Tree Traversals

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April 19, 2022

Admin

- Sign up to be a TA! Deadline Friday
- Masks for now in class
- Any questions?

Let’s look at last week’s quiz

Binary Tree

Binary Tree

- Binary Tree: A tree where each node has at most 2 children
- The degree of a node is the number of children it has. So a binary tree is a tree where all nodes have degree at most 2.
- Let's see an example of a binary tree. Then, we'll discuss the `BinaryTree` class that comes with `structure5`

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- Let's take a look at the code for `BinaryTree`
- Now, let's look at how we can evaluate a tree of expressions stored in a `BinaryTree`

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- The *depth* of a node *n* is the number of edges between *n* and the root.
- The *height* of a tree is the largest depth of any node in the tree.

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Correctness on Trees

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- Do we want strong induction or weak induction? What will our proof look like?
- Answer: strong induction. When proving correctness for a method call of size $n + 1$, the recursive calls may be on a tree of size less than n.

Base case

- $n = \mathtt{0}$

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- So size returns `0` correctly.

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- I.H. (strong induction): There exists some n such that for all k from `0` to n, `size()` returns k correctly on all trees of size k.

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- Putting these together, `size()` returns the size of the tree correctly.

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- Induction is often on the *size* or height of the tree
- Inductive proofs map very closely to correct recursive algorithms

Iterating Over Trees

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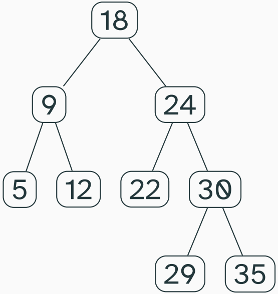

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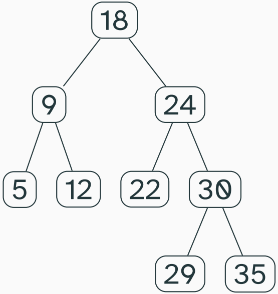

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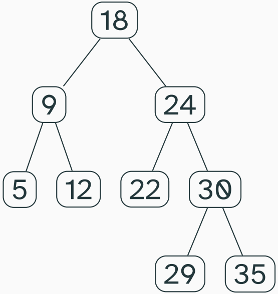

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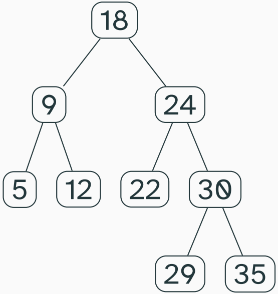

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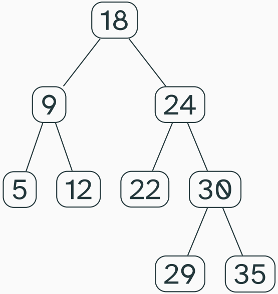

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- Any ideas?

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- Let’s see an example

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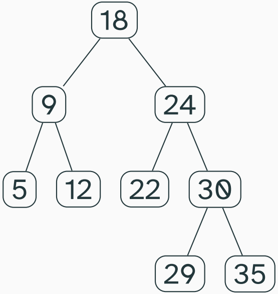

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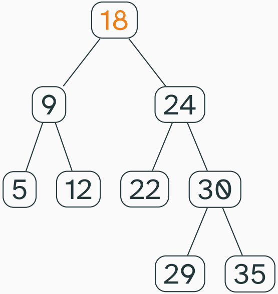

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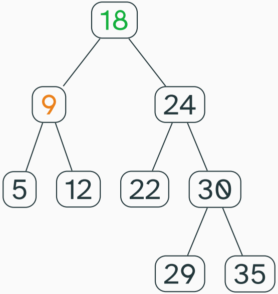

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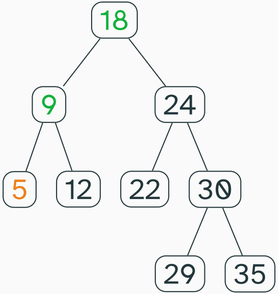

