CSCI 136:
Data Structures
and
Advanced Programming

Lecture 31

Graphs

Instructor: Dan Barowy

Williams

Topics

Graphs

Your to-dos

1. Read **before Fri**: *Bailey*, Ch. 13.4.
2. Lab 10 (partner lab), **due Tuesday 5/10 by 10pm**.

Announcements

Suresh Venkatasubramanian (White House; Brown U)

Friday, May 6 @ 2:35pm*
Computer Science Colloquium – Wege TCL 123
On Equity in Access
*Williams students, faculty and staff only.

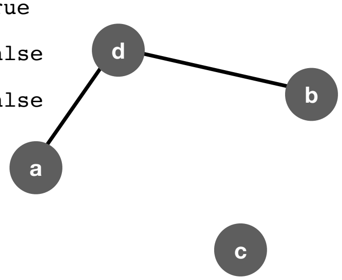
Suresh Venkatasubramanian is a professor in computer science and data science, currently at the White House in the Office of Science and Technology Policy. His background is in theoretical computer science, and he's taken a long and winding path through many areas of data science. For almost the past decade, he's been interested in algorithmic fairness, and more broadly the impact of automated decision-making systems in society.

Graphs

Graph operations

Fundamental graph ADT operations

```
adjacent(a, d) = true
adjacent(a, b) = false
adjacent(a, c) = false
```



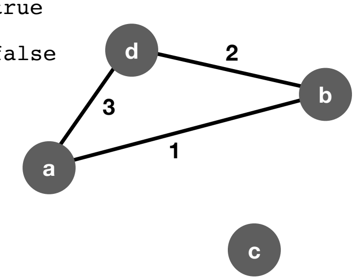

```
bool adjacent(Vertex u, Vextex v):
```

Given vertices **u** and **v**, are they **adjacent**?

(i.e., share an edge?)

Fundamental graph ADT operations

```
incident(a, 1) = true
incident(a, 2) = false
```



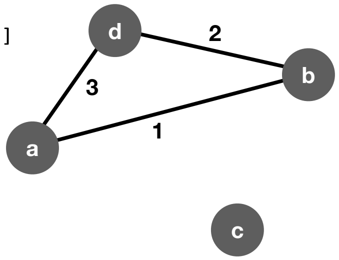

```
bool incident(Vertex v, Edge e):
```

Given vertex **v** and edge **e**, are they **incident**?

(i.e., is v an endpoint of edge e?)

Fundamental graph ADT operations

```
vertices(1) = [a, b]
vertices(2) = [d, b]
```

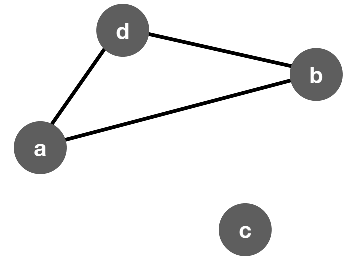



```
Vertex[] vertices(Edge e):
```

Given edge **e**, what are its **end points**?

Fundamental graph ADT operations

```
degree(a) = 2
degree(c) = 0
```

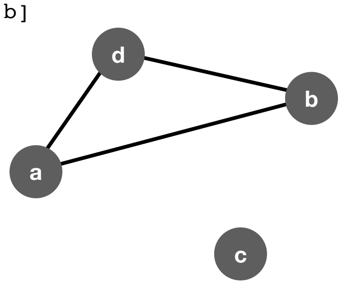



```
int degree(Vertex v):
```

Given vertex **v** how many vertices are **adjacent**?

Fundamental graph ADT operations

```
neighbors(a) = [d, b]
neighbors(c) = []
```



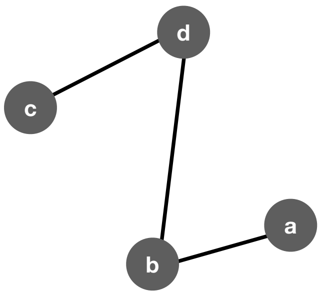

```
Vertex[] neighbors(Vertex v):
```

Given vertex **v** what other vertices are **adjacent**?

