Sample Final Exam Solutions

Handout 7
CSCI 136: Spring 2022
8 May

This is a *closed book* exam. You have 150*† minutes to complete the exam. You may use the back of the preceding page for additional space if necessary, but be sure to mark you answers clearly.

In some cases, there may be a variety of implementation choices. The most credit will be given to the most elegant, appropriate, and efficient solutions.

Problem	Points	Description	Score
1	10	Short Answer	
2	10	Queues	
3	10	StackSort	
4	10	Heaps	
5	10	Binary Trees	
6	10	Hashing	
7	10	Iterators	
8	10	Time Complexity	
9	10	Graphs	
10	10	Data Structure Design	
Total	100		

I have neither given nor received aid on this examination.

Signature: _____

Name: _____

*In fact, 150 minutes is too little time for the practice exam! We are providing a larger- and harder-than-usual set of questions to help you prepare for the exam. The actual final will be closer to 7 questions.

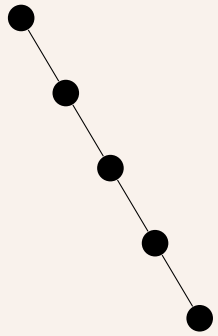
†Problem 10 is harder than any question you should expect to see on the final exam. But think about it anyway: you will be surprised at how well a little stretching prepares you!

1. (*10 points*) ..... Short Answer

Show your work and justify answers where appropriate.

a. A tree with n distinct elements is both a min-heap and a binary search tree. What must it look like?

By the min heap property, the smallest node must be at the top; by the binary search tree property all nodes must be right descendants of the root. This is then true recursively for every node. So the final tree must be a path; here is an example with 5 nodes:



b. Which tree traversal would you use to print an expression tree in human-readable form?

In-order traversal.

c. Which tree traversal would you use to evaluate an expression tree?

Post-order traversal. (You need the "solution" from both the left and right side before applying the operator.)

d. We applied sorting methods primarily to arrays and `Vector`s. Of the following sort algorithms, which are most appropriate to sort a `SinglyLinkedList`: insertion sort, selection sort, quicksort, merge sort? (Several of these can be used to sort the SLL—which works most easily? What obstacles need to be overcome when using the other sorting methods?)

The simplest is probably merge sort: breaking a linked list into two equal-sized parts can be $O(n)$ time, and merging two linked lists can be done in $O(n)$ time.

Quicksort would require modifying the partition method to work in $O(n)$ time on a linked list. (This can probably be done with a linear scan.) Then, after the two recursive quicksort calls, the two solutions and the pivot element must be carefully placed back together (note that this is an extra step not necessary in quicksort on an array). This is also a reasonable solution, but requires more modifications than merge sort.

For insertion sort or selection sort, the sorting method would have to be carefully modified to allow linked list items to be swapped efficiently. (Using `get()` to swap two items will lead to $O(n^3)$ rather than $O(n^2)$ running time.) This modification likely requires carefully maintaining two pointers, but is probably ultimately possible.

e. When we rewrite a recursive algorithm to be iterative, we generally must introduce which kind of data structure to aid in simulating the recursion?

A stack

2. (*10 points*) ..... Queues

Recall that the `Queue` interface may be implemented using an array to store the queue elements. Suppose that two `int` values are used to keep track of the ends of the queue. We treat the array as circular: adding or deleting an element may cause the head or tail to "wrap around" to the beginning of the array.

You are to provide a Java implementation of `class CircularQueueArray` by filling in the bodies of the methods below. Note that there is no instance variable which stored the number of elements currently in the queue; you must compute this from the values of `head` and `tail`. You may **not** add any additional instance variables.

```
public class CircularQueueArray {
  // instance variables
  protected int head, tail;
  protected Object[] data;

  // constructor: build an empty queue of capacity n
  public CircularQueueArray(int n) {
```

```
                head = 0;
                tail = 0;
                data = new Object[n];
                for(int i = 0; i < n; i++) {
                        data[i] = null;
                }
```

```
  }

  // pre: queue is not fill
  // post: adds value to the queue
  public void enqueue(Object value) {
```

```
        data[tail] = value;
        tail = (tail + 1) % data.length;
```

```
  }
```

Name: _____

```
  // pre: queue is not empty
  // post: removes value from the head of the queue
  public Object dequeue() {
```

```
        Object retVal = data[head];
        data[head] = null;
        head = (head + 1) % data.length;
        return retVal;
```

```
  }

  // post: return the number of elements in the queue
  public int size() {
```

```
        if (head <= tail) {
                return tail - head;
        }

        return data.length - (head - tail);
```

```
  }

  // post: returns true iff queue is empty
  public boolean isEmpty() {
```

```
          return head == tail && data[head] == null;
```

```
  }

  // post: returns true iff queue is full
  public boolean isFull() {
```

```
        return head == tail && data[head] != null;
```

```
  }
}
```

Name: _____

3. (*10 points*) ..... Stacks

Suppose you are given an iterator that will let you access a sequence of `Comparable` elements. You would like to sort them, but the only data structure available to you is an implementation of the `Stack` interface in the structure5 package (say, `StackList`), and memory constraints only allow you to make a small (constant) number of stacks. Because the elements are available only through an `Iterator`, so you must process each item as it is returned by the `next()` method of the `Iterator`. The sort method should return a `Stack` containing the sorted elements, with the smallest at the top of the stack. Please fill in the body of the method.

```
public static <E extends Comparable<E> > Stack<E> StackSort(Iterator<E> iter) {
  // pre: iter is an Iterator over a structure containing objects of type E
  // post: a Stack is returned with the elements sorted, smallest on top
```

