CSCI 136
Data Structures &
Advanced Programming

Lecture 31

Spring 2020

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Last Time

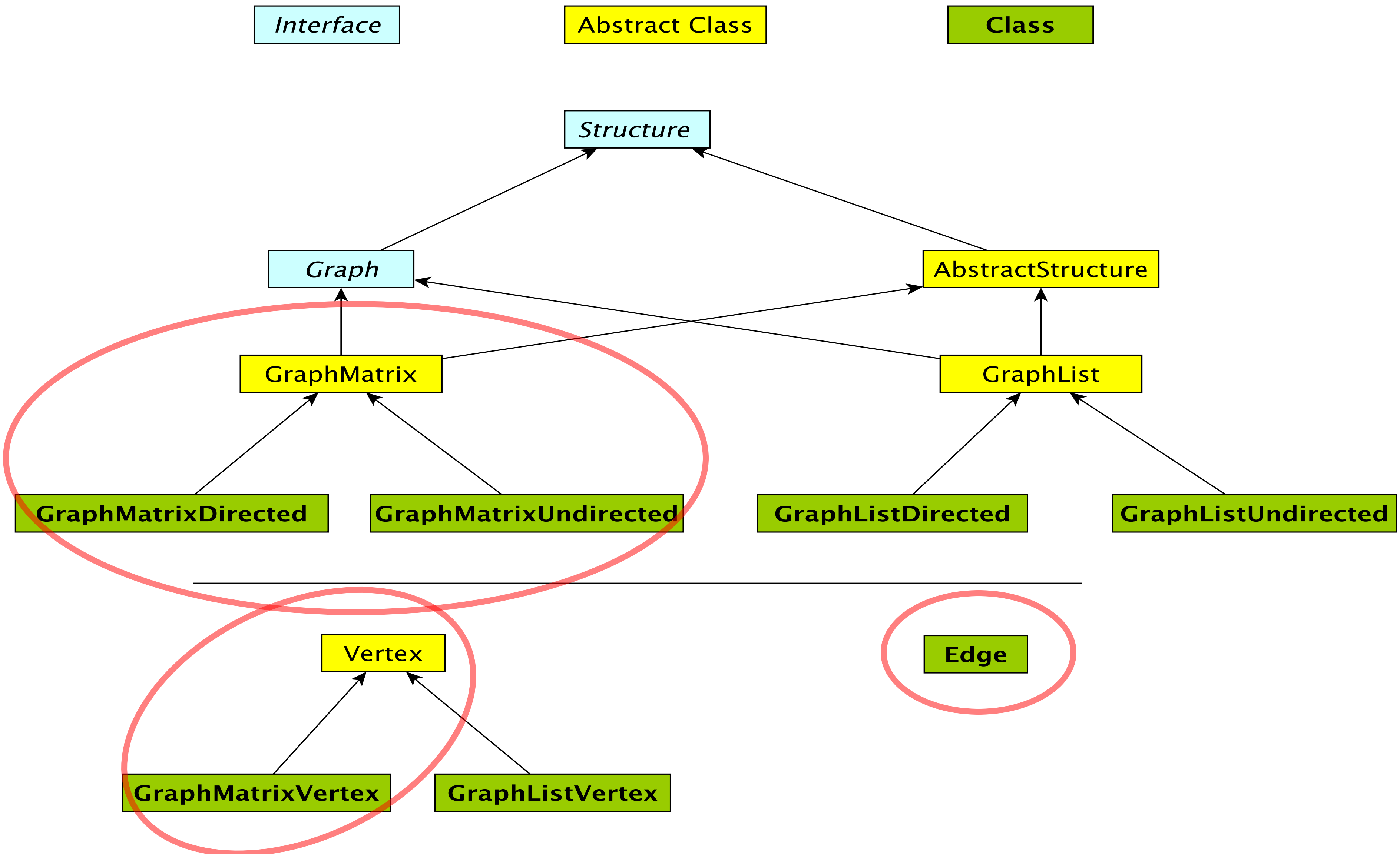
- Graph Intro & Vocab
  - Vertices, edges, neighbors, degree
  - Path, walk, circuit
  - Connectedness, reachability
  - Directedness
- API & Graph<V,E> Interface
- Traversals & Reachability
  - Breadth-First & Depth-First Searches

Today's Outline

- Graph Implementation Details
  - Adjacency Matrix
  - Adjacency List
- Time/Space Complexity

