CSCI 136:
Data Structures
and
Advanced Programming

Lecture 29

Hash collisions

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Williams

Topics

Hash collisions

Graphs

Your to-dos

1. Read **before Wed**: *Bailey*, Ch. 16.4.
2. Lab 9 (solo lab), **due Tuesday 5/3 by 10pm**.

Note about lab 9:

You may use the structure5 `Hashtable` implementation.

Practice Quiz

Hash tables

Hash codes

Hashing so important that every `Object` in Java has a built-in hash function.

hashCode

`public int hashCode()`

Returns a hash code value for the object. This method is supported for the benefit of hash tables such as those provided by `HashMap`.

The general contract of `hashCode` is:

- Whenever it is invoked on the same object more than once during an execution of a Java application, the `hashCode` method must consistently return the same integer, provided no information used in `equals` comparisons on the object is modified. This integer need not remain consistent from one execution of an application to another execution of the same application.
- If two objects are equal according to the `equals(Object)` method, then calling the `hashCode` method on each of the two objects must produce the same integer result.
- It is *not* required that if two objects are unequal according to the `equals(java.lang.Object)` method, then calling the `hashCode` method on each of the two objects must produce distinct integer results. However, the programmer should be aware that producing distinct integer results for unequal objects may improve the performance of hash tables.

As much as is reasonably practical, the hashCode method defined by class `Object` does return distinct integers for distinct objects. (This is typically implemented by converting the internal address of the object into an integer, but this implementation technique is not required by the Java™ programming language.)

Returns:

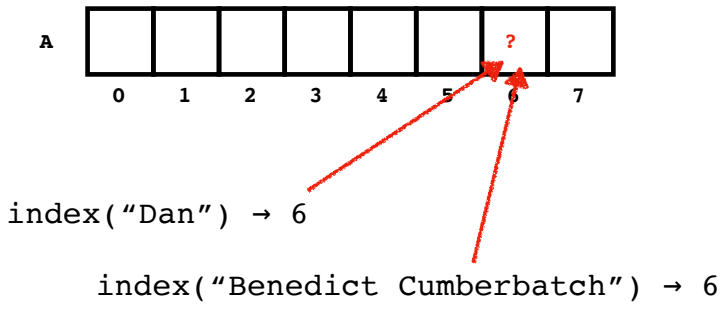
a hash code value for this object.

See Also:

`equals(java.lang.Object)`, `System.identityHashCode(java.lang.Object)`

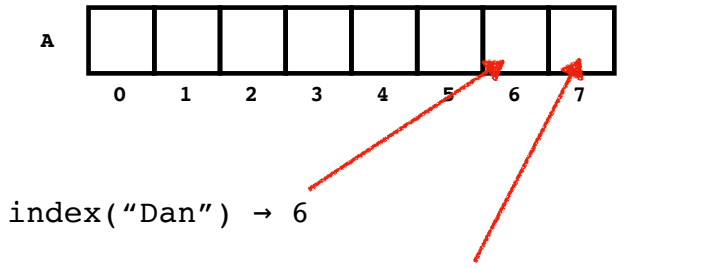
Hash collisions

A **hash collision** is when **two or more distinct keys** have the **same hash value**.



Perfect hash function

A **perfect hash function** is a hash function that ensures that **distinct keys** map to **distinct indices**. I.e., there are **no collisions**.



Perfect hash function

Problem: It's **pretty darn hard** to come up with a perfect hash function.

1. You need to know **all possible keys in advance**.
2. If the number of possible keys is large, it is **expensive to compute** ($O(n^2)$ time) and **expensive to store** ($O(n)$ space).

With a good hash table implementation, "imperfect" hash functions are usually **good enough**.

Dealing with collisions

There are **two approaches** to dealing with collisions:

1. Change your **hash function**.
2. Change your **hash table design**.

The easier of the two approaches turns out to be #2.

Open addressing

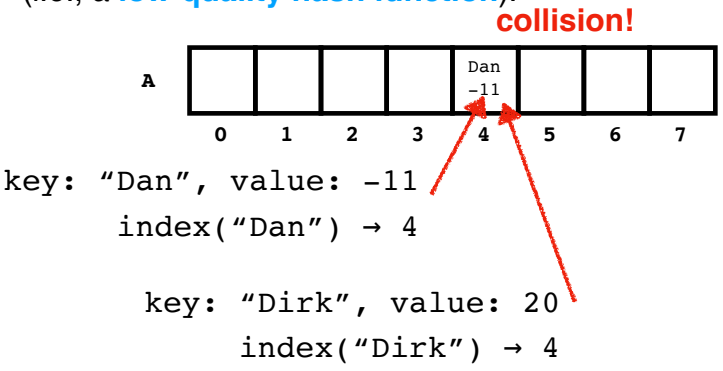
Open addressing is a method for resolving collisions in a hash table. Collisions are resolved by **probing**, which is a predetermined method for searching the hash table (aka **a probe sequence**). On **insertion**, probing finds the **first available bucket**. On **lookup**, probing searches until either the **key is found** or **an empty space** is found.

