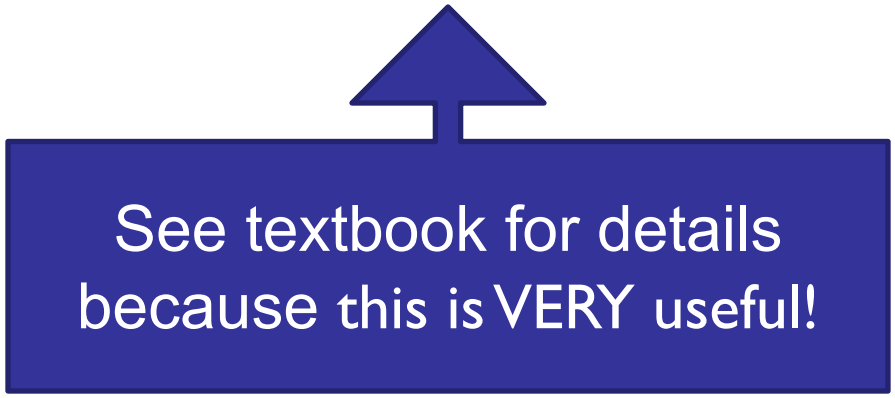
Stack Applications

- The `Stack` implementation is simple, but there are *many* applications, including:
  - Evaluating mathematical expressions
  - Searching (Depth-first search)
  - Removing recursion for optimization
  - …



Evaluating Arithmetic Expressions

- Computer programs regularly use stacks to evaluate arithmetic expressions
- Example: x*y+z
  - First rewrite as xy*z+
    - *we'll look at this rewriting process in more detail soon*
  - Then:
    - push x
    - push y
    - * (*pop twice, multiply popped items, push result*)
    - push z
    - + (*pop twice, add popped items, push result*)

Converting Expressions

- We (humans) primarily use infix notation to evaluate expressions
  - (x+y)*z
- Computers traditionally used postfix (also called Reverse Polish) notation
  - xy+z*
  - Operators appear after operands, parentheses are not necessary
- How do we convert between the two?
  - (Compilers do this for us)

Converting Expressions

- Example: x*y+z*w
- Conversion
  1) Add full parentheses to preserve order of operations

     ((x*y)+(z*w))
  2) Move all operators (+-*/) after operands

     ((xy*)(zw*)+)
  3) Remove parentheses

     xy*zw*+

Use Stack to Evaluate Postfix Exp

- While there are input "tokens" (i.e., symbols) left:
  - Read the next token from input.
  - If the token is a value, push it onto the stack.
  - Else, the token is an operator that takes n arguments.
    (It is known that an operator takes n arguments by its definition.)
    - If there are fewer than n values on the stack → error.
    - Else, pop the top n values from the stack and:
      - Evaluate the operator, with the values as arguments.
      - Push the returned result, if any, back onto the stack.
  - The top value on the stack is the result of the calculation.
  - Note that results can be left on stack to be used in future computations:
    - Eg: 3 2 * 4 + followed by 5 / yields 2 on top of stack

Symbolic Example: Converting then Evaluating

- (x*y)+(z*w) → xy*zw*+
- Evaluate `xy*zw*+` :
  - Push x
  - Push y
  - Mult: Pop y, Pop x, Push x*y
  - Push z
  - Push w
  - Mult: Pop w, Pop z, Push z*w
  - Add: Pop x*y, Pop z*w, Push (x*y)+(z*w)
  - Result is now on top of stack

Concrete Example: Converting then Evaluating

- (x*y)+(z*w) → xy*zw*+
- Evaluate `xy*zw*+` :
  - Push x
  - Push y
  - Mult: Pop y, Pop x, Push x*y
  - Push z
  - Push w
  - Mult: Pop w, Pop z, Push z*w
  - Add: Pop x*y, Pop z*w, Push (x*y)+(z*w)
  - Result is now on top of stack
- Try with: w=3, x=4, y=5, z=6

PostScript

- PostScript is a programming language used for generating vector graphics
  - Best-known application: describing pages to printers
- It is a stack-based language
  - Values are put on stack
  - Operators pop values from stack, put result back on
  - There are numeric, logic, string values
  - Many operators
- Let's try it: The 'gs' command runs a PostScript interpreter….
- Implementing a tiny part of gs is something we will do in lab... it's a lot of fun!