Graph Terminology

CSCI 136: Spring 2022

Definitions

Walk A *walk* from u to v in a graph $G = (V, E)$ is an alternating sequence of vertices and edges $u = v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k = v$ such that $e_i = \{v_i, v_{i+1}\}$ for $i = 1, \ldots, k$

Path A *path* is a walk with no repeated edge.

Simple Path A *simple path* is a path with no repeated vertex.

Closed Walk A *closed walk* in a graph $G = (V, E)$ is a walk $v_0, e_1, v_1, e_2, v_2, \ldots, v_{k-1}, e_k, v_k$ such that $v_0 = v_k$. In other words, in a closed walk, the ending vertex is the same as the starting vertex.

Circuit A *circuit* is a closed walk with no repeated edge. Alternatively, it is a path where $v_0 = v_k$.

Cycle A *cycle* is a closed walk with no repeated vertex except the starting vertex. Alternatively, it is a simple path where $v_0 = v_k$.

Walk Length The *length* of a walk is the number of edges in the sequence.

Degree The *degree* of a vertex v, deg(v), is the number of edges incident to v.

Adjacent Two vertices are *adjacent* if and only if they share an edge.

Incident A vertex is *incident* to an edge if the vertex is one of the endpoints of the edge.

In-degree The *in-degree* of a vertex v on a directed graph, in-deg(v), is the number of incoming edges incident to v.

Out-degree The *out-degree* of a vertex v on a directed graph, out-deg(v), is the number of outgoing edges incident to v.

Reachable A vertex v is *reachable* from vertex u in a graph $G = (V, E)$ if there is a path from u to v. In an undirected graph, if v is reachable from u, then u is reachable from v.

Connected An undirected graph $G = (V, E)$ is connected if for every pair of vertices u, v in G, v is reachable from u.

Connected Component The set of all vertices reachable from v in $G = (V, E)$, along with all edges of G connecting any two of them, is called the *connected component* of v. The connected component is itself a graph.