

Singly Linked Lists

Instructors: Sam McCauley and Dan Barowy

March 2, 2022

Admin

- Lab 3 in!
- Lab 4 out this afternoon
- Partners again! Fill out opt-out form *by this afternoon*
- New kind of lab: refactoring existing code
- Masks still required
- Remember to come on time Friday for the quiz
 - Look at graded responses; solutions on website under “Handouts”

Linked Lists

Motivating Example

- Let's say we go to the movies



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- How can I keep track of where all of you are sitting?
- One solution: keep a list of all of your seats
- Another option: I'll keep track of one student. They keep track of the seat of the next student. So on and so forth
- Each student remembers the location of the next student (or none if they are the last). But I can still traverse all students!

Linked List

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- A new kind of `List`.
 - So: will implement all the operations that a `Vector` implements
 - (Looking ahead: will be faster for some operations; slower for others.)
- Uses the principle from the theater example: each piece of data remembers the location of the next

First step: Node

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- Needs to have the actual information
 - Probably of generic type
- Also needs to store the location of the next piece of data
 - That is to say: needs to hold the next Node

Node

```
public class Node<E> {  
    protected E data  
    protected Node<E> nextElement;  
    public Node(E v, Node<E> next) {  
        data = v;  
        nextElement = next;  
    }  
    public void setNext(Node<E> next) {  
        nextElement = next;  
    }  
    public void setValue(E value) {  
        data = value;  
    }  
}
```

```
    public Node<E> next() {  
        return nextElement;  
    }  
    public E value() {  
        return data;  
    }  
}
```

Creating a (Singly) Linked List

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 - First `Node`
 - Maybe some other useful `List` information? Perhaps the number of stored items

Creating a (Singly) Linked List

- We have a way to get from one Node to the next
- What else do we need to store?
 - First Node
 - Maybe some other useful List information? Perhaps the number of stored items
- And, then, need to implement methods

SinglyLinkedList

```
public class SinglyLinkedList<E>
{
    protected int count;           // list size
    public Node<E> head; // ref. to first element

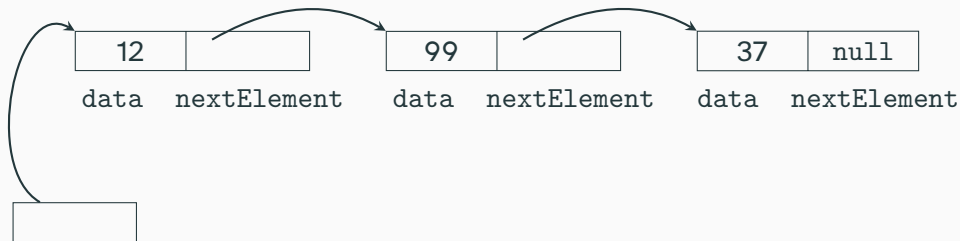
    public SinglyLinkedList()
    {
        head = null;
        count = 0;
    }

    //to fill in: methods...
```