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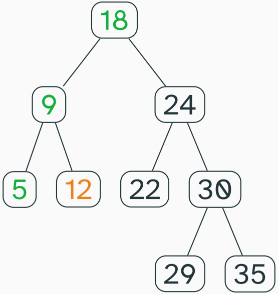

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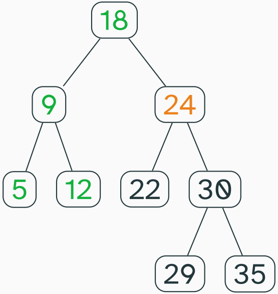

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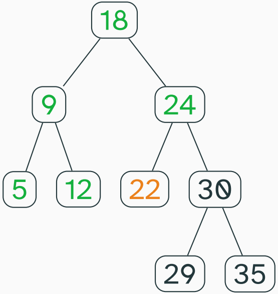

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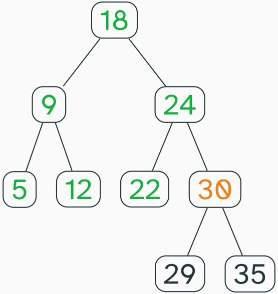

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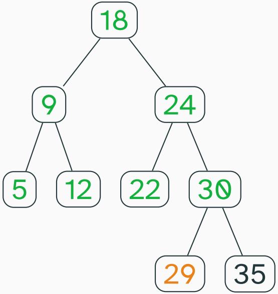

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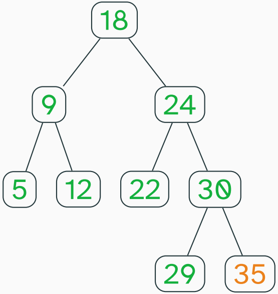

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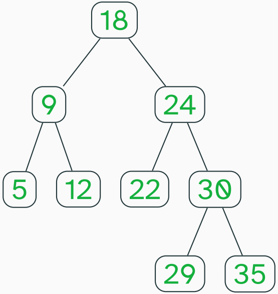

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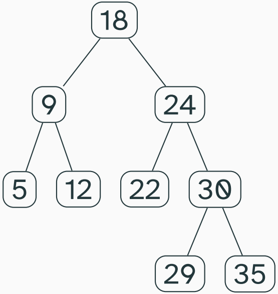

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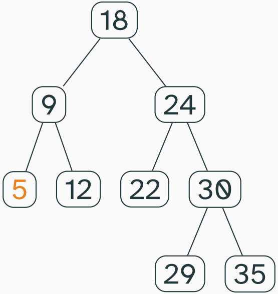

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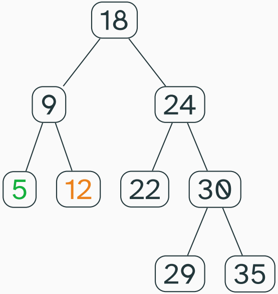

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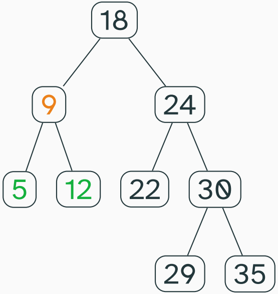

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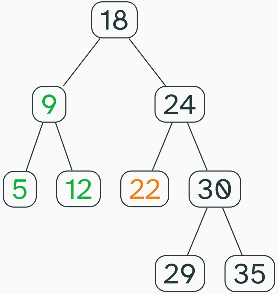

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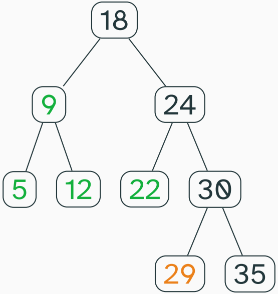

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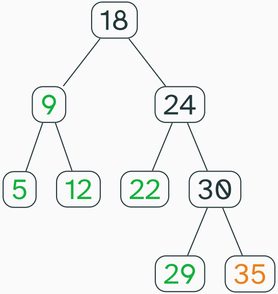