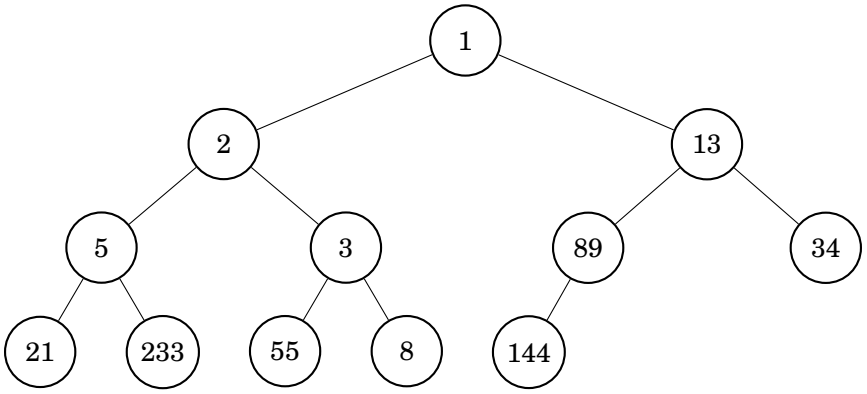
```
  Stack<E> retStack = new StackList<E>();
  Stack<E> tempStack = new StackList<E>();
  // strategy: keep retStack sorted each iteration by putting
  // next() into its proper location in the stack.
  // We use tempStack to temporarily hold everything smaller than next(),
  // then push next, then restore the smaller elements from tempStack.
  // (draw a picture!)
  while (iter.hasNext()) {
        E cur = iter.next();
        // pop everything smaller than cur from retStack
        while (!retStack.empty() && cur.compareTo(retstack.peek()) > 0) {
                tempStack.push(retStack.pop());
        }
        retStack.push(cur);
        while (!tempStack.empty()) {
                retStack.push(tempStack.pop());
        }
  }
  return retStack;
```

```
}
```

Name: _____

4. (*10 points*) ..... Heaps

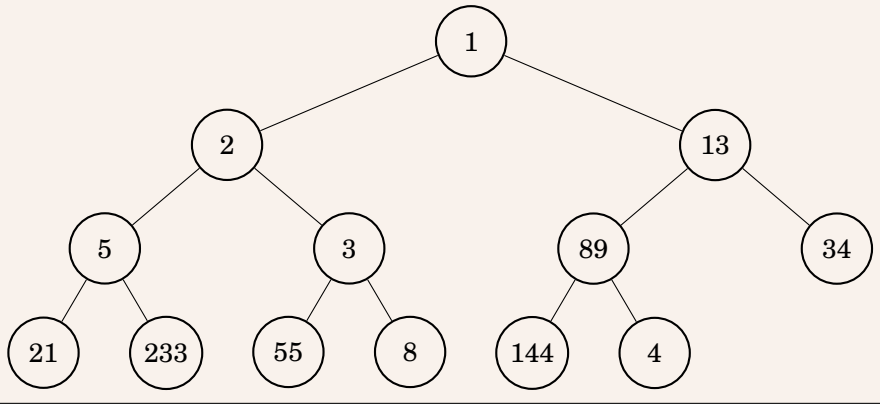
Recall the definition of a min-heap, a binary tree in which each node's value is no bigger than that of each of its descendants. For the rest of this question, we presume the Vector implementation of heaps (`class VectorHeap`). Consider the following tree, which is a min-heap.