Linear probing

Suppose our keys are `Strings` and our hash function is

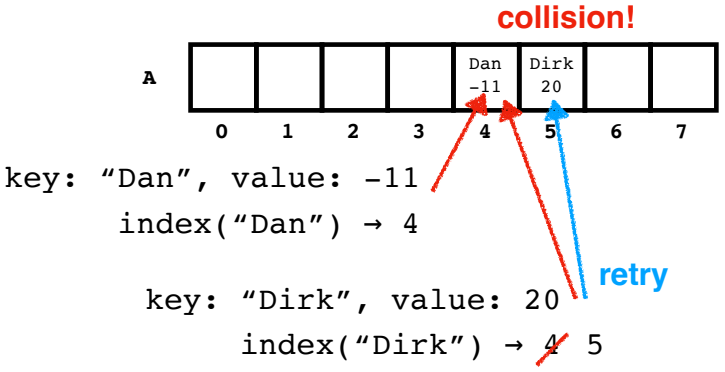
```
((int) key.charAt(0)) % A.length
```

(i.e., a **low-quality hash function**).



Linear probing

Linear probing works by scanning for *h*(`key`) + *c* × *i*, where *c* is a constant (usually 1) and *i* is the *i*th attempt.



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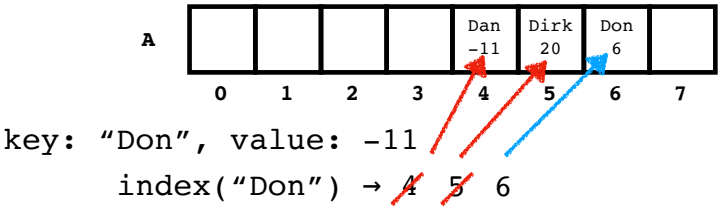
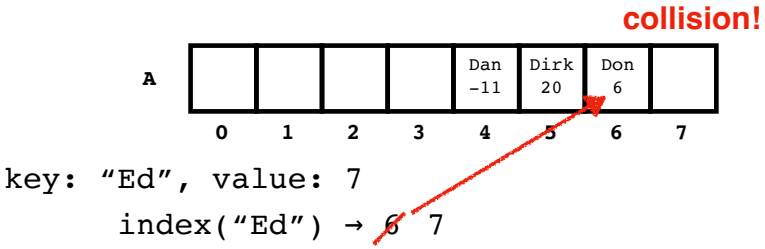


Linear probing

Downside: values **cluster** around **collisions**.



Likelihood of collisions grows as cluster grows.

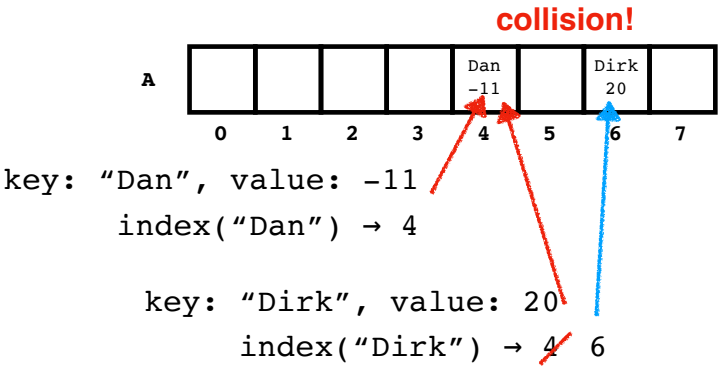
Our table is **still half empty**! This is **bad**!

Linear probing

h(key) + *c* × *i*

Changing *c* helps some.

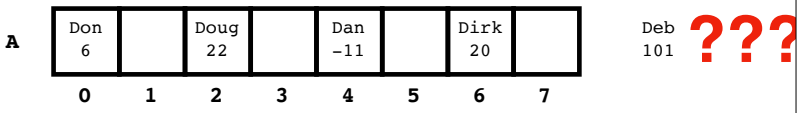
E.g., *c* = 2.



Linear probing

Changing *c* helps some.

But it can also **make the problem worse**.



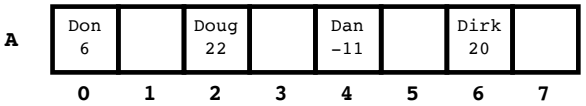
key: "Dan", value: -11
index("Dan") → 4

key: "Dirk", value: 20
index("Dirk") → 4 6

Now we are only using *1/c* buckets!