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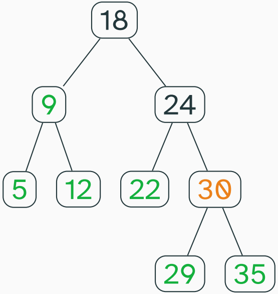

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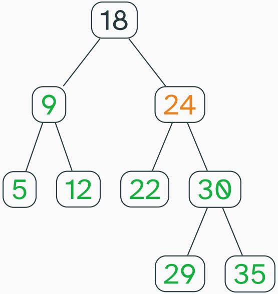

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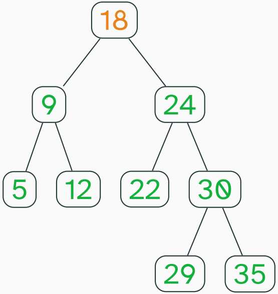

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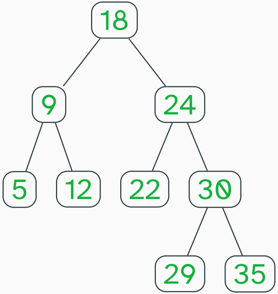

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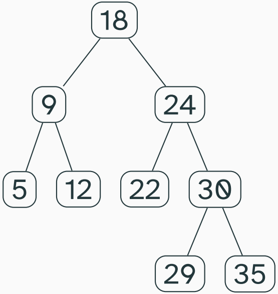

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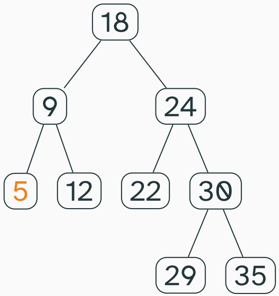

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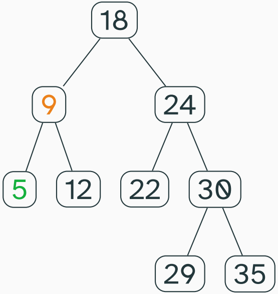

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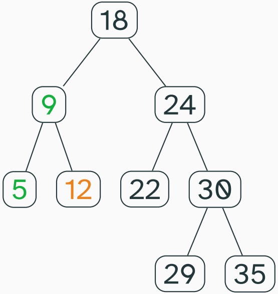

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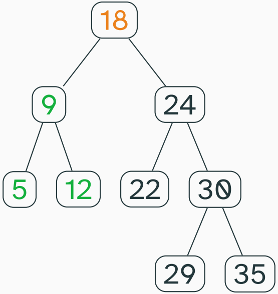

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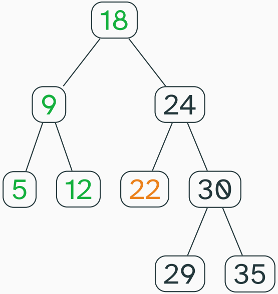

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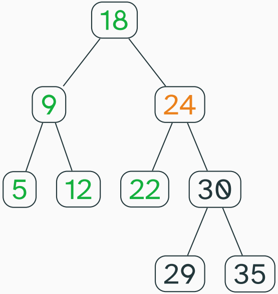

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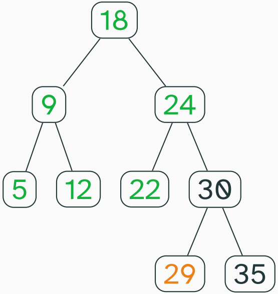

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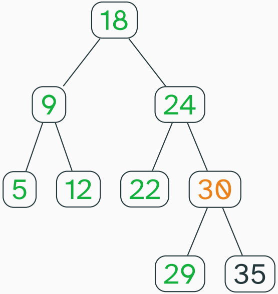

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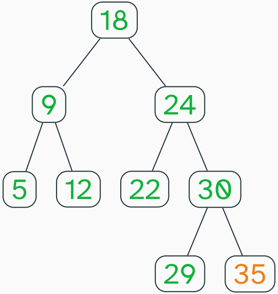