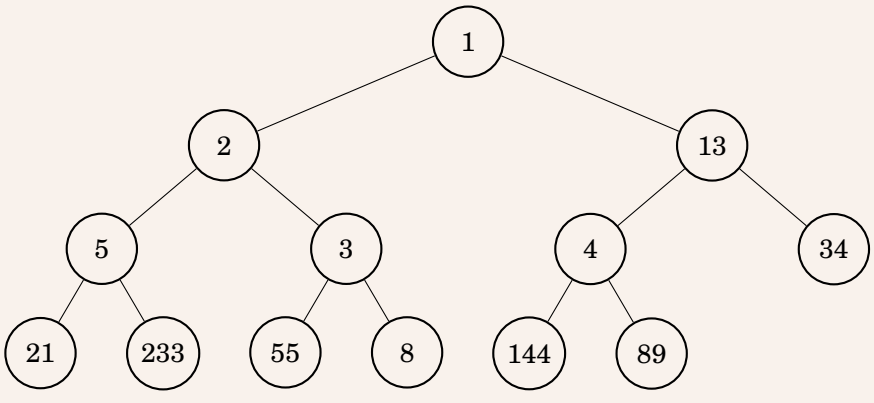


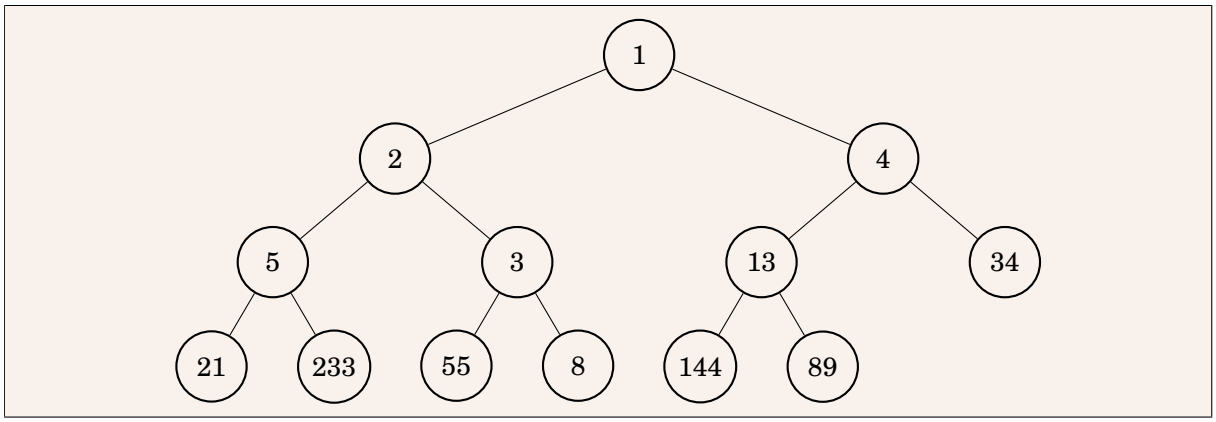
a. Show the order in which the elements would be stored in the `Vector` underlying our `VectorHeap`.

1	2	13	5	3	89	34	21	233	55	8	144
0	1	2	3	4	5	6	7	8	9	10	11

b. Show the steps involved in adding the value 4 to the heap. **Use drawings of the tree, not the vector.**