Where we are now

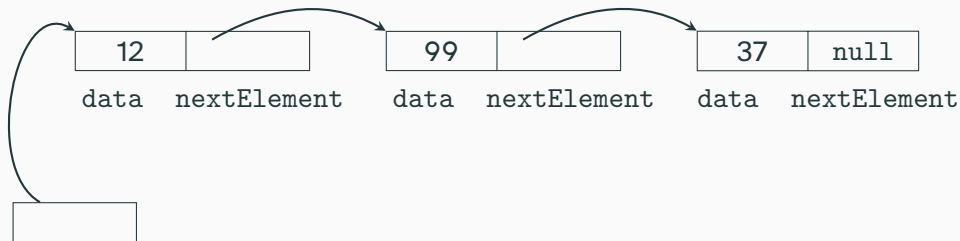


Where we are now



`SinglyLinkedList.head`

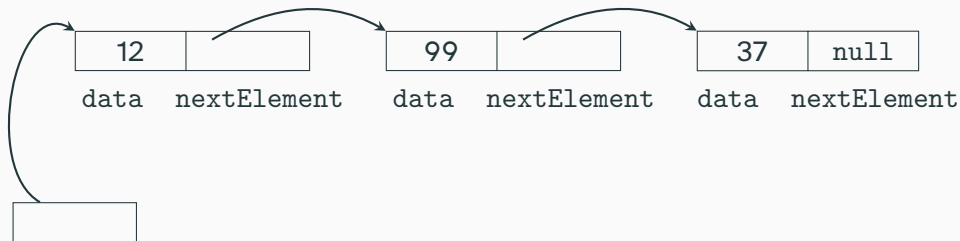
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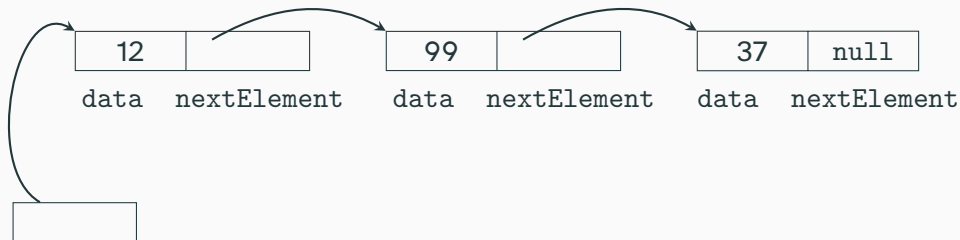
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- We just need to store the address of the first one (head).

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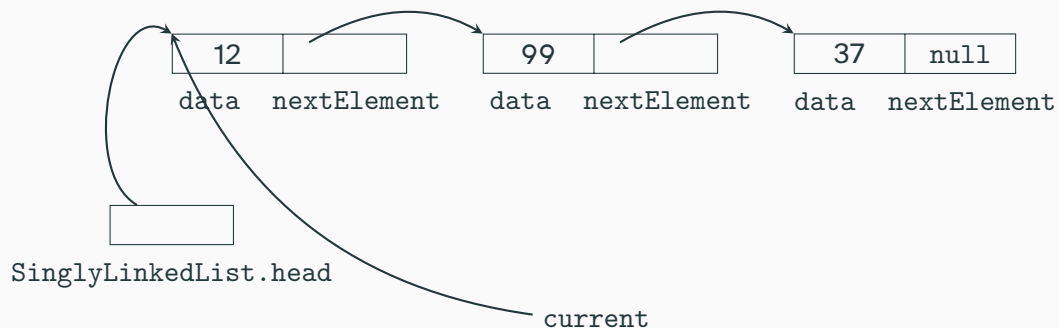
`SinglyLinkedList.head`

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- How can we perform `contains()`?

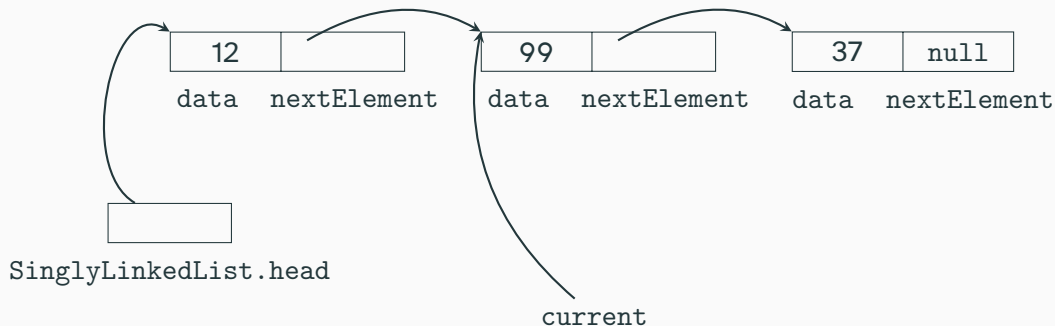
Draft of contains

```
public boolean contains(E value) {
    Node<E> current = head;
    while(current != null) { //why == if these are objects?
        if(value.equals(current.value())) {
            return true;
        }
    }
    return false;
}
```

