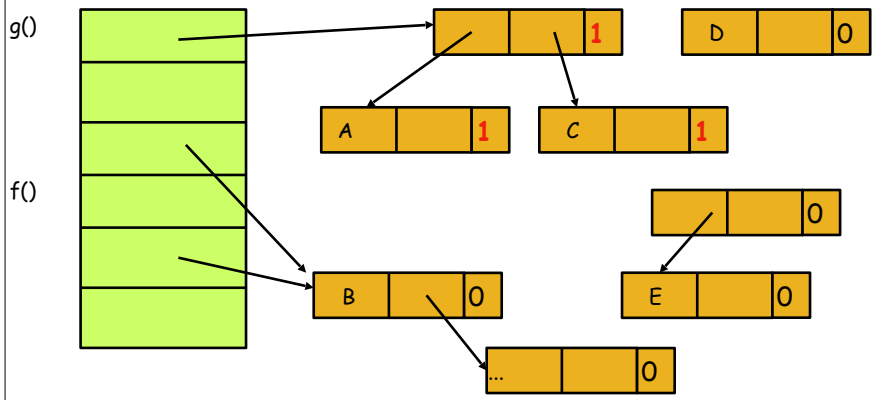

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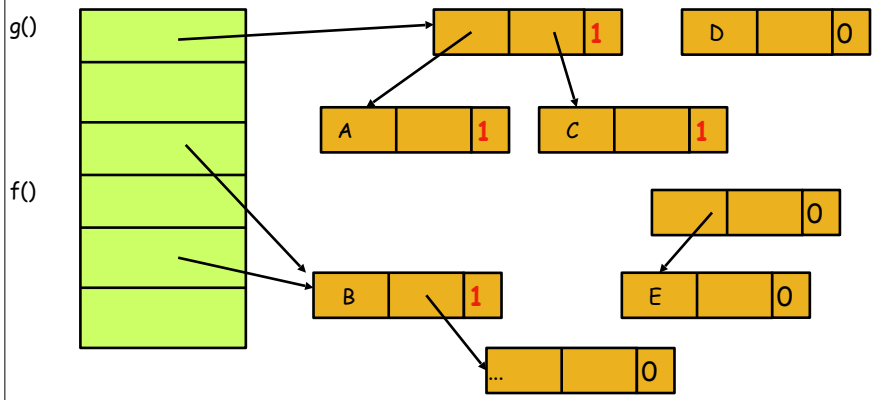

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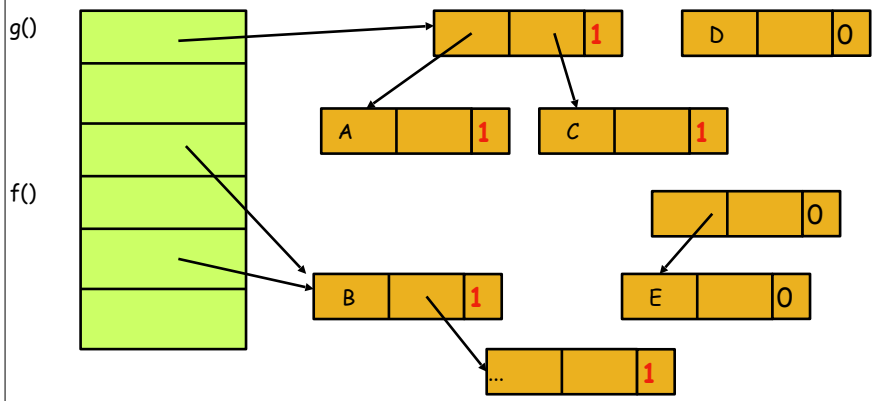
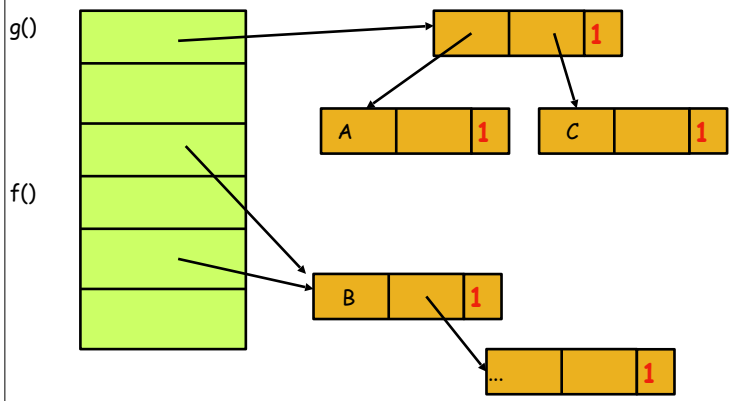
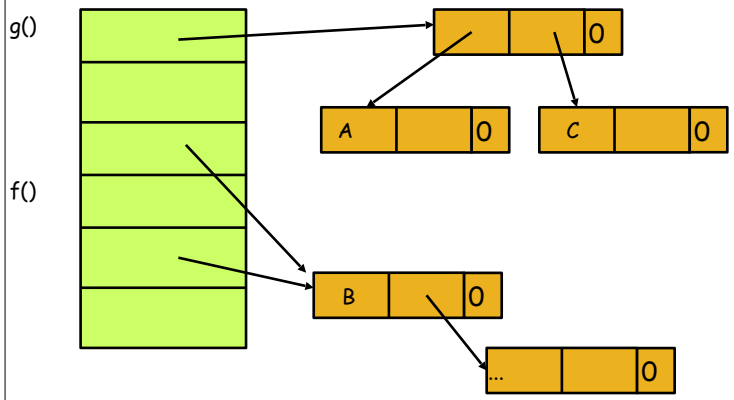