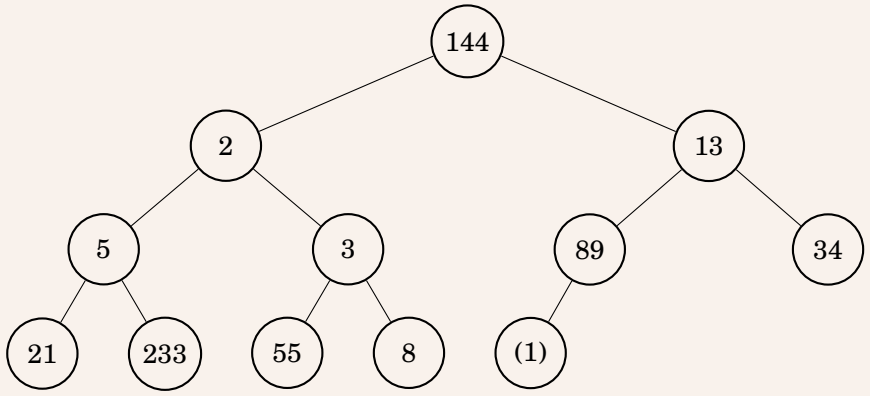


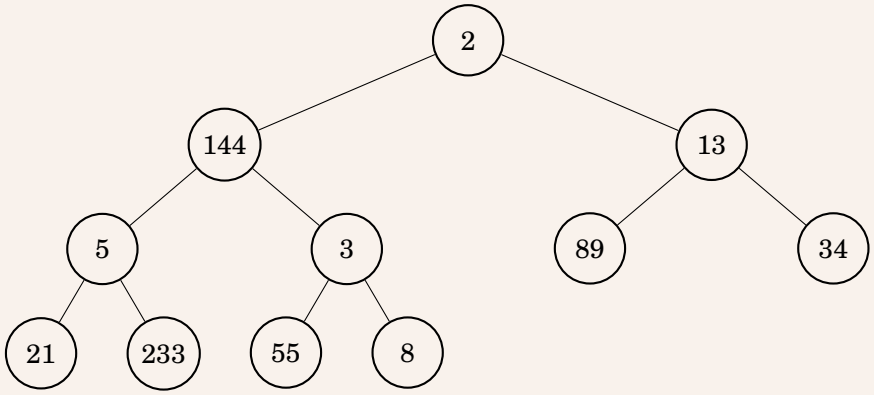

1
2
4
5
3
13
34
21
233
55
8
144
89

Name: _____

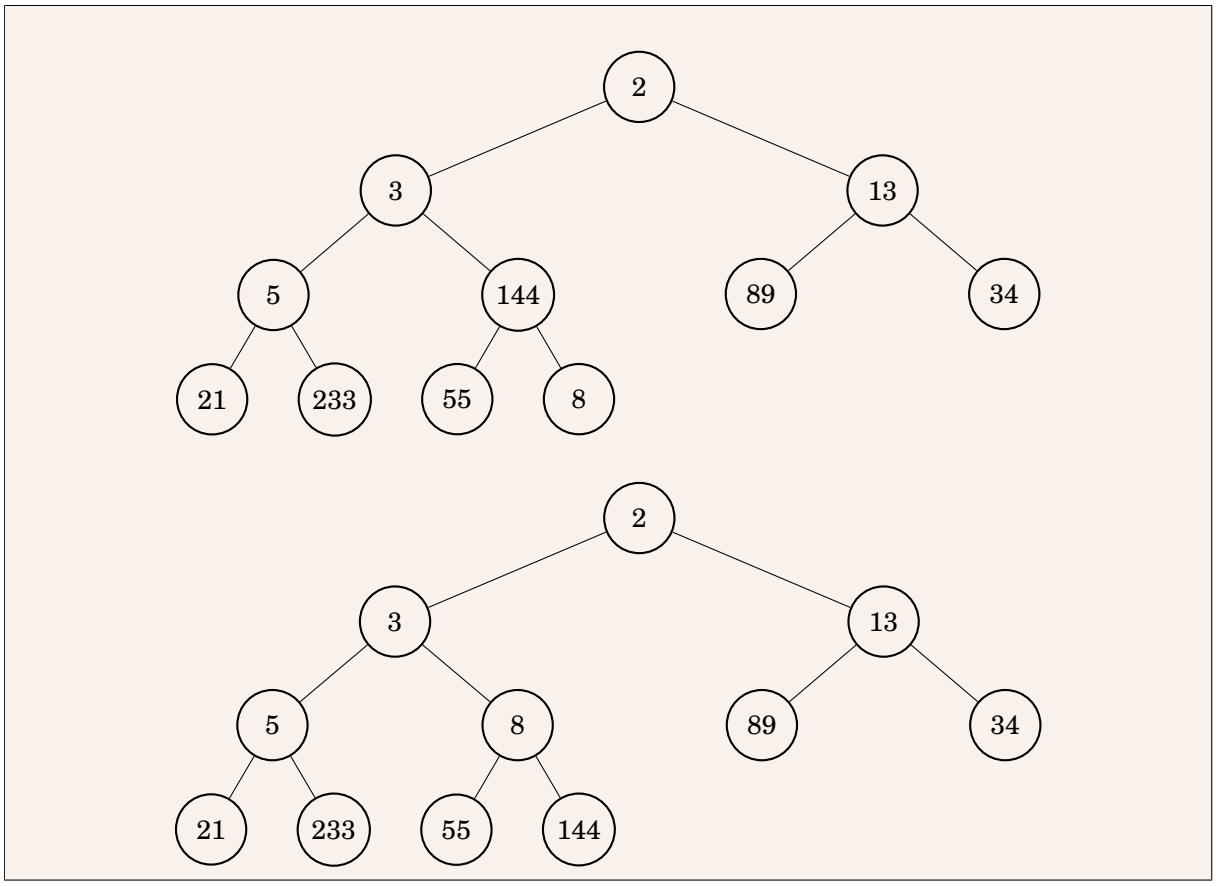
c. Using the original tree (not the one with the 4 added), show the steps involved in removing the minimum value of the heap.

1, the minimum value, is immediately removed after this step (so will not be in future trees):





(Continued on next page:)


2
3
13
5
144
89
34
21
233
55
8
2
3
13
5
8
89
34
21
233
55
144

d. Why is the `VectorHeap` implementation of a priority queue better than one that uses a linked list implementation of regular queues, modified to keep all items in order by priority? Hint: Your answer should compare the complexities of the add and remove operations.

The `VectorHeap` implementation requires $O(\log n)$ time to add and remove.

A linked list that keeps items in order requires $O(n)$ time to add, since all items must be scanned to find the correct location. However, removing the minimum-priority item is $O(1)$ time.

If we add and then remove $O(n)$ items, the `VectorHeap` requires $O(n \log n)$ total time for all operations, whereas the sorted linked list requires $O(n^2)$ time.

Name: _____

5. (*10 points*) ..... Binary Trees

Suppose we have a `BinaryTree` that contains only `Comparable` values.

a. It is often useful to find the minimum and maximum values in the tree. Implement the method `maximum` as a member of `class BinaryTree`. As a guide, relevant sections of `BinaryTree.java` from the `structure5` package are included after this question. Your method should return the `Comparable` that is the maximum value in the tree. It should return `null` if called on an empty tree.

```
public Comparable maximum() {
  // pre: the values in this tree are all Comparable
  // post: the maximum value in the tree is returned
```

```
        if(isEmpty()) {
                return null;
        }

        Comparable maxLeft = left.maximum();
        Comparable maxRight = right.maximum();

        Comparable maxChild;
        if(maxLeft != null && maxLeft.compareTo(maxRight) > 0 ) {
                maxChild = maxLeft;
        else {
                maxChild = maxRight;
        }

        Comparable thisVal = (Comparable) val;

        if(maxChild != null && maxChild.compareTo(thisVal) > 0) {
                return maxChild;
        } else {
                return thisVal;
        }
```

```
}
```

b. What is the worst-case complexity of `maximum` on a tree containing n values?

$O(n)$

c. What is the complexity of `maximum` on a full tree containing n values?

$O(n)$

Name: _____

d. Consider the following method, which I propose as a member of `class BinaryTree`:

```
public boolean isBST() {
  // post: returns true iff the tree rooted here is a binary search tree
  if (this == EMPTY) return true;
  return left().isBST() && right().isBST();
}
```

As written, this method will not always return the correct value. Explain why, then provide a correct method. You may use `minimum()` and `maximum()` from part (a), as well as any other methods of `BinaryTree`.

A binary search tree must have value $\geq$ all of its left descendants, and value $<$ all of its right descendants. This method does not compare any values.

```
public boolean isBST() {

        if(this == EMPTY) {
                return true;
        }

        Comparable compVal = (Comparable) val;

        if(compVal.compareTo(left.maximum()) < 0) {
                return false;
        }

        if(compVal.compareTo(right.minimum()) >= 0) {
                return false;
        }

        return left.isBST() && right.isBST();
}
```

e. In `class BinaryTree`, why is the `setLeft()` method `public`, but the `setParent()` method is `protected`?