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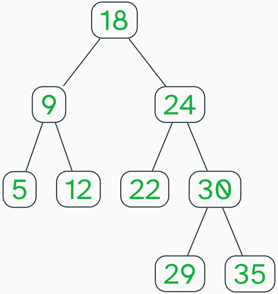

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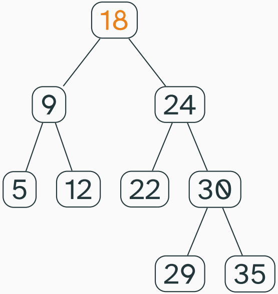

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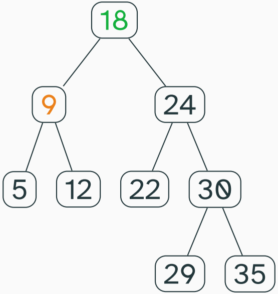

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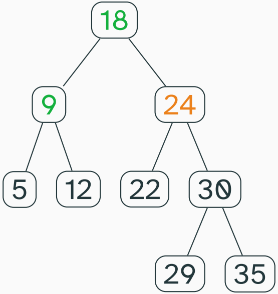

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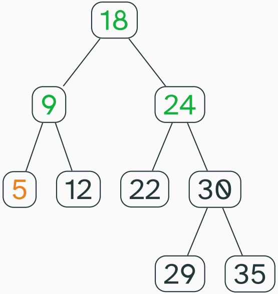

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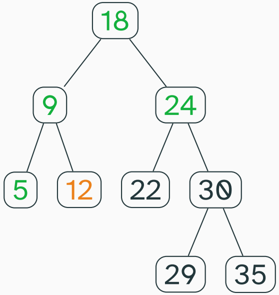

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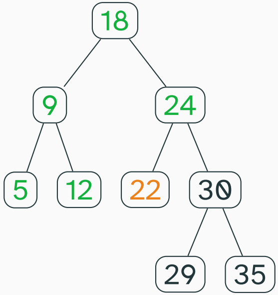

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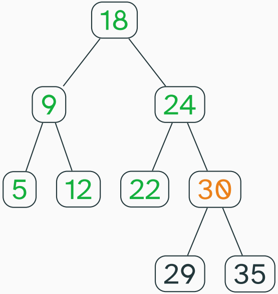

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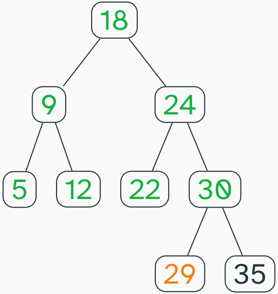

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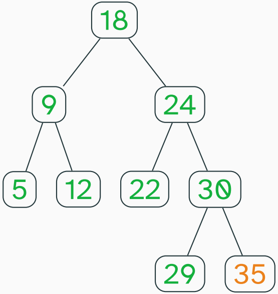

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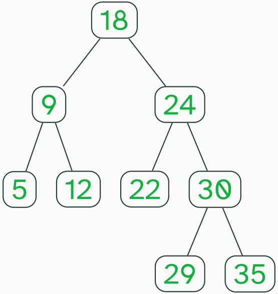

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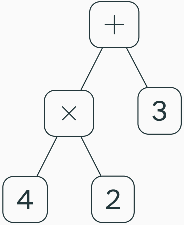
A Question



- We saw how to evaluate an expression tree.

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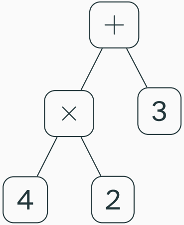

- We saw how to evaluate an expression tree.
- We had to traverse all of the tree to evaluate the expression. What kind of traversal was that?

Tree Iterators

Implementing Tree Iterators

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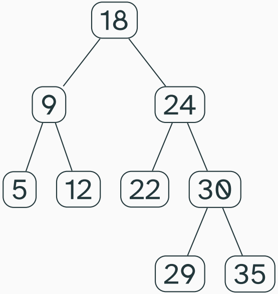
- Challenge: implementing a recursive traversal piece-by-piece
- To think about: what data structure helps with recursion?