2. Free ("sweep") unreachable cells



3. Clear tags



The Halting Problem: Proof

Suppose:

`Halt(P,x)` = { returns `true` if `P(x)` halts; returns `false` otherwise } — `Halt` always halts!

Construct:

`DNH(P)` = { if `Halt(P,P)` is `true`, `while(1){}`; returns `false` otherwise }

`DNH` does not always halt!

The Halting Problem

Isn't `DNH` itself a program?

What happens if we call `DNH(DNH)`?

`P = DNH`

`DNH( P )` will run forever if `P ( P )` halts.
`DNH( P )` will halt if `P ( P )` runs forever.

The Halting Problem

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`DNH(DNH)` will run forever if `DNH(DNH)` halts.
`DNH(DNH)` will halt if `DNH(DNH)` runs forever.

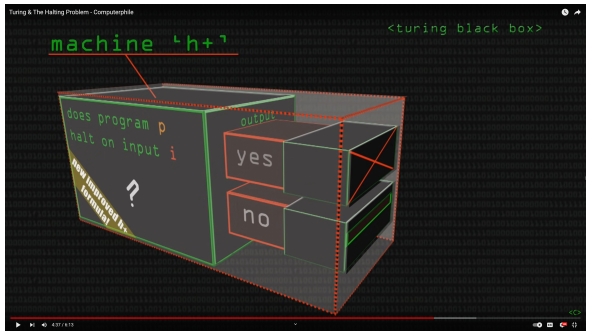
This literally makes no sense. **Contradiction!**

What was our one assumption? `Halt` **exists**.

Therefore, the `Halt` function **cannot exist**.

Need more explanation?

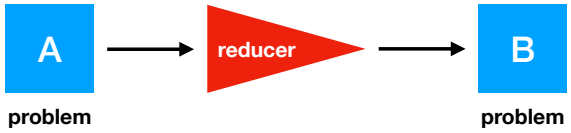
Watch this!



https://youtu.be/macM_MtS_w4

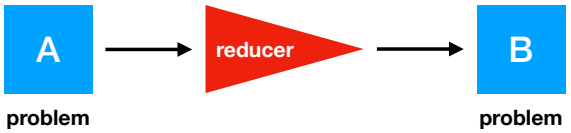
Reductions

A **reduction** is an **algorithm** that transforms an instance of one problem into an instance of another. Reductions are often **employed to prove something** about a problem given a similar problem.



Reductions

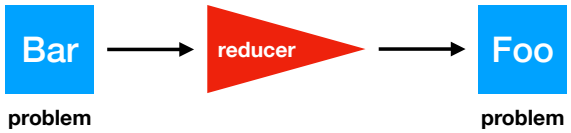
In this class, we will focus on proving things about **impossibility**, but reductions are much more general. In other cases, we prove things about **complexity**.



Reductions

Reductions are often used in a **counterintuitive** way.

For example, if we **want to know whether problem Foo is impossible**, we assume Foo is possible, and then use that fact to show that problem Bar (which **we already know** to be impossible) **appears to be possible**.