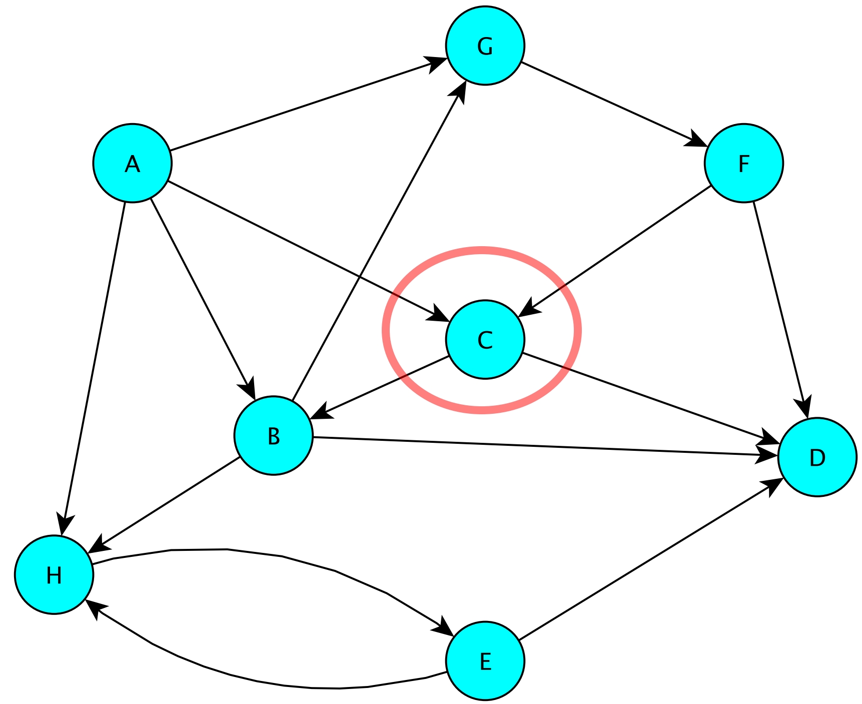
- Last Graph day, so may go fast! Focus on the high level ideas and the ***tradeoffs***

Graph Classes in structure5



Adjacency Array: Directed Graph

	A	B	C	D	E	F	G	H
A	0	1	1	0	0	0	1	1
B	0	0	0	1	0	0	1	1
C	0	1	0	1	0	0	0	0
D	0	0	0	0	0	0	0	0
E	0	0	0	1	0	0	0	1
F	0	0	1	1	0	0	0	0
G	0	0	0	0	0	1	0	0
H	0	0	0	0	1	0	0	0



Challenges to having our rows/columns be "vertices"

- Can't use Objects as array indices
- How does adding/deleting a vertex work?!

Vertex and GraphMatrixVertex

- We need to define a Vertex class
  - Unlike the Edge class, Vertex class **is not public**
  - Useful Vertex methods:

    ```
    V label(), boolean visit(),
    boolean isVisited(), void reset()
    ```

  - `GraphMatrixVertex` adds one more useful attribute to the abstract `Vertex` class:
    - Index of node (int) in adjacency matrix

