Hashing Continued

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Admin

- Talk today: gerrymandering and how it relates to computer science (2:30 in Wege)
- Any questions?

Linear Probing

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- General idea: store each key-value pair in the first open slot on or after its canonical slot
- Insertion: if a collision occurs at the bin, just scan forward (linearly) until an empty slot is available; store the item there
    - We “wrap around” at the end of the array
    - Let’s call a contiguous region of full bins a *run*
- Lookup: to find a key-value pair, calculate the bin. Then, scan linearly until the item is found or you reach the end of the run.

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    - Now `get()` won't work correctly

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    - Must still scan the whole run to make sure the key isn't present later on

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- Finally, `Hashtable.java`

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- The length of a run dictates the performance
- Removing elements does not shrink the run–it defers the work to other operations
  - Keeping runs small is important, so we may want to reconsider some design decisions if we expect a lot of deletions

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    - Never scans more items than linear probing!
    - But: worse cache behavior (locality)

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- Related: how do we actually choose a hash function?

Hashtable Size

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    - We want our table size to be small to minimize wasted space (empty slots): leads to *good space*, bad performance
- Some flexibility (like with `Vectors`): we don't know the size up front

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- A smaller load factor means the hashtable is less full, which likely gives better performance

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- Does this work for hashtables?

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- Example: suppose `key.hashCode() == 11`
- Then `11 % 8 == 3` but `11 % 16 == 11`
- How can we handle this?
- To grow our hashtable, we must recompute the canonical slot for each item, then reinsert the item into the new array

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- `structure5` `Hashtable` uses `.6`

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- Downside: need to find a prime size when doubling
- We won’t worry about this in this class; just a heads up. You’ll often see a hash table of size 997 or something—this is why.

Choosing Hash Functions

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- Rules of thumb to make good hash functions?
  - Not really. We almost always have to test “goodness” empirically

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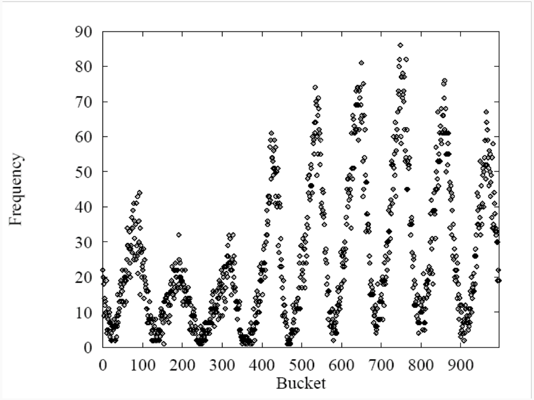
    - Doesn't work well for large hashtables
    - Not good at avoiding collisions: smile, limes, miles, and slime are all the same

Sum of Unicode Values



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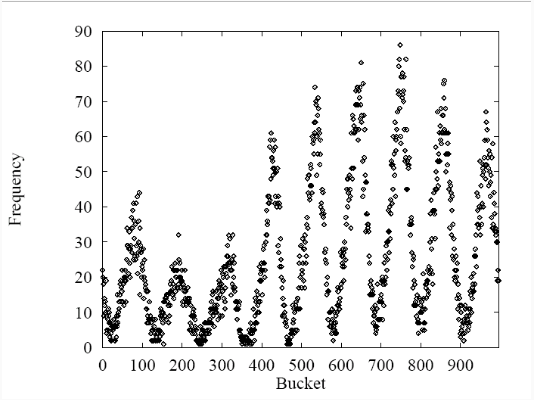

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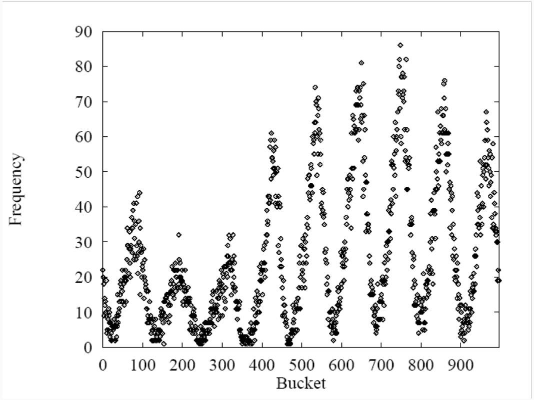

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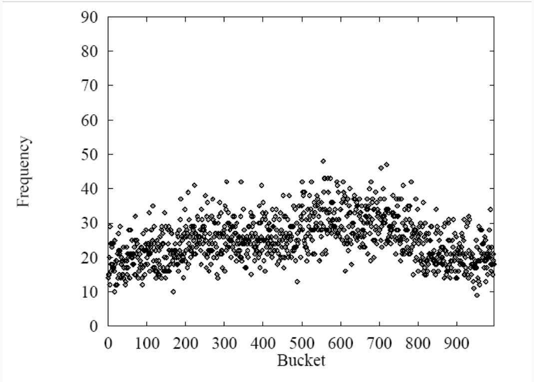