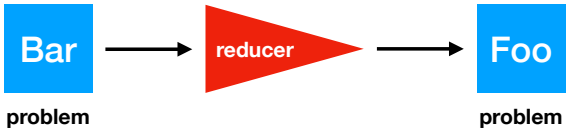


The above is a **contradiction**, meaning that **Foo is not possible**.

Reductions

Observe the **direction** of the reduction.

For computability proofs, we reduce the **problem we already know something about** to the **problem we want to understand**.

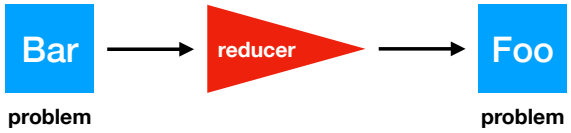


If the reduction is possible, it means that Foo is impossible **because we already know that Bar is impossible**.

Reductions

This direction **seems backward** to most people.

However, this counterintuitive direction is a feature common to many reductions.

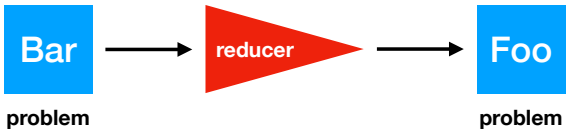


See the reading *Proof by Reduction* to understand the logic in more detail.

Reductions

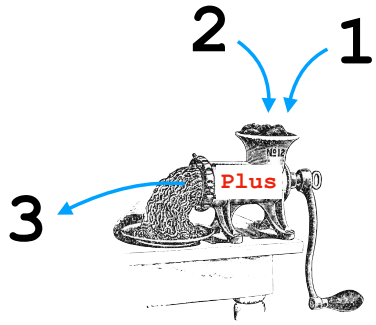
An important part of a reduction is that the reducer be an **ordinary algorithm**.

The reducer **should not solve the problem**. A reducer just converts problems from one form to another.



You will get **a lot** more exposure to reductions in CSCI 361.

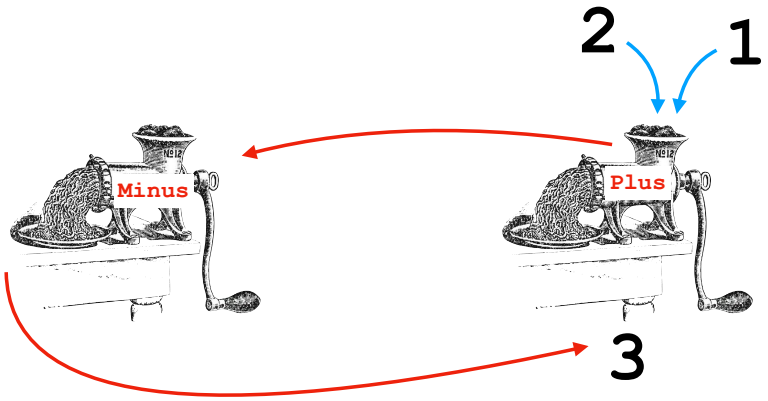
Reductions



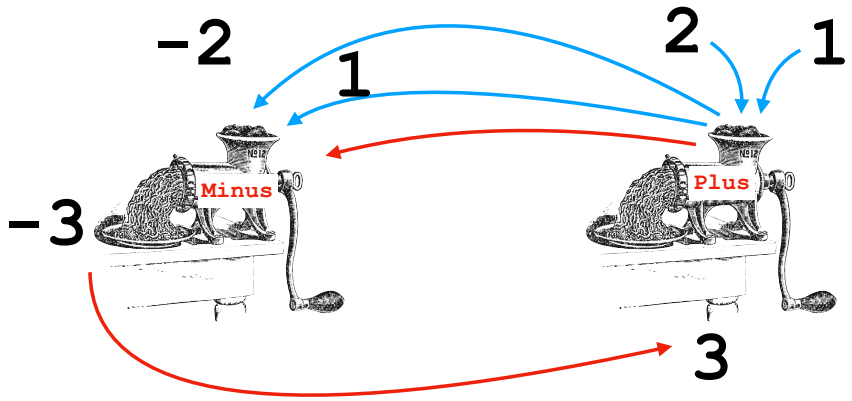
The humble algorithm.