In short: the `BinaryTree` class maintains the invariant that if node `n` has node `m` as its child, then node `m` must have node `n` as its parent.

The `setLeft()` method changes the child of a given node—but then it automatically calls `setParent()` to ensure that the invariant is maintained.

The `setParent()` method must be protected so that other classes cannot cause this invariant to be violated. (Note that `setParent()` cannot call `setLeft()`, since `setLeft()` calls `setParent()`.)

Name: _____

```
public class BinaryTree {
    protected Object val; // value associated with node
    protected BinaryTree parent; // parent of node
    protected BinaryTree left; // left child of node
    protected BinaryTree right; // right child of node
    // The unique empty node
    public static final BinaryTree EMPTY = new BinaryTree();

    // A one-time constructor, for constructing empty trees.
    private BinaryTree() {
        val = null; parent = null; left = right = this;
    }

    // Constructs a tree node with no children.  Value of the node
    // is provided by the user
    public BinaryTree(Object value) {
        val = value; parent = null; left = right = EMPTY;
    }

    // Constructs a tree node with no children.  Value of the node
    // and subtrees are provided by the user
    public BinaryTree(Object value, BinaryTree left, BinaryTree right) {
        this(value);
        setLeft(left);
        setRight(right);
    }

    // Get left subtree of current node
    public BinaryTree left() {
        return left;
    }

    // Get right subtree of current node
    public BinaryTree right() {
        return right;
    }

    // Get reference to parent of this node
    public BinaryTree parent() {
        return parent;
    }

    // Update the left subtree of this node.  Parent of the left subtree
    // is updated consistently.  Existing subtree is detached
    public void setLeft(BinaryTree newLeft) {
        if (isEmpty()) return;
        if (left.parent() == this) left.setParent(null);
        left = newLeft;
        left.setParent(this);
    }

    // Update the right subtree of this node.  Parent of the right subtree
    // is updated consistently.  Existing subtree is detached
    public void setRight(BinaryTree newRight) {
        if (isEmpty()) return;
        if (right.parent() == this) right.setParent(null);
        right = newRight;
        right.setParent(this);
```

```
}

// Update the parent of this node
protected void setParent(BinaryTree newParent) {
    parent = newParent;
}

// Returns the number of descendants of node
public int size() {
    if (this == EMPTY) return 0;
    return left().size() + right.size() + 1;
}

// Returns reference to root of tree containing n
public BinaryTree root() {
    if (parent() == null) return this;
    else return parent().root();
}

// Returns height of node in tree.  Height is maximum path
// length to descendant
public int height() {
    if (this == EMPTY) return -1;
    return 1 + Math.max(left.height(),right.height());
}

// Compute the depth of a node.  The depth is the path length
// from node to root
public int depth() {
    if (parent() == null) return 0;
    return 1 + parent.depth();
}

// Returns true if tree is full.  A tree is full if adding a node
// to tree would necessarily increase its height
public boolean isFull() {
    if (this == EMPTY) return true;
    if (left().height() != right().height()) return false;
    return left().isFull() && right().isFull();
}

// Returns true if tree is empty.
public boolean isEmpty() {
    return this == EMPTY;
}

// Return whether tree is complete.  A complete tree has minimal height
// and any holes in tree would appear in last level to right.
public boolean isComplete() {
    int leftHeight, rightHeight;
    boolean leftIsFull, rightIsFull, leftIsComplete, rightIsComplete;
    if (this == EMPTY) return true;
    leftHeight = left().height();
    rightHeight = right().height();
    leftIsFull = left().isFull();
    rightIsFull = right().isFull();
    leftIsComplete = left().isComplete();
    rightIsComplete = right().isComplete();
```

```
        // case 1: left is full, right is complete, heights same
        if (leftIsFull && rightIsComplete &&
            (leftHeight == rightHeight)) return true;
        // case 2: left is complete, right is full, heights differ
        if (leftIsComplete && rightIsFull &&
            (leftHeight == (rightHeight + 1))) return true;
        return false;
    }

    // Return true iff the tree is height balanced.  A tree is height
    // balanced iff at every node the difference in heights of subtrees is
    // no greater than one
    public boolean isBalanced() {
        if (this == EMPTY) return true;
        return (Math.abs(left().height()-right().height()) <= 1) &&
                left().isBalanced() && right().isBalanced();
    }

    // Returns value associated with this node
    public Object value() {
        return val;
    }
}
```

Name: _____

6. (*10 points*) ..... Hashing

a. What is meant by the "load factor" of a hash table?

Number of elements divided by the number of slots.

b. We take care to make sure our hash functions return the same hash code for any two equivalent (by the `equals()` method) objects. Why?

Hash codes are often used to implement a `Map` using a hash table. The `get` method of `Map` asks the user to look up a key that is `.equals()` to the requested key. We must ensure that the hash codes are the same so that `get()` looks in the correct position.

(There would be issues with `contains()` as well, and even `put()`, which searches to ensure no duplicate keys.)

c. A hash table with *ordered linear probing* maintains an order among keys considered during the rehashing process. For example, we may store the keys in a run in sorted order in the hash table (while ensuring that each key occurs on or after the slot it initially hashed to).

