The following equation can be proven by induction. For all $n \geq 1$,

$$1 + 4 + 16 + 64 + \ldots + 4^n = (4^{n+1} - 1)/3$$

We’ve completed part of an inductive proof below. Fill in the missing details.

Prove the **base case**:

Your answer: \(\text{For } n = 0, \text{ we have that } 1 = (4^1 - 1)/3.\)

(Alternative solution: \(\text{For } n = 1, \text{ we have that } 1 + 4 = 5 = (4^2 - 1)/3.\)

State the **inductive hypothesis**: For some \(k\),

Your answer: \(1 + 4 + 16 + \ldots + 4^k = (4^{k+1} - 1)/3.\)

Now, we prove the **inductive step**. As a first step in our proof, we show that

$$1 + 4 + 16 + \ldots + 4^k + 4^{k+1} = (4^{k+1} - 1)/3 + 4^{k+1}$$

Why is the above statement true?

Your answer: \(\text{We assume the inductive hypothesis; then, we add } 4^{k+1} \text{ to both sides.}\)

With the above equation in hand, we can manipulate the right hand side algebraically, obtaining

$$1 + 4 + 16 + \ldots + 4^k + 4^{k+1} = (4^{k+1} - 1)/3 + 4^{k+1}$$
$$= (4^{k+1} - 1 + 3 \cdot 4^{k+1})/3$$
$$= (4 \cdot 4^{k+1} - 1)/3$$
$$= (4^{k+2} - 1)/3$$

Since our final equation matches the form of the original equation, where \(n = k + 1\), we have shown by induction for all \(n \geq 1\), that

$$1 + 4 + 16 + 64 + \ldots + 4^n = (4^{n+1} - 1)/3$$