Linear probing: deletion

Deletions are also problematic.



`delete("Dan")`

`lookup("Dirk")`

We can no longer find `Dirk`.

Linear probing: deletion

Deletions are also problematic.

Addressed by leaving a sentinel value at deleted location.

`delete("Dan")`

`lookup("Dirk")`

Doesn't reclaim space until all colliding entries deleted.

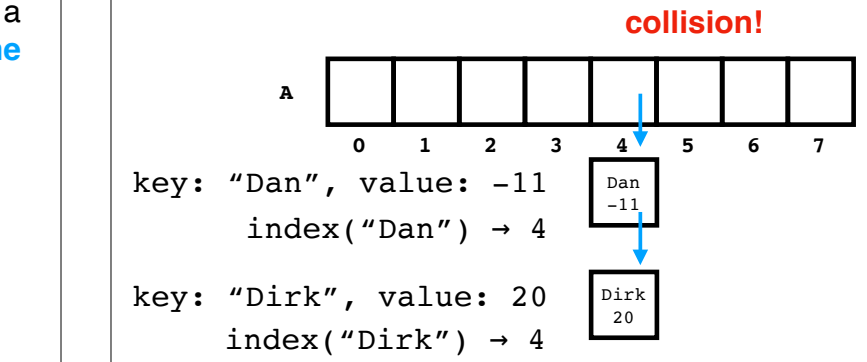
External chaining

External chaining is a method for resolving collisions in a hash table. Collisions are resolved by storing **more than one value in a bucket**, e.g., using a **list**.

External chaining

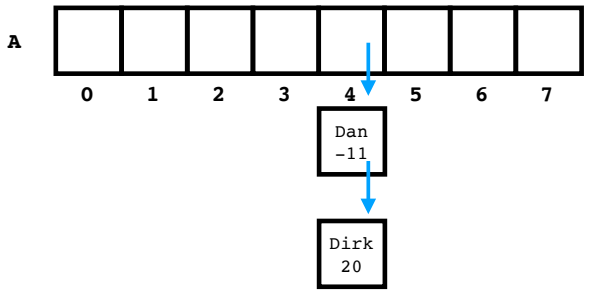
Same **bad hash function**:

```
((int) key.charAt(0)) % A.length
```



External chaining: deletion

Deletion is trivial.



Complexity

Method	Successful	Unsuccessful
Linear probes	$\frac{1}{2}\left(1+\frac{1}{(1-\alpha)}\right)$	$\frac{1}{2}\left(1+\frac{1}{(1-\alpha)^2}\right)$
Double hashing	$\frac{1}{\alpha}\ln\frac{1}{(1-\alpha)}$	$\frac{1}{1-\alpha}$
External chaining	$1+\frac{1}{2}\alpha$	$\alpha+e^{-\alpha}$

Figure 15.11 Expected theoretical performance of hashing methods, as a function of α, the current load factor. Formulas are for the number of association compares needed to locate the correct value or to demonstrate that the value cannot be found.

Hash Table Expansion

When a hash table **fills up**, we should **expand**, just as with a Vector. But there are some problems…

Hash Table Expansion

Virtually every hash function relies on **the size of the underlying array** to do the hashing. Recall:

```
int index(K key) {
  return abs(h(key) % A.length);
}
```

When a hash table expands, we usually address this by **rehashing** all elements during a copy. **Why is this OK?**

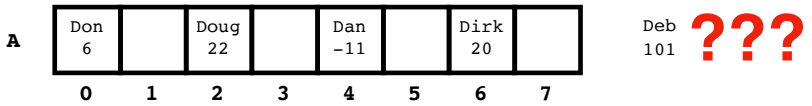
Hash Table Expansion

Another issue: hash table performance **degrades severely** as it **fills up**.

Recall that we can have an **effectively full** hash table even when there is actually space.

$h(\text{key}) + c \times i$

where $c = 2$



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Hash Table Expansion

Therefore, we resize before the table is likely to be full.

Let `n` be the **number of elements** stored in a hash table.

Let `m` be the **number of buckets**.

Load factor = `n / m`

When the load factor is reached, the hash table is **resized**.

Hash Table Expansion

There are two ways to find a good load factor.

1. Careful analysis of the probability of attempting to insert more than one element into the same bucket, combined with a preference for acceptable average slowdown.
2. Empirical measurement, combined with a preference for acceptable average slowdown.

A load factor of 0.7-0.8 is generally accepted to be a good threshold.

Recap & Next Class

Today:

Hash collisions

Graphs

Next class:

Graph algorithms