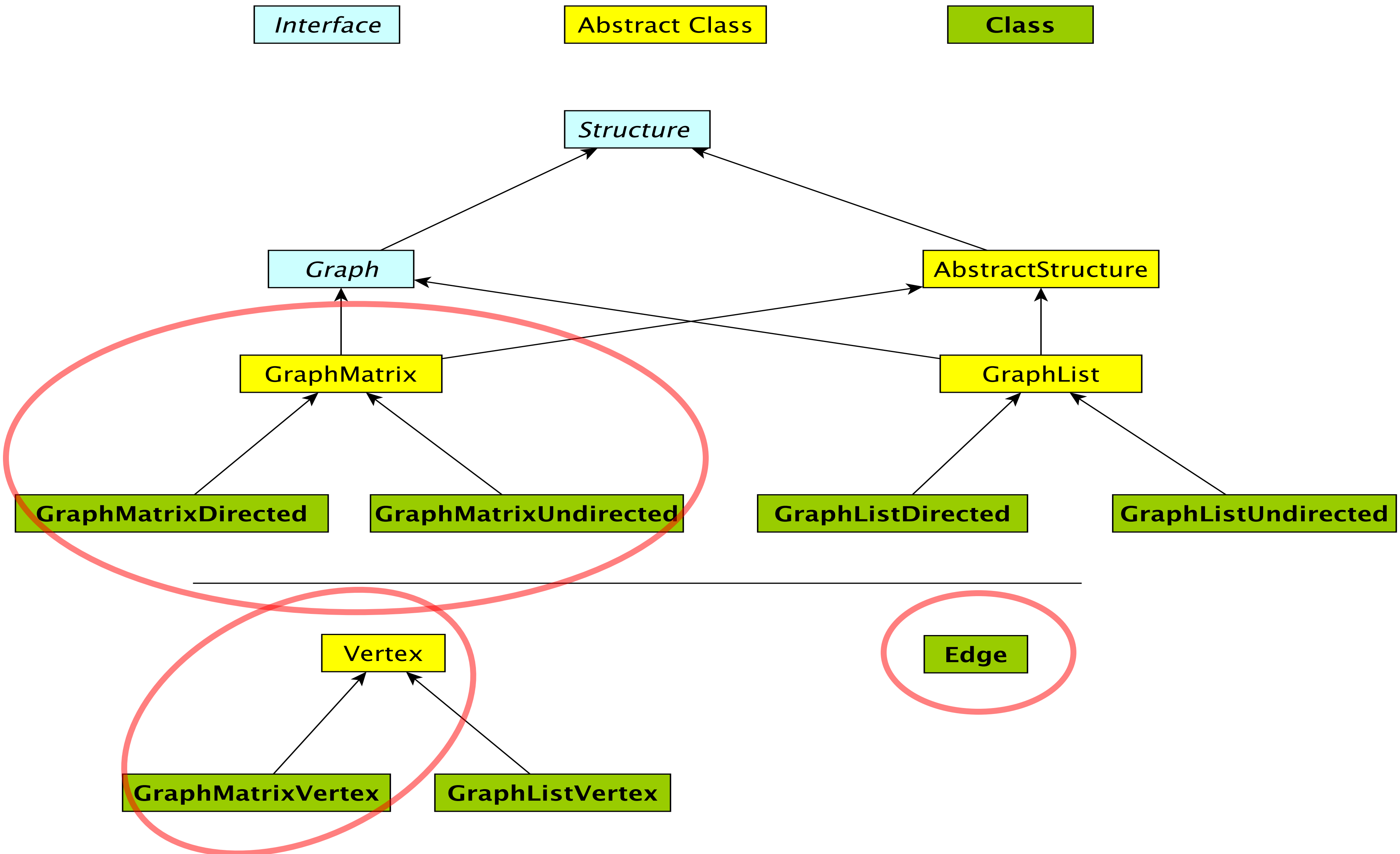
      ```
      int index()
      ```

    - Why do we only need one `int` to represent index?
      - Our matrix is NxN: a vertex is at the same row and column offset

Choosing a Dictionary Structure

- We need a structure that will let us retrieve the index of a vertex given the vertex label (a dictionary)
- Many choices
  - `Vector` of `Associations`:
    - `Vector<Association<V, GraphMatrixVertex<V>>>`
  - `OrderedVector` of `Associations`
  - `BinarySearchTree` of `Associations`
- Problem: We don't want to allow multiple vertices with same label.... [Why?]
- We'll use the Map Interface [Hashtable!]
  - Maps require a unique key for each entry

Graph Classes in structure5



Implementing the Matrix Model

- Abstract class – partially implements Graph

```
public abstract class GraphMatrix<V,E> implements Graph<V,E>
```

- This class will implement features common to directed and undirected graphs
- Instance variables

```
protected int size;  //max size of matrix
protected Object data[][];  //matrix of edges
protected Map<V, GMV<V>> dict; //labels -> vertices
// This is structure5.Map, NOT java.util.Map!
protected List<Integer> freeList; //available indices
protected boolean directed;
```

GraphMatrix Constructor

(Yes, abstract classes can have constructors!)

```
protected GraphMatrix(int size, boolean directed) {
    this.size = size; // set maximum size
    this.directed = directed; // fix direction of edges

    // the following constructs a size x size matrix
    // (the “Objects” will be “Edges”)
    // (can’t use generics with arrays!)
    data = new Object[size][size];

    // label→index translation table
    dict = new Hashtable<V,GraphMatrixVertex<V>>(size);

    // put all indices in the free list
    freeList = new SinglyLinkedList<Integer>();
    for (int row = size - 1; row >= 0; row--)
        freeList.add(new Integer(row));
}
```