Pre-order traversal



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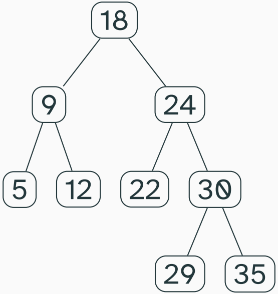

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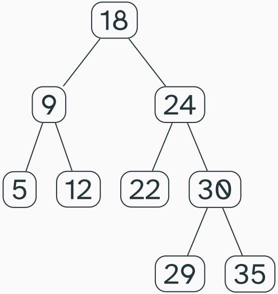

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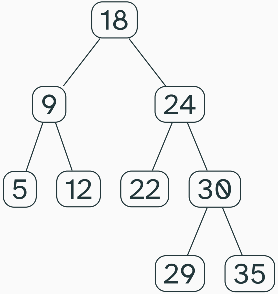

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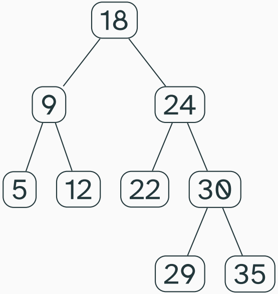

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- Instead: maintain nodes to visit on a stack!

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  - Just returns if the stack is empty

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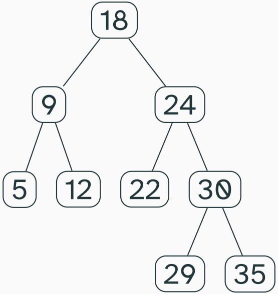

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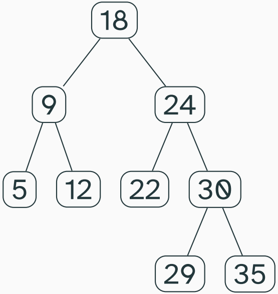

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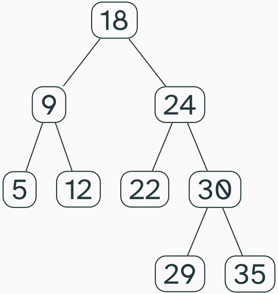

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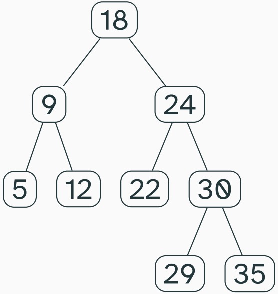

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- Nice idea, but it takes some care. Let's be a bit more specific

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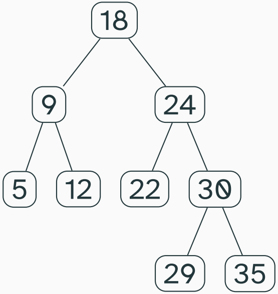

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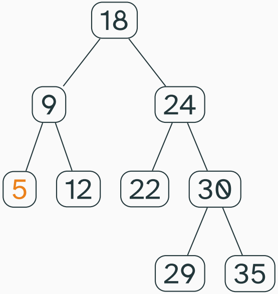

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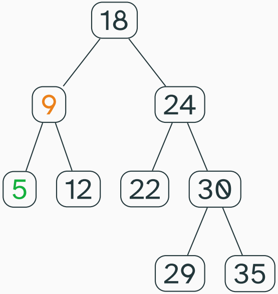

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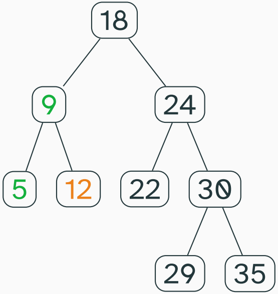

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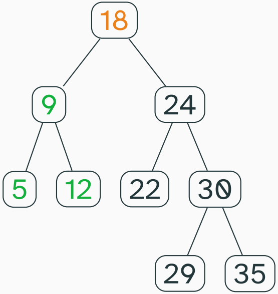

