CS256-01: Algorithm Design and Analysis

Problem Set 0 (due 02/12/2025)

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Note. This problem set will not be graded on correctness but on completion—to get full points, you must attempt all the questions (except the optional feedback question). This problem set is done individually.

The goal of this problem set is for you to check your familiarity with the background material from Data Structures (CS136) and Discrete Math (MATH200), and for you to get comfortable with LaTeX. It is your responsibility to fill in the gaps in your knowledge.

Submission guidelines. When submitting your solution PDF on Gradescope, you must match questions to pages in the PDF. This takes less than a minute and is crucial for efficient and anonymous grading. Sometimes, you may need to mark pages approximately, e.g., for multi-part questions such as Problem 4.

There is one question per page of this problem set. Make sure you scroll to see all pages.

Problem 1. Please complete the following intro form.

Intro Form: https://forms.gle/9JbS8ptvhxYJPE9n6

Problem 2. Let A, B be sets. Prove by contradiction that $A \cap B = \emptyset \implies A \subseteq \overline{B}$.

> **Hint.** When we prove by contradiction, we assume that the statement is false. Here we have an implication: $A \cap B = \emptyset$ implies $A \subseteq \overline{B}$. What does it mean for this implication to be false?

Solution.

□

Problem 3. Prove by induction that $1 + 2 + 3 + 4 + \ldots + n = n(n+1)/2$.

Written another way, prove that

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

Solution.

□

Note. Problem 4 relies on material you've likely seen before, but that we won't go over in class until Monday, 02/10/25.

Problem 4. For each of the following, answer with the tightest upper bound from this list: $O(\log n)$, $O(n)$, $O(n \log n)$, $O(n^2)$, $O(2^n)$. Briefly justify your answer.

(a) The *depth* of a complete binary tree with n nodes:

Solution.

□

(b) The worst-case run time to sort n items using merge sort:

Solution.

□

(c) The number of distinct subsets of a set of n items:

Solution.

□

(d) The number of bits needed to represent the positive integer n:

Solution.

□

(e) The time to find the second largest number in a set of n (not necessarily sorted) numbers:

Solution.

□