Graph representations

Adjacency matrix

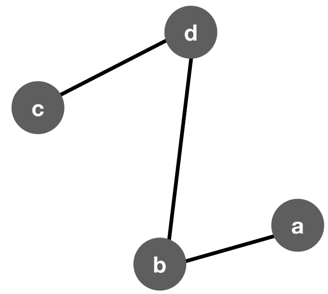
An **adjacency matrix** is a data structure for representing a finite graph. It consists of a **square matrix** (usually implemented as an array of arrays). In the simplest case, the **elements** of the matrix indicate **whether an edge is present**. Elements on the diagonal are **defined as zero**.



	a	b	c	d
a	0	1	0	0
b	1	0	0	1
c	0	0	0	1
d	0	1	1	0

Adjacency matrix

In an **undirected graph**, the adjacency matrix is **symmetric**.



	a	b	c	d
a	0	1	0	0
b	1	0	0	1
c	0	0	0	1
d	0	1	1	0

Adjacency matrix

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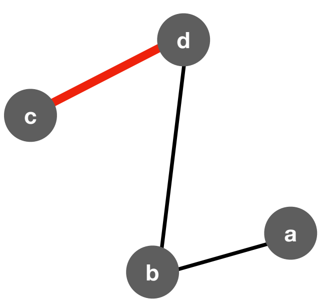
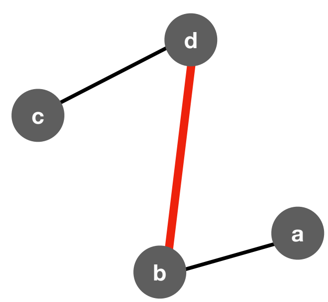


	a	b	c	d
a	0	1	0	0
b	1	0	0	1
c	0	0	0	**1**
d	0	1	**1**	0

Adjacency matrix

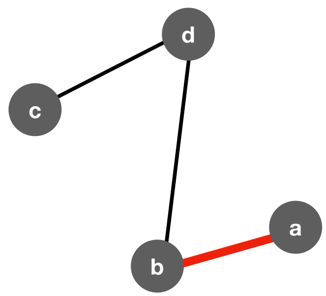
In an **undirected graph**, the adjacency matrix is **symmetric**.



	a	b	c	d
a	0	1	0	0
b	1	0	0	**1**
c	0	0	0	1
d	0	**1**	1	0

Adjacency matrix

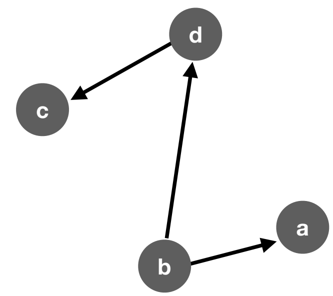
In an **undirected graph**, the adjacency matrix is **symmetric**.



	a	b	c	d
a	0	**1**	0	0
b	**1**	0	0	1
c	0	0	0	1
d	0	1	1	0

Adjacency matrix

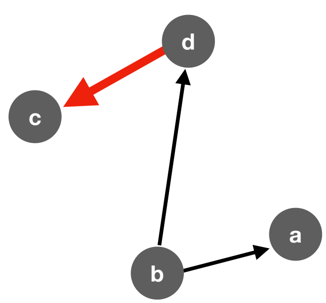
In a **directed graph**, the adjacency matrix is **not symmetric** because edges are directed. A directed edge, **from→to**, is conventionally encoded in **row-major** form.



	a	b	c	d
a	0	0	0	0
b	1	0	0	1
c	0	0	0	0
d	0	0	1	0

Adjacency matrix

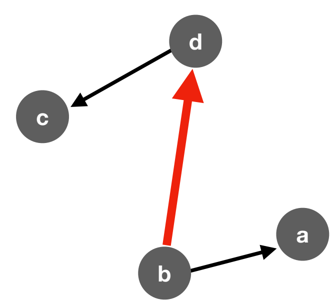
In a **directed graph**, the adjacency matrix is **not symmetric** because edges are directed. A directed edge, **from→to**, is conventionally encoded in **row-major** form.