GraphMatrix add()

```
public void add(V label) {
     // if there already, do nothing
     if (dict.containsKey(label)) return;

     Assert.pre(!freeList.isEmpty(), "Matrix not full");

     // allocate a free row and column
     int row = freeList.removeFirst().intValue();

     // add vertex to dictionary
     dict.put(label, new GraphMatrixVertex<V>(label, row));
}
```

GraphMatrix remove()

```
public V remove(V label) {
        // find and extract vertex
        GraphMatrixVertex<V> vert = dict.remove(label);
        if (vert == null) return null;
        // remove vertex from matrix
        int index = vert.index();
        // clear row and column entries
        for (int row=0; row<size; row++) {
            data[row][index] = null;
            data[index][row] = null;
        }
        // add node index to free list
        freeList.add(new Integer(index));
        return vert.label();
}
```

Neighbors Iterator : GraphMatrix

neighbors() Iterator: Construct a list of all of a Node’s neighbors and return that list’s iterator

```
public Iterator<V> neighbors(V label) {
      GraphMatrixVertex<V> vert = dict.get(label);
      List<V> list = new SinglyLinkedList<V>();
      for (int row=size-1; row>=0; row--) {
            Edge<V,E> e = (Edge<V,E>)data[vert.index()][row];
            if (e != null)
                  if (e.here().equals(vert.label()))
                        list.add(e.there());
                        else list.add(e.here());
      }
      return list.iterator();
   }
```

GraphMatrixDirected

- Completes the implementation of GraphMatrix to ensure graph is directed
- GraphMatrixUndirected is very similar…
- How do we implement GraphMatrixDirected?
  - We'll discuss some methods
  - Read Ch 16 for complete details…

GraphMatrixDirected

- Constructor

```
public GraphMatrixDirected(int size) {
   // pre: size > 0
   // post: constructs an empty graph that may be
   //       expanded to at most size vertices. Graph
   //       is directed if dir true and undirected
   //       otherwise

   // call GraphMatrix constructor
   super(size,true);
}
```

GraphMatrixDirected

- addEdge

```
// pre: vLabel1 and vLabel2 are labels of existing vertices
public void addEdge(V vLabel1, V vLabel2, E label) {
    // get indices
    GraphMatrixVertex<V> vtx1, vtx2;
    vtx1 = dict.get(vLabel1);
    vtx2 = dict.get(vLabel2);
    // Construct edge (since directed, order matters)
    Edge<V,E> e = new Edge<V,E>(vtx1.label(), vtx2.label(),
                                label, true);
    // add edge to matrix
    data[vtx1.index()][vtx2.index()] = e;
}
```

GraphMatrixDirected

- removeEdge

```
// pre: vLabel1 and vLabel2 are labels of existing vertices
public E removeEdge(V vLabel1, Vlabel2) {
    // get indices
    int row = dict.get(vLabel1).index();
    int col = dict.get(vLabel2).index();
    // cache old value
    Edge<V,E> e = (Edge<V,E>)data[row][col];
    // update matrix
    data[row][col] = null;
    if (e == null) return null;
    else   return e.label(); // return old value
}
```