When the keys are encountered, say, in increasing order, the performance of a failed lookup approaches that of a successful search. Describe how a key might be inserted into the ordered sequence of values that compete for the same initial table entry.

When we detect a collision, we will shift the remainder of the run to make room for the new object so that it can be stored in sorted order. (Another way of saying this: we will treat the run as an OrderedVector, and place each new item into the correct place within the run as if it were shorted, shifting the other elements down.)

d. Is the hash table constructed using ordered linear probing as described in part (c) really just an ordered vector? Why or why not?

No; the hash table will not be sorted by the keys. It will only be sorted within runs.

e. One means of potentially reducing the complexity of computing the hash code for `String`s is to compute it once – when the `String` is constructed. Future calls to `hashCode()` would return this precomputed value. Since Java `String`s are immutable, that is, they cannot change once constructed, this could work. Do you think this is a good idea? Why or why not?

Probably not. Many (if not most) `String`s are created and used without ever being stored in a hash table. If we compute the hash code every time we create the string, then all strings are forced to pay this high cost.

(That said, of course, in a use case where most `String`s get hashed repeatedly, this would save time.)

7. (*10 points*) ..... Iterators

Given a pair of iterators that return data in sorted order, a `Mergerator` merges them into a single iterator that returns all data in sorted order. Here is a simple example of how it might be used:

```
String a[] = { "brown", "cow", "moo" };
String b[] = { "another", "sentence", "sorted" };

AbstractIterator<String>> iter1 = ArrayIterator<String>(a);
AbstractIterator<String>> iter2 = ArrayIterator<String>(b);

Iterator<String> merger = new Mergerator(iter1,iter2);
while (merger.hasNext()) {
  System.out.println(merger.next());
}
```

This code prints out:

```
another brown cow moo sentence sorted
```

This problem asks about implementing the `Mergerator` class to provide the `AbstractIterator` methods: `reset()`, `next()`, `hasNext()`. (You do not need to implement `get()`; also, your `next()` method should not rely on `get()`.)

Complete the code for the `Mergerator` class. Declare any instance variables you will use in your class and implement the constructor and methods (`reset`, `hasNext`, and `next`). Your class should be parameterized by the type `E` of elements that will be returned by the underlying iterators.

```
public class Mergerator <E extends Comparable<E>> extends AbstractIterator<E> {
```

```
AbstractIterator<E> iter1;
AbstractIterator<E> iter2;

public Mergerator(AbstractIterator<E> theIter1, AbstractIterator<E> theIter2) {
    iter1 = theIter1;
    iter2 = theIter2;

}
```

```
  public void reset() {
```

```
    iter1.reset();
    iter2.reset();
```

```
  }
  public boolean hasNext() {
```

```
    return iter1.hasNext() || iter2.hasNext();
```

```
  }
  public E next() {
```

```
    if(iter1.hasNext() &&
            (!iter2.hasNext() || iter1.get().compareTo(iter2.get())) <= 0) {
      return iter1.next();
    } else {
      return iter2.next();
    }
```

```
  }
  public E get() {
```

```
    if(iter1.hasNext() &&
            (!iter2.hasNext() || iter1.get().compareTo(iter2.get())) <= 0) {
      return iter1.get();
    } else {
      return iter2.get();
    }
```

```
  }
  }
```

Name: _____

8. (*10 points*) ..... Time Complexity

Suppose you are given n lists, each of which is of size n and each of which is sorted in increasing order. We wish to merge these lists into a single sorted list L, with all n^2 elements. For each algorithm below, determine its time complexity (Big O) and justify your result.

a. At each step, examine the smallest element from each list; take the smallest of those elements, remove it from its list and add it to the end of L. Repeat until all input lists are empty.

Finding the smallest element of all lists is $O(n)$ time. We repeat this $O(n^2)$ times, for $O(n^3)$ total time.

b. Merge the lists in pairs, obtaining $\frac{n}{2}$ lists of size $2n$. Repeat, obtaining $\frac{n}{4}$ lists of size $4n$, and so on, until one list remains.

Merging two lists of size k requires $O(k)$ time (as discussed in class). Therefore, merging *all* pairs of lists (to obtain $n/2$ lists of size $2n$) requires $O(n^2$ time; similarly, merging all pairs of lists of size $2n$ to obtain $n/4$ lists of size $4n$ requires $O(n^2)$ time, and so on.

Each time we merge all pairs, the number of lists is divided by two. We begin with n lists, and end up with 1 list, so we need to merge all pairs $O(\log n)$ times.

We perform $O(\log n)$ merges, each of which takes $O(n^2)$ time, for $O(n^2 \log n)$ time in total.

Name: _____

9. (*10 points*) ..... Graphs

a. Recall the `Trie` structure you implemented for the Lexicon lab. It was a general tree, where a node in the tree could have an arbitrary number of children. Trees are nothing more than graphs with some restrictions on the edges allowed. You could store the same information in a `Graph` by making a `Vertex` for each tree node and adding `Edge`s representing the links to the children. Which `Graph` implementation would you use for this, and why? How does its time and space complexity compare to your `Trie` implementation?

I would use an adjacency list implementation. The number of edges in this graph is generally going to be much less than the number of vertices squared, so this is the space efficient choice. Furthermore, for the lexicon lab, we traverse the tree by iterating over the children of a node; the child whose stored letter is equal to the next character in the word is the next child we visit. This corresponds to iterating over the neighbors of a vertex in a graph. The adjacency list implementation is significantly more efficient at iterating over all neighbors of a vertex, so it is more time efficient as well.