(sorry, vegetarians)

Reductions

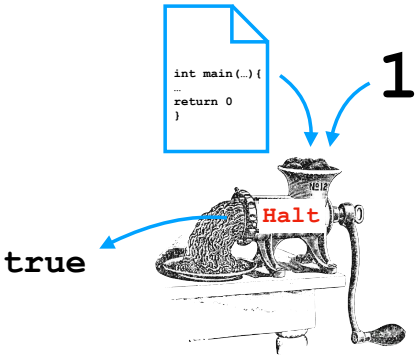


Reductions



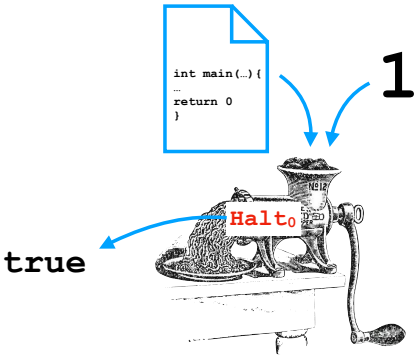

```
let reducer(x: int)(y: int) = 0-(0-x-y)
```

Reductions



We **know** that `Halt` is not computable.

Reductions



A function `f(i)` **halts not** if and only if `f` **does not halt** on input `i`.

Is $\texttt{Halt}_0$ computable?

Reductions

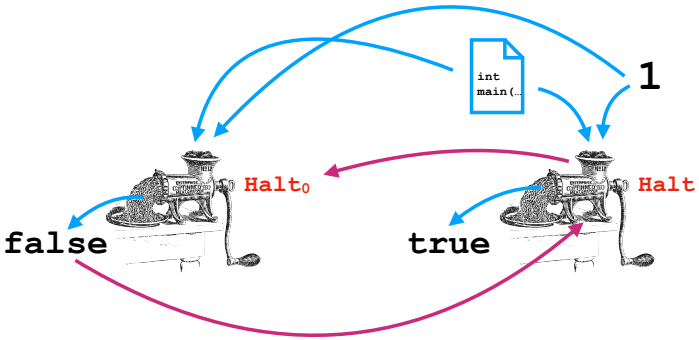
A function `f(i)` **halts not** if and only if `f` **does not halt** on input `i`.

If $\texttt{Halt}_0$ is computable, couldn't we do this?

Assume that $\texttt{Halt}_0$ is computable.
(e.g., it's in your standard library)

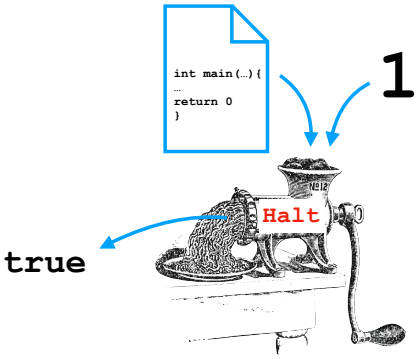
```
def halt(f, i):
  return not halt₀(f, i);
```

Reductions



Reduction: **Construct** $\texttt{Halt}$ using $\texttt{Halt}_0$.

Reductions



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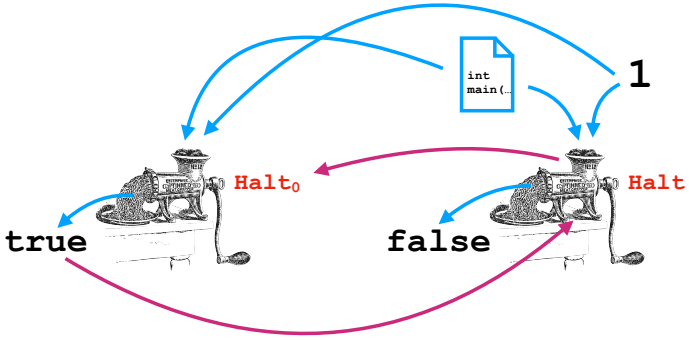
Reductions



If we can build this new machine, what does that mean for **Halt₀**?

Halt₀ is **not computable**.

Activity

Prove that the Halting-No-Input problem is **undecidable**.

Problem: given a program P that requires no input, does P halt?

The Halting Problem

… helps us to understand the difficulty of many other problems.

Reductions

We can use the Halting Problem to show that other problems cannot be solved **by reduction** to the Halting Problem.

We cannot tell, in general…

- … if a program will **run forever**.
- … if a program will **eventually produce an error**.
- … if a program **is done using a variable**.
- … if a program **is a virus**!

Generality

```
def myprog(x):
  return 0

def Halt(f,i):
  if(f = "def myprog(x):\n\treturn 0"):
    return true
  else
    return false
```

The Halting Problem is about **an arbitrary program**.

Recap & Next Class

Today:

Halting problem

Reduction proofs

Next class:

Midterm review