GraphMatrix Efficiency

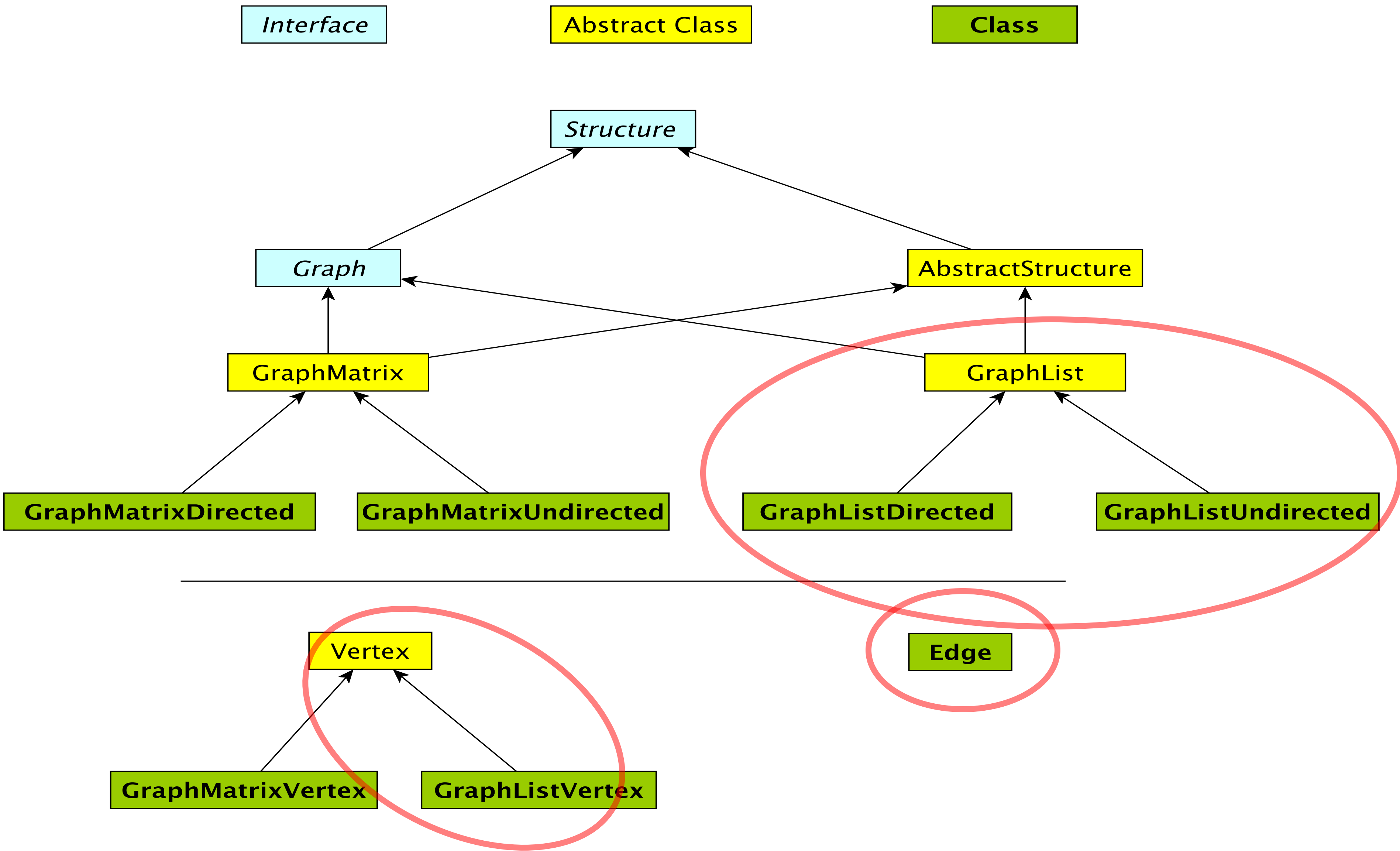
- Assuming Map operations are O(1), and
  - |E| = number of edges
  - |V| = number of vertices
- Runtime of add, addEdge, getEdge, removeEdge, remove?
- Space usage?
- Conclusions
  - Matrix is good for dense graphs
  - Have to commit to maximum # of vertices in advance

Efficiency : Assuming Fast Map

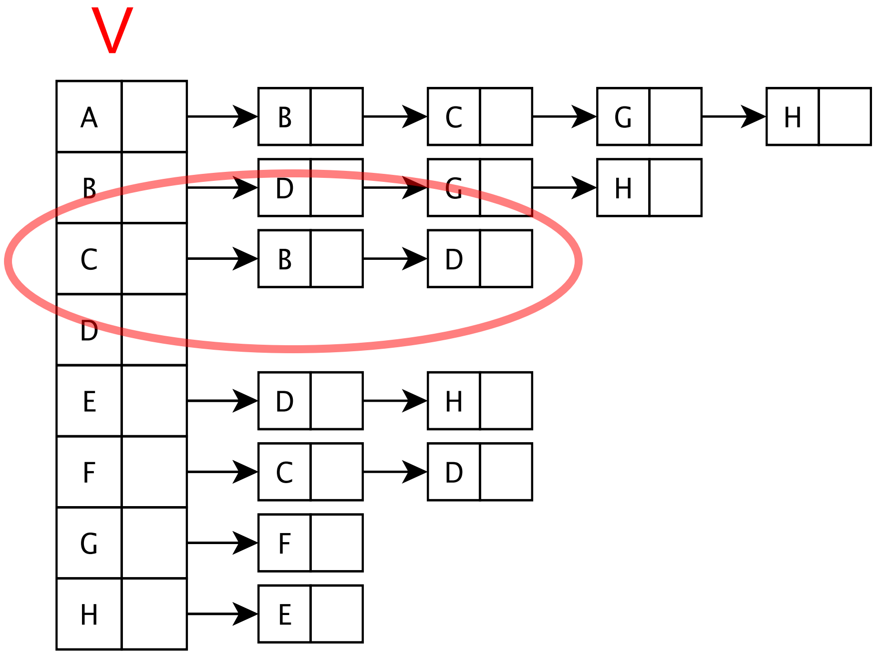
	GraphMatrix
add	$O(1)$
addEdge	$O(1)$
getEdge	$O(1)$
removeEdge	$O(1)$
remove	$O(\|V\|)$
space	$O(\|V\|^2)$