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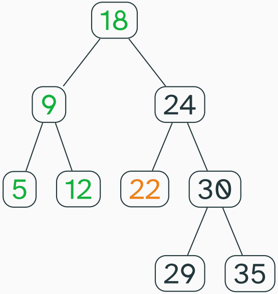

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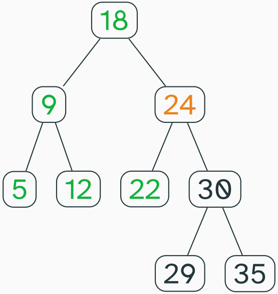

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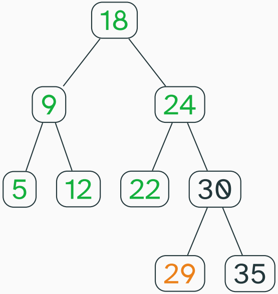

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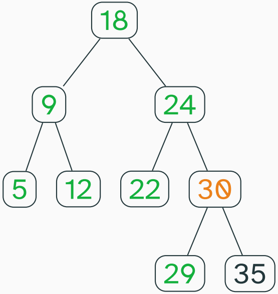

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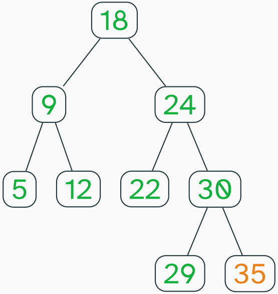

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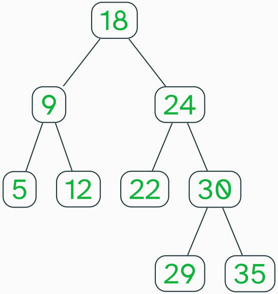

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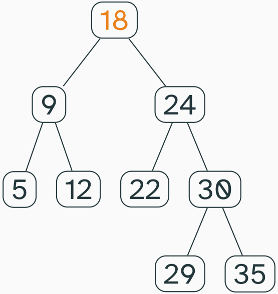

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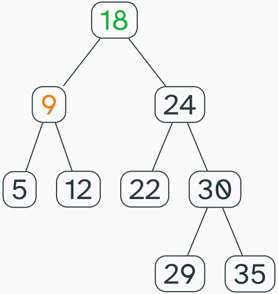

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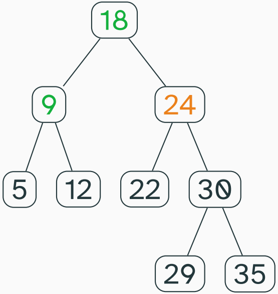

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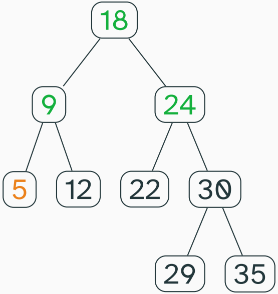

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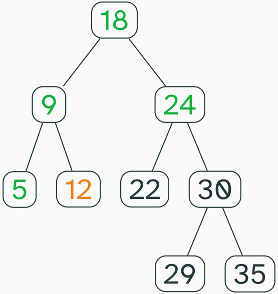

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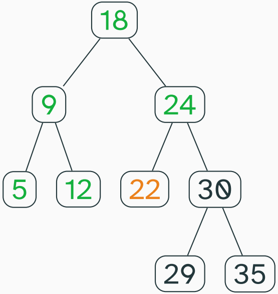
Level-order Traversal



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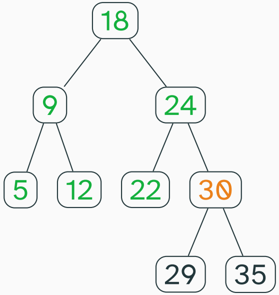

