Practice Quiz 5

CSCI 136: Spring 2022

Your name: ____________________________

This week's quizzes cover readings, handouts, labs, and lecture materials up to and including Monday 3/7. Answer the following questions as practice for your graded quiz on Friday.

The following equation can be proven by induction. For all $n \geq 1$,

$$1 + 4 + 16 + 64 + \ldots + 4^n = (4^{n+1} - 1)/3$$

We've completed part of an inductive proof below. Fill in the missing details.

Prove the **base case:**

Your answer: For $n = 0$, we have that $1 = (4^1 - 1)/3$.

(**Alternative solution:**) For $n = 1$, we have that $1 + 4 = 5 = (4^2 - 1)/3$.

State the **inductive hypothesis:** For some k,

Your answer: $1 + 4 + 16 + \ldots + 4^k = (4^{k+1} - 1)/3$.

Now, we prove the **inductive step.** As a first step in our proof, we show that

$$1 + 4 + 16 + \ldots + 4^k + 4^{k+1} = (4^{k+1} - 1)/3 + 4^{k+1}$$

Why is the above statement true?

Your answer: We assume the inductive hypothesis; then, we add 4^{k+1} to both sides.

With the above equation in hand, we can manipulate the right hand side algebraically, obtaining

$$\begin{aligned} 1 + 4 + 16 + \ldots + 4^k + 4^{k+1} &= (4^{k+1} - 1)/3 + 4^{k+1} \\ &= (4^{k+1} - 1 + 3 \cdot 4^{k+1})/3 \\ &= (4 \cdot 4^{k+1} - 1)/3 \\ &= (4^{k+2} - 1)/3 \end{aligned}$$

Since our final equation matches the form of the original equation, where $n = k + 1$, we have shown by induction for all $n \geq 1$, that

$$1 + 4 + 16 + 64 + \ldots + 4^n = (4^{n+1} - 1)/3$$