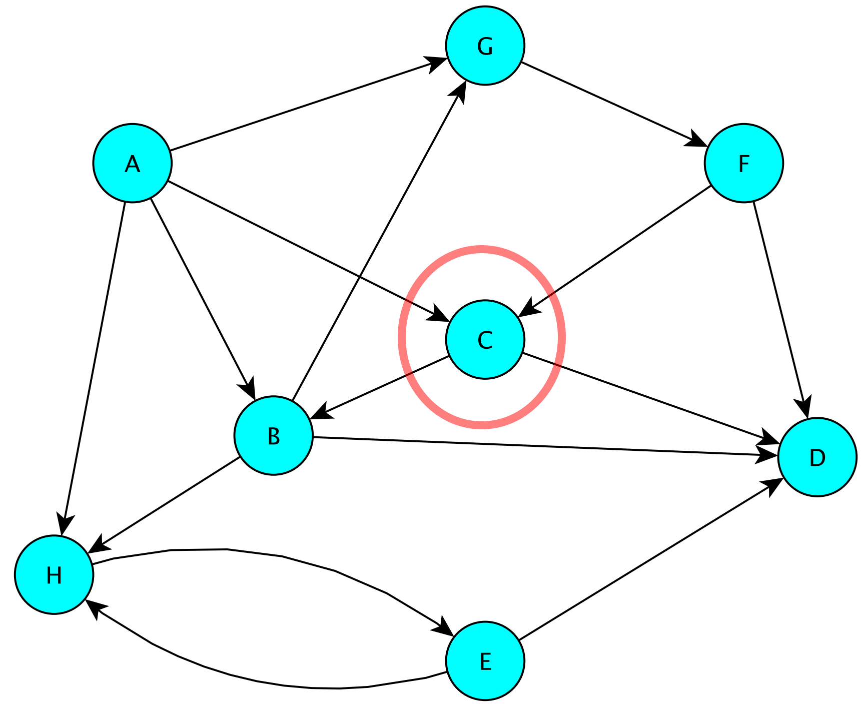
Must delete all edges associated with the deleted vertex.

