

In this example, I always choose the *normal order* reduction. Normal order is always “outermost leftmost” in the AST. Note that I did this quickly. Please let me know if you spot mistakes.

expression	AST	derivation rule
$(\lambda f.\lambda x.f(fx))(\lambda z.(+xz))2$		given
$(\lambda f.\lambda a.f(fa))(\lambda z.(+xz))2$		α reduce a for x to avoid “capturing” the free x
$(\lambda a.(\lambda z.(+xz))((\lambda z.(+xz))a))2$		β reduce $(\lambda z.(+xz))$ for f

expression	AST	derivation rule
	<pre> graph TD A[⊖] --- B[λ] A --- C[⊖] B --- D[z] B --- E[+] E --- F[x] E --- G[z] C --- H[λ] C --- I[2] H --- J[z] H --- K[+] K --- L[x] K --- M[z] </pre>	
$((\lambda z.(+xz))((\lambda z.(+xz))2))$		β reduce 2 for a
$(\lambda z.(+xz))((\lambda z.(+xz))2)$	Same tree.	eliminate parens
	<pre> graph TD A[+] --- B[x] A --- C[⊖] C --- D[λ] C --- E[2] D --- F[z] D --- G[+] G --- H[x] G --- I[z] </pre>	
$((+x((\lambda z.(+xz))2)))$		β reduce $((\lambda z.(+xz))2)$ for z
$(+x((\lambda z.(+xz))2))$	Same tree.	Eliminate parens.
	<pre> graph TD A[+] --- B[x] A --- C[+] C --- D[x] C --- E[2] </pre>	
$(+x(((+x2))))$		β reduce 2 for z
$(+x(+x2))$	Same tree.	Eliminate parens. Done.