Consider the following code to compute the square of a number \( n \geq 0 \).

```java
/** computes the square of a number
 * @param n the number to be squared
 * @return n*n
 * @pre n >= 0
 * @post returns n*n
 */
public static int square(int n) {
    int ret = 0;
    int odd = 1;
    for (int i = 1; i <= n; i++) {
        ret += odd;
        odd += 2;
    }
    return ret;
}
```

1. What can you say about the value of \( ret \) after the loop has iterated \( k \) times?

_**Hint:** trace the loop execution for some small values of \( n \)

_**Hint #2:** we’re looking for a very simple statement about the loop

Your answer: After the loop iterates \( k \) times, \( ret \) holds the value \( k^2 \).

2. Prove by induction that the above method correctly returns the square of \( n \). Remember to write all parts of an inductive proof. (You can use the back of the paper if you run out of room)

_**Hint:** you may want to use the fact that \((k+1)^2 = k^2 + (2k + 1)\)

_**Answer below:**

We’ll prove by induction that after the loop iterates \( n \) times, \( ret \) stores the value \( n^2 \). Since we then return \( ret \), the correct value is returned.

**Base case:** after the loop iterates 0 times, \( ret \) stores 0, which is \( 0^2 \).

**Inductive Hypothesis:** For some \( k \geq 0 \), after the loop iterates \( k \) times, \( ret \) stores the value \( k^2 \).

**Inductive Step:** By the inductive hypothesis, after the loop iterates \( k \) times, \( ret \) stores the value \( k^2 \). Let’s say it iterates one more time.

Since the loop has iterated \( k \) times, \( odd \) stores the value \( 1 + 2k \) at the beginning of the \( k + 1 \)th loop iteration. In the \( k + 1 \)st iteration, the only change to \( ret \) is \( ret += odd \). Since \( ret \) is \( k^2 \) (by the inductive hypothesis) and \( odd \) is \( 1 + 2k \) at the beginning of the loop, after this line is run \( ret \) stores \( k^2 + (2k + 1) = (k + 1)^2 \). Therefore, after the \( k + 1 \)st iteration, \( ret \) stores the value \((k + 1)^2\), and the inductive hypothesis is proven for \( k + 1 \).