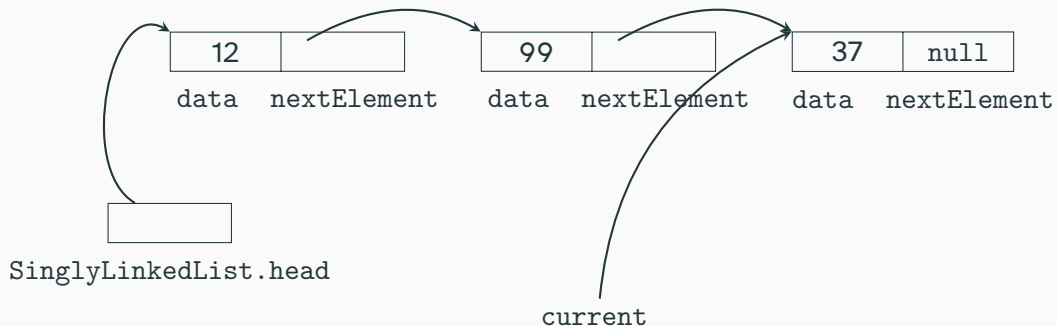
Contains Diagram



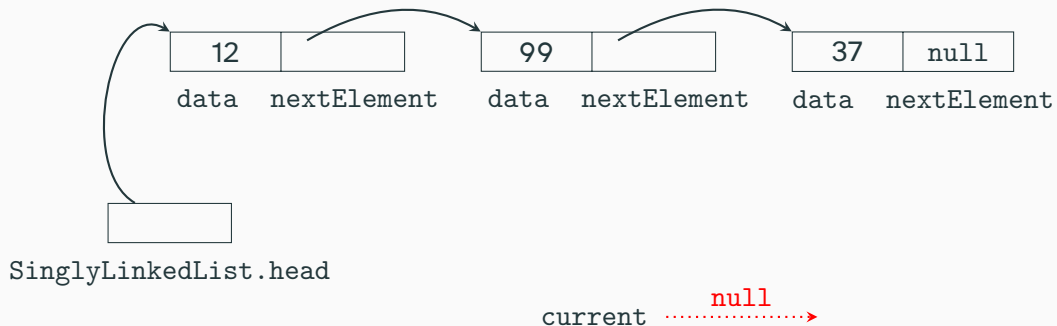
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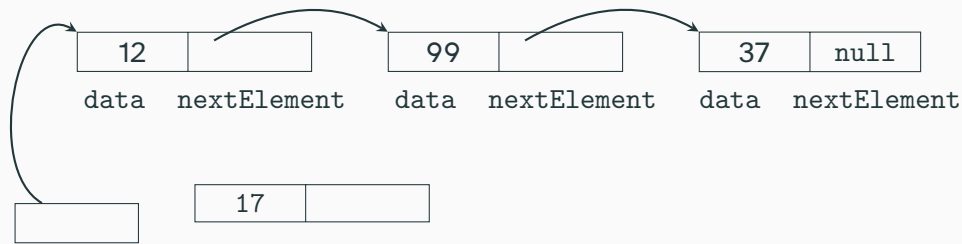
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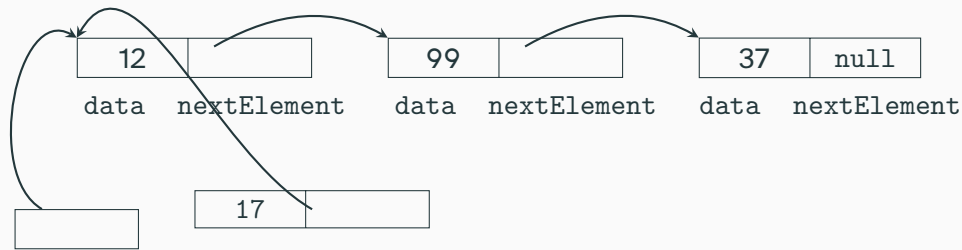
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 - Create a `Node<E>` that holds `value`. Let's call it `newNode`
 - Set the `nextElement` of `newNode` to be the previous head
 - Set the head to now point to `nextElement`
- Also: update `count`

Add Diagram (Adding 17)