Graph Classes in structure5



Adjacency List : Directed Graph

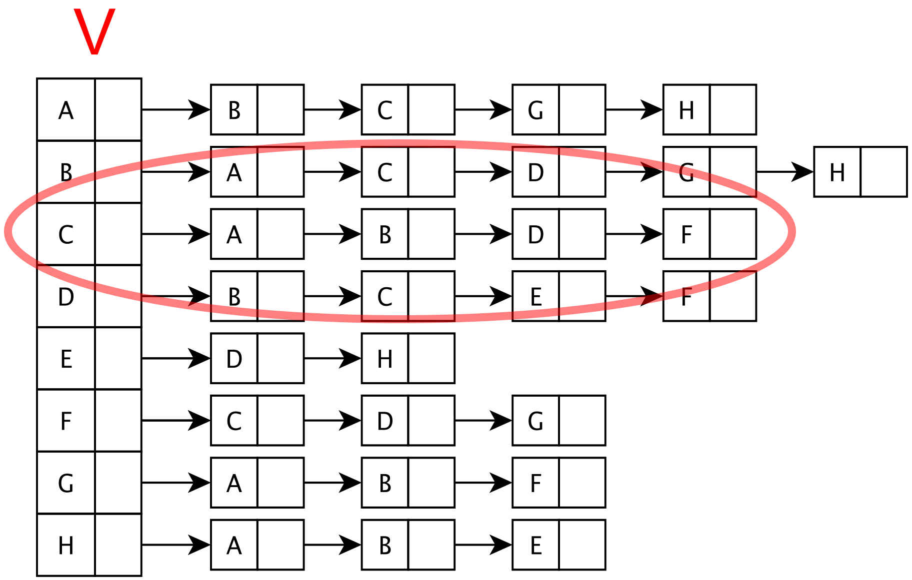


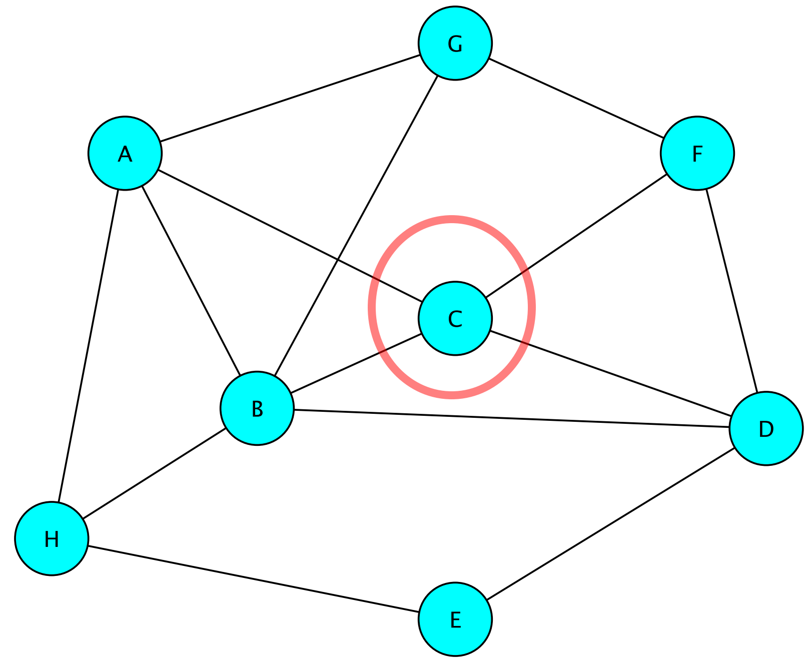


The vertices are stored in an array V[]

V[i] contains a linked list of all edges with a given source

Adjacency List : Undirected Graph





The vertices are stored in an array V[]

V[i] contains a linked list of all edges incident to a given vertex

GraphList: Big Picture

- Maintain an *adjacency list* of edges at each vertex (no adjacency matrix)
  - Keep only outgoing edges for directed graphs
- Support both directed and undirected graphs (GraphListDirected, GraphListUndirected)

Vertex and GraphListVertex

- We use the same Edge class for all graph types
- We extend Vertex to include an Edge list
- GraphListVertex class adds to Vertex class:
  - A Structure to store edges adjacent to the vertex

    protected Structure<Edge<V,E>> adjacencies; // adjacent edges

    – adjacencies is created as a SinglyLinkedList of edges
  - Several methods

```
public void addEdge(Edge<V,E> e)
public boolean containsEdge(Edge<V,E> e)
public Edge<V,E> removeEdge(Edge<V,E> e)
public Edge<V,E> getEdge(Edge<V,E> e)
public int degree()
// and methods to produce Iterators...
```

GraphListVertex

```
public GraphListVertex(V label){
        super(label); // init Vertex fields
        adjacencies = new SinglyLinkedList<Edge<V,E>>();
}

public boolean containsEdge(Edge<V,E> e){
        return adjacencies.contains(e);
}

public void addEdge(Edge<V,E> e){
        if (!containsEdge(e)) adjacencies.add(e);
}

public Edge<V,E> removeEdge(Edge<V,E> e) {
        return adjacencies.remove(e);
}
```

GraphListVertex Iterators

```
// Iterator for incident edges
public Iterator<Edge<V,E>> adjacentEdges() {
        return adjacencies.iterator();
}

// Iterator for adjacent vertices
public Iterator<V> adjacentVertices() {
        return new GraphListAIterator<V,E>
            (adjacentEdges(), label());
}
```

GraphListAIterator creates an Iterator over *vertices* based on the Iterator over *edges* produced by `adjacentEdges()`

(Details in the book and on posted slides)

GraphList (Abstract base class)

- To implement `GraphList`, what data structures do we need?
  - (Maintain an *adjacency list* of edges at each vertex)
- `GraphListVertex` class
  - Instance vars: `label`, `visited` flag, linked list of edges
- "Array V[]" of `GraphListVertex`
  - I Lied! We actually use a Map from V to GraphListVertex:
    Map<V,GraphListVertex<V,E>> dict; // label -> vertex
- Do we need a free list like `GraphMatrix`?
- Do we need to know |V| ahead of time?

