Definitions

Walk A walk from $u$ to $v$ in a graph $G = (V, E)$ is an alternating sequence of vertices and edges $u = v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k = v$ such that $e_i = \{v_i, v_{i+1}\}$ for $i = 1, \ldots, k$.

Path A path is a walk with no repeated edge.

Simple Path A simple path is a path with no repeated vertex.

Closed Walk A closed walk in a graph $G = (V, E)$ is a walk $v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k$ such that $v_0 = v_k$. In other words, in a closed walk, the ending vertex is the same as the starting vertex.

Circuit A circuit is a closed walk with no repeated edge. Alternatively, it is a path where $v_0 = v_k$.

Cycle A cycle is a closed walk with no repeated vertex except the starting vertex. Alternatively, it is a simple path where $v_0 = v_k$.

Walk Length The length of a walk is the number of edges in the sequence.

Degree The degree of a vertex $v$, $\deg(v)$, is the number of edges incident to $v$.

Adjacent Two vertices are adjacent if and only if they share an edge.

Incident A vertex is incident to an edge if the vertex is one of the endpoints of the edge.

In-degree The in-degree of a vertex $v$ on a directed graph, $\text{in-deg}(v)$, is the number of incoming edges incident to $v$.

Out-degree The out-degree of a vertex $v$ on a directed graph, $\text{out-deg}(v)$, is the number of outgoing edges incident to $v$.

Reachable A vertex $v$ is reachable from vertex $u$ in a graph $G = (V, E)$ if there is a path from $u$ to $v$. In an undirected graph, if $v$ is reachable from $u$, then $u$ is reachable from $v$.

Connected An undirected graph $G = (V, E)$ is connected if for every pair of vertices $u, v$ in $G$, $v$ is reachable from $u$.

Connected Component The set of all vertices reachable from $v$ in $G = (V, E)$, along with all edges of $G$ connecting any two of them, is called the connected component of $v$. The connected component is itself a graph.