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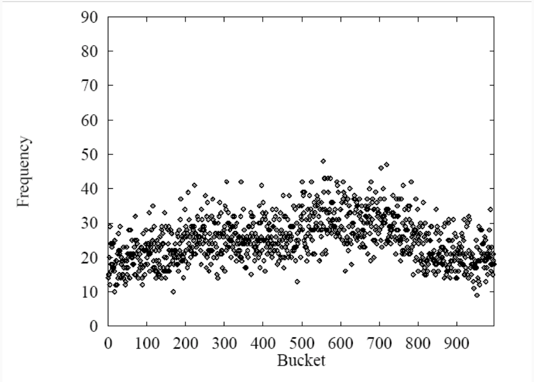

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- Better! But still not great.

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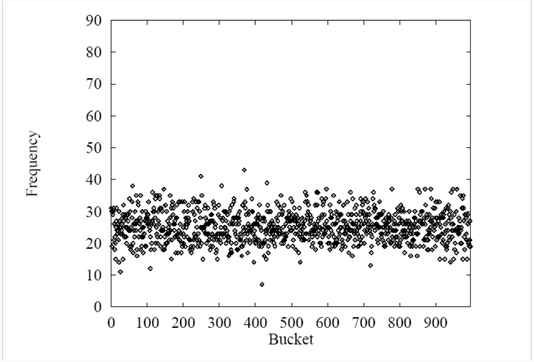

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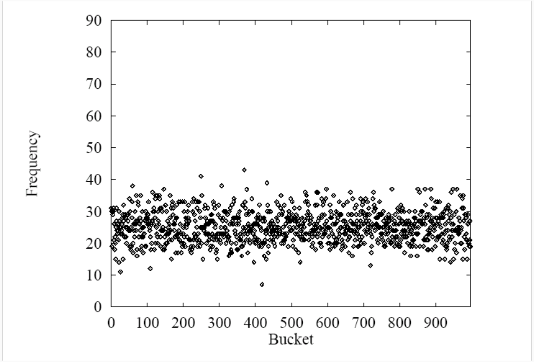

- Hash of a string s: $\sum_{i=0}^{s.length} 256^i \cdot$ `s.charAt(i)`
- Really good! But do we need numbers as big as 256^i?

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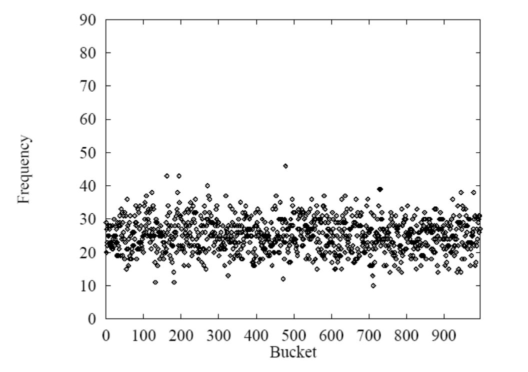

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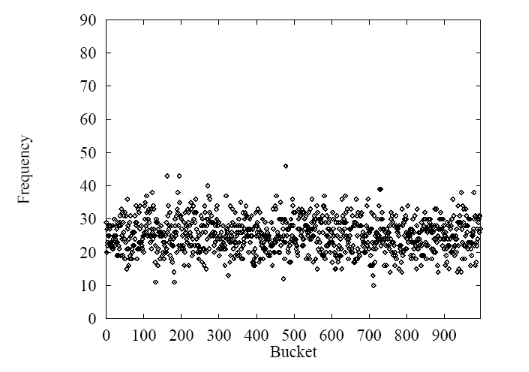

- Hash of a string *s*: $\sum_{i=0}^{s.length} 31^i \cdot$ `s.charAt(i)`
- This is (essentially) what Java uses to hash strings!

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- What about other classes?
- Write your own (probably similar) `hashCode()` methods. Test empirically to make sure elements are spread out

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  - $O(1)$ in terms of the number of items in the hash table
  - Another example of being careful about how we're stating our running time. Usually: in terms of number of strings in the table. But do we care about the length of our strings?

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- An *average* bucket is in a run of constant length
- (With overwhelming probability, never gets worse than $O(\log n)$ for any bucket)
- Usually we say we have $O(1)$ performance. True on average; the actual worst case might be a bit worse

Summary of Map Performance

	`put`	`get`	space
Unsorted Vector	$O(n)$	$O(n)$	$O(n)$
Unsorted List	$O(n)$	$O(n)$	$O(n)$
Sorted Vector	$O(n)$	$O(\log n)$	$O(n)$
Balanced BST	$O(\log n)$	$O(\log n)$	$O(n)$
Hashtable (average)	$O(1)$	$O(1)$	$O(n)$