GraphList

```
protected Map<V,GraphListVertex<V,E>> dict;
protected boolean directed;

protected GraphList(boolean dir){
      dict = new Hashtable<V,GraphListVertex<V,E>>();
      directed = dir;
}

public void add(V label) {
      if (dict.containsKey(label)) return;
      GraphListVertex<V,E> v = new
            GraphListVertex<V,E>(label);
      dict.put(label,v);
}
```

```
public Edge<V,E> getEdge(V label1, V label2) {
      Edge<V,E> e = new Edge<V,E>(get(label1),
                                  get(label2), null, directed);
      return dict.get(label1).getEdge(e);
}
```

(in GraphListVertex)

```
public Edge<V,E> getEdge(Edge<V,E> e) {
      Iterator<Edge<V,E>> edges = adjacencies.iterator();
      while (edges.hasNext()) {
            Edge<V,E> adjE = edges.next();
            if (e.equals(adjE))
                  return adjE;
      }
      return null;
}
```

GraphListDirected

- GraphListDirected (GraphListUndirected) implements the methods requiring different treatment due to (un)directedness of edges
  - addEdge, remove, removeEdge, …
- (We will only look at GraphListDirected in class)

```java
// addEdge in GraphListDirected.java
// first vertex is source, second is destination
 public void addEdge(V vLabel1, V vLabel2, E label) {
   // first get the vertices
   GraphListVertex<V,E> v1 = dict.get(vLabel1);
   GraphListVertex<V,E> v2 = dict.get(vLabel2);
   // create the new edge
   Edge<V,E> e = new Edge<V,E>(v1.label(), v2.label(), label, true);
   // add edge only to source vertex linked list (aka adjacency list)
   v1.addEdge(e);
}
```

```java
public V remove(V label) {
   //Get vertex out of map/dictionary
   GraphListVertex<V,E> v = dict.get(label);

   //Iterate over all vertex labels (called the map "keyset")
   Iterator<V> vi = iterator();
   while (vi.hasNext()) {
        //Get next vertex label in iterator
        V v2 = vi.next();

        //Skip over the vertex label we're removing
        //(Nodes don't have edges to themselves...)
        if (!label.equals(v2)) {
            //Remove all edges to "label"
            //If edge does not exist, removeEdge returns null
            removeEdge(v2,label);
        }
    }
    //Remove vertex from map
    dict.remove(label);
    return v.label();
}
```

```java
public E removeEdge(V vLabel1, V vLabel2) {
   //Get vertices out of map
   GraphListVertex<V,E> v1 = dict.get(vLabel1);
   GraphListVertex<V,E> v2 = dict.get(vLabel2);

   //Create a “temporary” edge connecting two vertices
   Edge<V,E> e = new Edge<V,E>(v1.label(), v2.label(), null, true);

   //Remove edge from source vertex linked list
   e = v1.removeEdge(e);
   if (e == null) return null;
   else return e.label();
}
```

Efficiency Revisited

- Assume Map operations are O(1) (for now)
  - |E| = number of edges
  - |V| = number of vertices
- Runtime of add, addEdge, getEdge, removeEdge, remove?
- Space usage?
- Conclusions
  - Matrix is better for dense graphs
  - List is better for sparse graphs
  - For graphs "in the middle" there is no clear winner

Efficiency : Assuming Fast Map

	Matrix	GraphList
add	O(1)	O(1)
addEdge	O(1)	O(1)
getEdge	O(1)	O(|V|)
removeEdge	O(1)	O(|V|)
remove	O(|V|)	O(|V|+|E|)
space	O(|V|²)	O(|V|+|E|)

Musing About Real World Graphs

- For each, directed/undirected? sparse/dense?
  - Airports
    - (known as a “hub-and-spoke” system)
  - Social Network (Facebook)
  - Social Network (Twitter)

Next Class

- Wrap up graphs with revisiting the start of the semester (Traveling Salesperson)
- Wrap up course with a stroll down data structure lane
- Some notes about the final and the course
- Some thoughts for the next steps