

In this example, I always choose the *normal order* reduction. Normal order is always “outermost leftmost” in the AST. The **highlight** identifies the parts of the AST reduced in the next step.

expression	AST	derivation rule
$(\lambda f.\lambda x.f(fx))(\lambda z.(+xz))2$		given
$(\lambda f.\lambda a.f(fa))(\lambda z.(+xz))2$		$\alpha$ reduce $a$ for $x$ to avoid “capturing” the free $x$
$(\lambda a.(\lambda z.(+xz))((\lambda z.(+xz))a))2$		$\beta$ reduce $(\lambda z.(+xz))$ for $f$

expression	AST	derivation rule
	<p>The diagram shows an AST for the expression ((λz.(+xz))((λz.(+xz))2)). The root node is an application node 'a'. Its left child is a lambda node 'λ' with a single parameter 'z'. Its right child is another application node 'a'. This second application node has a lambda node 'λ' as its left child (with parameter 'z') and a constant node '2' as its right child. The lambda node 'λ' has an addition node '+' as its child, which has 'x' and 'z' as children. The lambda node 'λ' also has an addition node '+' as its child, which has 'x' and 'z' as children.</p>	
$((\lambda z.(+xz))((\lambda z.(+xz))2))$ $(\lambda z.(+xz))((\lambda z.(+xz))2)$	Same tree.	$\beta$ reduce 2 for a eliminate parens
	<p>The diagram shows an AST for the expression ((+x((λz.(+xz))2))). The root node is an addition node '+'. Its left child is 'x' and its right child is an application node 'a'. The application node 'a' has a lambda node 'λ' as its left child (with parameter 'z') and a constant node '2' as its right child. The lambda node 'λ' has an addition node '+' as its child, which has 'x' and 'z' as children.</p>	
$((+x((\lambda z.(+xz))2)))$ $(+x((\lambda z.(+xz))2))$	Same tree.	$\beta$ reduce ((λz.(+xz))2) for z Eliminate parens.
	<p>The diagram shows an AST for the expression (+x(((+x2))). The root node is an addition node '+'. Its left child is 'x' and its right child is another addition node '+'. This second addition node has 'x' as its left child and a constant node '2' as its right child.</p>	
$(+x(((+x2))))$ $(+x(+x2))$	Same tree.	$\beta$ reduce 2 for z Eliminate parens. Done.