The space efficiency of an adjacency list graph with n vertices and m edges is $O(n+m)$. The space efficiency of a trie with n' nodes, that have a total of m' children, is $O(n' + m')$. Note that $n = n'$ and $m = m'$, so the space efficiency is the same.

The time to obtain a child of a node in the `Trie` with d children is $O(d)$. The time to iterate through all neighbors of a node in an adjacency list with d incident edges is $O(d)$. Therefore, the time to traverse also remains the same.

Name: _____

10. (*10 points*) ..... Data Structure Design

Be careful to answer all parts of this ridiculously long question!

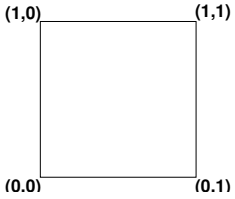
No answers for this one—it's just for fun! Good luck.

The structures we have studied this semester all store Java `Object`s. Many structures place no restrictions on the types of the objects that can be stored. `OrderedStructure`s limit the types of objects that may be store to those which implement the `Comparable` interface, to allow storage based on an ordering of the objects using the `compareTo()` method. If our objects have Cartesian coordinates, perhaps they could be stored in a structure that uses this information. These objects might be mesh elements in a scientific computation using the finite element method, particles in a particle-in-cell simulation, or coordinates of cities or other points of interest on a map. Much of what we look at in this problem works for coordinates of any dimension, but for simplicity, we will consider objects with two-dimensional coordinates. Such `Object`s can implement the following interface.

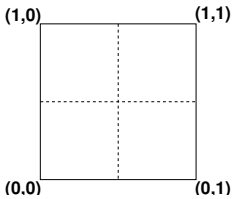
```
public interface CoordObject {
    // pre: none
    // post: returns object's x coordinate
    public double getX();

    // pre: none
    // post: returns object's y coordinate
    public double getY();
}
```

Now we wish to store n `CoordObject`s in a structure. Suppose we know that all of these objects have coordinates some range. Again for simplicity, we will say they are all in the square bounded by $(0, 0)$ and $(1, 1)$.



Suppose we break the square into 4 smaller squares:



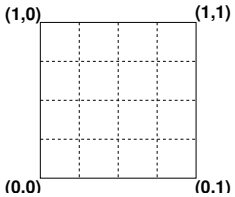
Name: _____

Objects now can be placed into one of four groups (bins) based on coordinates. We can define methods `add()`, `contains()`, *etc.* that insert into or search in the right bin.

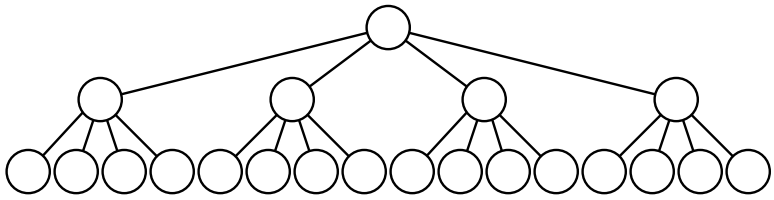
a. What structure would you use to store the objects that belong in each bin?

b. Suppose the n objects are inserted and are approximately evenly distributed among the four bins. How many comparisons will be needed for a call to `contains()` on an object that is not found? Be sure to show how you get your answer.

Perhaps we should break into more squares:



We could continue adding more and more bins until a reasonably small number of objects ends up in each bin. We can view the structure that we are creating as a *quadtree* (or in three dimensions, an *octree*). Consider this tree:



The root represents the square we started with (the universe). The next level represents the four-way subdivision, and the leaves represent the 16-way subdivision. The nodes are called `quadrants`. Only the leaf nodes contain objects stored in our quadtree. The others implicitly "contain" the objects contained by their descendants. During a search, the coordinates of an object are used to guide a search through the tree to the appropriate leaf quadrant.

c. Describe how a call to `contains()` would work on a quadtree of height h.

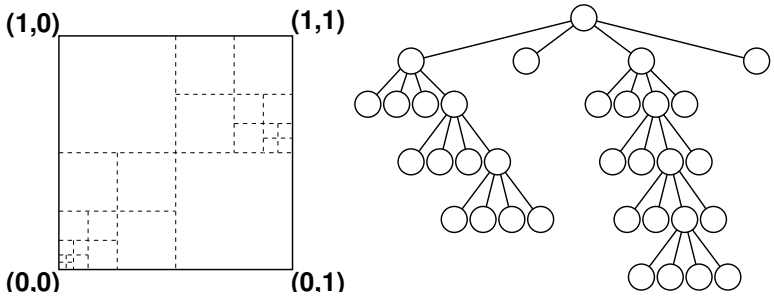
d. Again assuming a regular distribution of the objects among the bins, what is the complexity of an unsuccessful `contains()` method call? Explain briefly.

e. Now suppose that the distribution is not regular at all – for the tree of height h, the n objects end up with half in one bin and half in another. How does this affect the complexity of an unsuccessful `contains()` call?