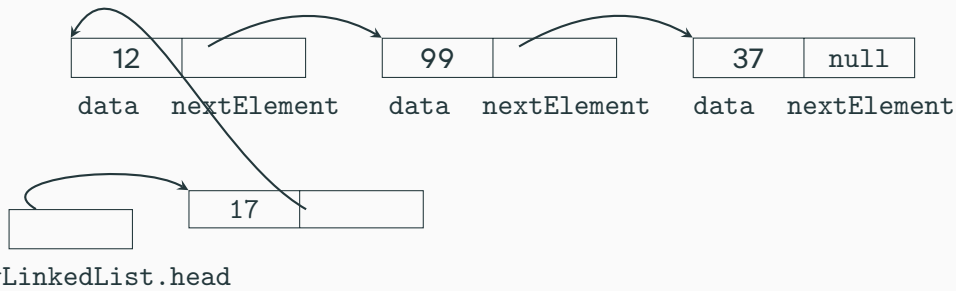
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- How can we implement `set(int, E)` and `get(int)`?
- How long do `set(int, E)` and `get(int)` take?
- If I have a `Node<E>` in the middle of my list, how long does it take to add a new node *after* it?

Motivating Recursion (Induction Intro)

Goal of this section

- Chat about what makes a recursive algorithm correct

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- We'll get more formal about this on Friday

Finding number of X in a string

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public static int numX(String s) {
    if(s.length() == 0) {
        return 0;
    }
    if(s.charAt(s.length()-1) == 'X') {
        return 1 + numX(s.substring(0,s.length() - 1));
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How do we know that this method works correctly?

Where to start?

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- If s has length 0 , then this algorithm correctly returns 0 .

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- If s has length 0 , then this algorithm correctly returns 0 .
- What if s has length 1 ?

Proving recursive correctness

- If s has length 1:

Proving recursive correctness

- If `s` has length 1:
 - We know that `numX(s.substring(0,0))` returns `0` because we know that `numX` is correct on strings of length `0`.

Proving recursive correctness

- If s has length 1:
 - We know that `numX(s.substring(0,0))` returns \emptyset because we know that `numX` is correct on strings of length \emptyset .
 - If the first character is X , we return $1 + \emptyset = 1$.

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- If s has length 1:
 - We know that `numX(s.substring(0,0))` returns 0 because we know that `numX` is correct on strings of length 0 .
 - If the first character is X , we return $1 + 0 = 1$.
 - If the first character is not X , we return $0 + 0 = 0$.

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- If s has length 1:
 - We know that `numX(s.substring(0,0))` returns \emptyset because we know that `numX` is correct on strings of length \emptyset .
 - If the first character is X , we return $1 + \emptyset = 1$.
 - If the first character is not X , we return $\emptyset + \emptyset = \emptyset$.
 - In both cases we're correct.

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- If `s` has length 1:
 - We know that `numX(s.substring(0,0))` returns `0` because we know that `numX` is correct on strings of length `0`.
 - If the first character is `X`, we return $1 + 0 = 1$.
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 - In both cases we're correct.
- So `numX()` works on strings of length at most 1.

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Proving recursive correctness

- If `s` has length 2:
 - We know that `numX(s.substring(0,1))` is correct because we know that `numX` is correct on strings of length 1.
 - We add 1 if the first character is 'X', 0 otherwise. So we are correct for strings of length at most 2
- So `numX()` works on strings of length at most 2.

Our strategy

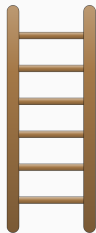
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Our strategy

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- Slowly argue that it works for larger and larger strings

Visualizing our Strategy

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Visualizing our Strategy



- How can we climb to the top of a ladder?
- Here's a two step process:
 - Figure out how to get on some rung of the ladder
 - Figure out a method to get from one rung to the next rung
- If I do both of these, will I always make it to the top of the ladder?

Our strategy

Ladder analogy: each step of the ladder is the length of the string in our recursive method. We want to show that our method is correct for a string of a certain length.

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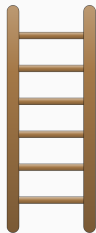
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rung of the ladder, how can we get to the next rung?

From one