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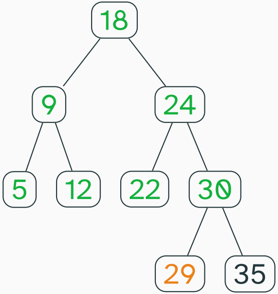

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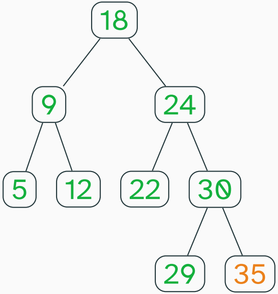

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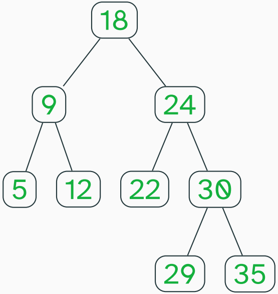

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- …Can we use a queue?

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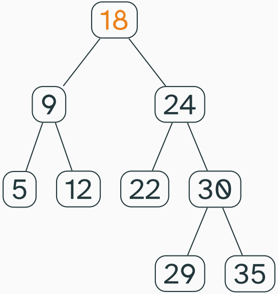
- Let’s look at the code

Level-order Traversal



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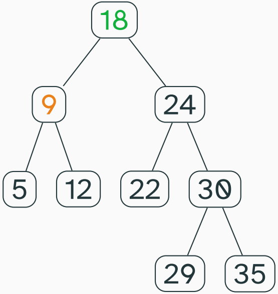
Queue: 9 24

Level-order Traversal



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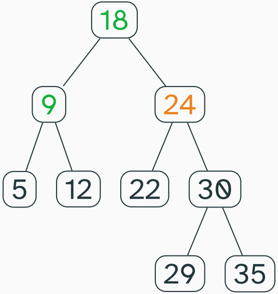
Queue: 24 5 12

Level-order Traversal



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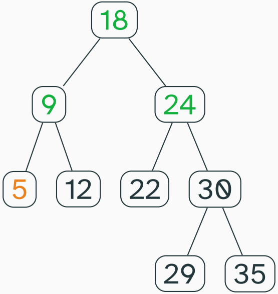
Queue: 5 12 22 30

Level-order Traversal



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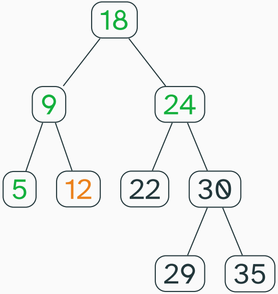

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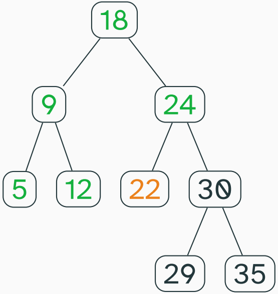

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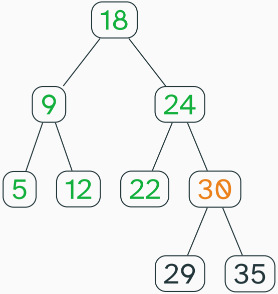
Level-order Traversal



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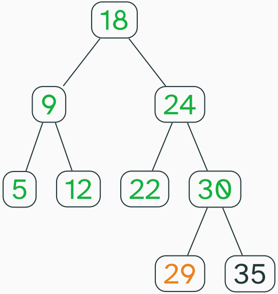
Queue: 29 35

Level-order Traversal



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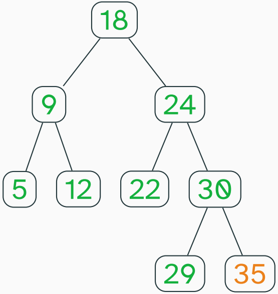
Queue: 35

Level-order Traversal



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Queue:

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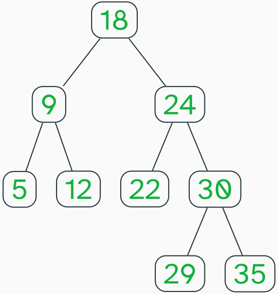

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