CSCI 136:
Data Structures
and
Advanced Programming

Lecture 9

Recursion, part 3

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Announcements

- Save the date:
  Final exam: May 15, 1:30pm
- Partners: no, can't go solo
- Partners: next time in your section

Outline

Quiz

Study tip

Mathematical induction

Activity

Mathematical induction

Example

Prove that the sum of the first n integers is:

$$\frac{n(n+1)}{2}$$

Example

Put another way, prove

$$\textbf{P(n)} : 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

For all $n \geq 1$.

We have an unbounded number of hypotheses ("for all $n \geq 1$").

Use mathematical induction.

Example

Step 1: Prove **P(a)**

What would a good **a** be?

$$P(n) : 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

The "simplest" instance is **a = 1**. Let's start there.

Example

Step 1: Prove **P(a)**

$$P(a) : 1 = \frac{1(1+1)}{2}$$

Is this statement true? Yes.

Proof: $\frac{1(1+1)}{2} = \frac{2}{2} = 1$

Example

Step 2: Prove **P(k) ⇒ P(k+1)**

Assume the following is true:

$$\textbf{P(k)} : 1 + 2 + 3 + ... + k = \frac{k(k+1)}{2}$$

Prove:

$$\textbf{P(k+1)} : 1 + 2 + 3 + ... + (k + 1) = \frac{(k+1)((k+1)+1)}{2}$$

Example

Step 2: Prove **P(k) ⇒ P(k+1)**

$$\textbf{P(k+1)} : 1 + 2 + 3 + ... + (k + 1) = \frac{(k+1)((k+1)+1)}{2}$$

Let's handle the left side first.

$$1 + 2 + 3 + ... + (k + 1)$$

Looks familiar. Isn't it the same as:

$$(1 + 2 + 3 + ... + k) + (k + 1)$$

Example

Step 2: Prove **P(k) ⇒ P(k+1)**

$$(1 + 2 + 3 + ... + k) + (k + 1)$$

According to P(k), which is true,
it must be equal to:

$$(1 + 2 + 3 + ... + k) + (k + 1) = \frac{k(k+1)}{2} + (k + 1)$$

Example

Step 2: Prove **P(k) ⇒ P(k+1)**

Simplify

$$= \frac{k(k+1)}{2} + (k + 1)$$

$$= \frac{k(k+1)}{2} + \frac{2(k+1)}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

Let's stop here.
The left side is

$$= \frac{(k+1)(k+2)}{2}$$

Example

Step 2: Prove **P(k) ⇒ P(k+1)**

P(k+1) : $1 + 2 + 3 + ... + (k + 1) = \frac{(k+1)((k+1)+1)}{2}$

Let's handle the right side now.

$$\frac{(k+1)((k+1)+1)}{2}$$

Simplify

$$\frac{(k+1)(k+2)}{2}$$

Let's stop here.

Example

Step 2: Prove **P(k) ⇒ P(k+1)**

P(k+1) : $1 + 2 + 3 + ... + (k + 1) = \frac{(k+1)((k+1)+1)}{2}$

We just showed that the left side

$$\frac{(k+1)(k+2)}{2}$$

equals the right side

$$\frac{(k+1)(k+2)}{2}$$

Example

Step 1: Prove **P(a)** ✓

Step 2: Prove **P(k) ⇒ P(k+1)** ✓

Therefore,

P(n) : $1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

For all $n \geq 1$.

Is **true**. ✓

Activity

Prove: n cents can be obtained by using only 3-cent and 8-cent coins, for all $n \geq 15$.

Activity

Now write a program that gives you the correct change for all $n \geq 15$.

Recap & Next Class

Today we learned:

Mathematical induction

Next class:

Asymptotic analysis (aka “Big-O”)