	a	b	c	d
a	0	0	0	0
b	1	0	0	1
c	0	0	0	0
d	0	0	**1**	0

Adjacency matrix

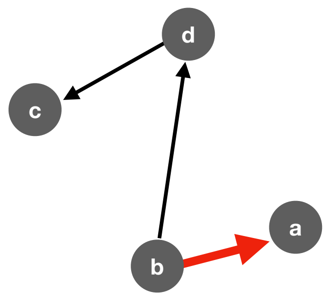
In a **directed graph**, the adjacency matrix is **not symmetric** because edges are directed. A directed edge, **from→to**, is conventionally encoded in **row-major** form.



	a	b	c	d
a	0	0	0	0
b	1	0	0	**1**
c	0	0	0	0
d	0	0	1	0

Adjacency matrix

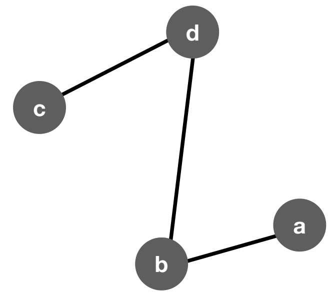
In a **directed graph**, the adjacency matrix is **not symmetric** because edges are directed. A directed edge, **from→to**, is conventionally encoded in **row-major** form.



	a	b	c	d
a	0	0	0	0
b	**1**	0	0	1
c	0	0	0	0
d	0	0	1	0

Adjacency list

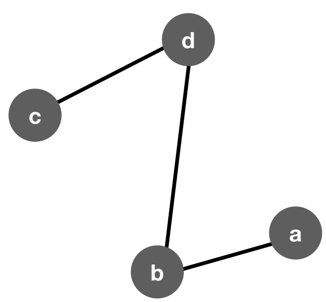
An **adjacency list** is a data structure for representing a finite graph. It consists of a **list of unordered lists**.




```
[[c,d],[d,b],[a,b]]
```

Adjacency list

There are many variants on adjacency lists. The most common is the **object-oriented adjacency list** that stores **a list of adjacent vertices** in each vertex object.

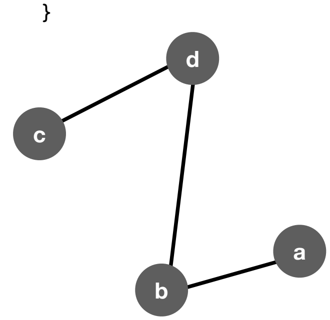


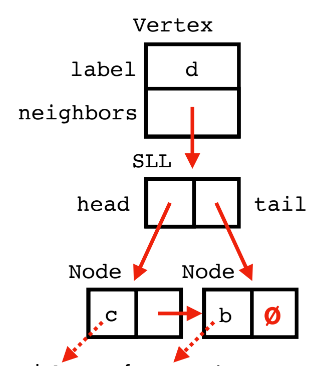

```
a:  [b]
b:  [a,d]
c:  [d]
d:  [b,c]
```

Adjacency list

Object-oriented adjacency list:

```
public class Vertex<T> {
    T label;
    List<Vertex<T>> neighbors = new SinglyLinkedList<>();
    …
}
```

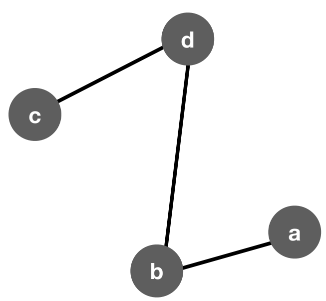




(strictly speaking, `c` and `d` are references to `Vertex` objects)

Adjacency list

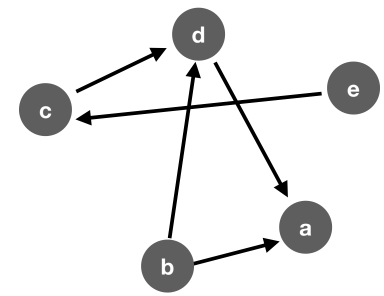
This latter version is **especially thrifty** for **directed graphs**.




```
a: []
b: [a,d]
c: []
d: [c]
```

Activity

Write down both **adjacency matrix** and **adjacency list** representations for this graph.



Which one is better for this graph? Why? (think Big-O)

Recap & Next Class

Today:

Graph operations

Graph representations

Next class:

Heaps and priority queues