Perhaps we should add more levels to the tree. If we do this "quadrant refinement" only for the part of the tree that becomes overcrowded, we can say our tree is *adaptive*. We start with only a root node. Objects are added to the root until a capacity threshold is exceeded. The root is then divided into four child quadrants, and the objects it contains are distributed among the four children. If subsequent additions of objects cause one of the children to exceed its capacity, it does the same. Parts of the tree corresponding to parts of the universe that contain most of the objects are refined more, while less crowded parts remain coarse.

Name: _____

If object coordinates are most likely to be near the origin and near $(0.9, 0.6)$, we might end up with a tree that looks like this:



f. In both figures above, mark with an X the quadrant which would contain an object with coordinates $(0.3, 0.3)$.

g. The following is an implementation of an adaptive quadtree structure. Like the `BinaryTree` from the structure package, it is defined recursively, so subtrees are themselves valid quadtrees. Some methods required by `AbstractStructure` have been left out for simplicity. Please fill in the bodies of the methods marked with `// ** WRITE THIS METHOD`. You need not explicitly enforce preconditions. Be sure to make use of the `protected` methods I have provided for you as appropriate.

```
public class Quadrant extends AbstractStructure {
    // the bounding coordinates of the space occupied by this quadrant
    protected double minX, minY, maxX, maxY;

    // child quadrants, all non-null for interior quadrants
    // all null for leaf quadrants
    protected Quadrant children[];

    // constants to refer to the child quadrants in meaningful ways
    // these are the indices into the children array
    public static final int NorthEast=0;
    public static final int NorthWest=1;
    public static final int SouthWest=2;
    public static final int SouthEast=3;

    // Threshold at which the quadrant should refine into four new
    // children and distribute its contents among them.
    // We could make this a parameter, but it is constant for simplicity
    protected static final int capacity=64;
```

Name: _____

```
// The contents
protected Structure contents;

// create a new quadrant.  Only leaf quadrants are created by the
// constructor.  Leaf quadrants may become interior quadrants
// if they are later refined during an add operation
public Quadrant(double minX, double minY, double maxX, double maxY) {
    this.minX=minX; this.minY=minY; this.maxX=maxX; this.maxY=maxY;
    children[0]=null; children[1]=null; children[2]=null; children[3]=null;
    // we can use any Structure here, for simplicity, a SinglyLinkedList
    contents=new SinglyLinkedList();
}

// pre: none
// post: returns whether this is a leaf quadrant
protected boolean isLeaf() {
    // contents is non-null only at leaf quadrants
    return contents!=null;
}

// pre: this must be an interior quadrant
//      (x,y) of object must lie within the bounding box of this quadrant
// post: return child Quadrant containing the coordinates of the object
protected Quadrant getChildQuadrant(Object obj) {
    CoordObject coord=(CoordObject)obj;
    double x=coord.getX(); double y=coord.getY();
    double midX=(minX+maxX)/2; double midY=(minY+maxY)/2;
    if (x < midX) {
        if (y < midY) return children[SouthWest];
        else return children[NorthWest];
    } else {
        if (y < midY) return children[SouthEast];
        else return children[NorthEast];
    }
}

// refine this quadrant
// pre: the quadrant is a leaf and has exceeded its capacity
// post: four child quadrants are created, the contents of this
//       quadrant are dispersed among them.  This quadrant
//       is transformed into an interior quadrant
protected void refine() {
    // first create the children
    children=new Quadrant[4];
    double midX=(minX+maxX)/2;
    double midY=(minY+maxY)/2;
    children[SouthWest]=new Quadrant(minX, minY, midX, midY);
    children[NorthWest]=new Quadrant(minX, midY, midX, maxY);
    children[SouthEast]=new Quadrant(midX, minY, maxX, midY);
    children[NorthEast]=new Quadrant(midX, midY, maxX, maxY);

    // loop over the contents, insert as appropriate
    Iterator i=contents.iterator();
    while (i.hasNext()) {
        Object obj=i.next();
        Quadrant child=getChildQuadrant(obj);
        child.add(obj);
```

```
    }

    // destroy contents by letting the garbage collector have at it
    contents=null;
}

// pre: object being added is a CoordObject, whose coordinates
//      lie within the bounding box of this quadrant
// post: the object is inserted into this Quadrant or one of its
//       children.  If this is an interior quadrant, the request
//       is passed to a child quadrant.  If this is a leaf, it is
//       inserted here.  If this add causes the quadrant's capacity
//       to be exceeded, the quadrant will be refined.
public void add(Object obj) {
    // ** WRITE THIS METHOD









}
```

Name: _____

```
// pre: none
// post: returns number of objects contained in this quadrant for a leaf,
//       sum of sizes of children for non-leaf
public int size() {
    // ** WRITE THIS METHOD









}

// pre: none
// post: clears this quadrant (for a leaf) or all of its children
//       (for interior)
public void clear() {
    // ** WRITE THIS METHOD









}
```

Name: _____

```
    // pre: none
    // post: removes the specified object from this quadrant (for a leaf)
    //       or the appropriate child (for interior)
    //       object removed is returned, or null is returned if the object
    //       was not found
    public Object remove(Object obj) {
        // ** WRITE THIS METHOD









    }

    // pre: none
    // post: return true iff the object is contained in this quadrant's
    //       contents (for a leaf) or in the appropriate child quadrant
    //       (for interior)
    public boolean contains(Object obj) {
        // since this method is almost the same as remove, you need
        // not provide it.
    }
}
```

h. What is the complexity of your `add()` method? Explain.