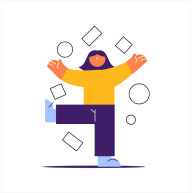
Lec 30: Hash Tables Continued

May 6, 2026

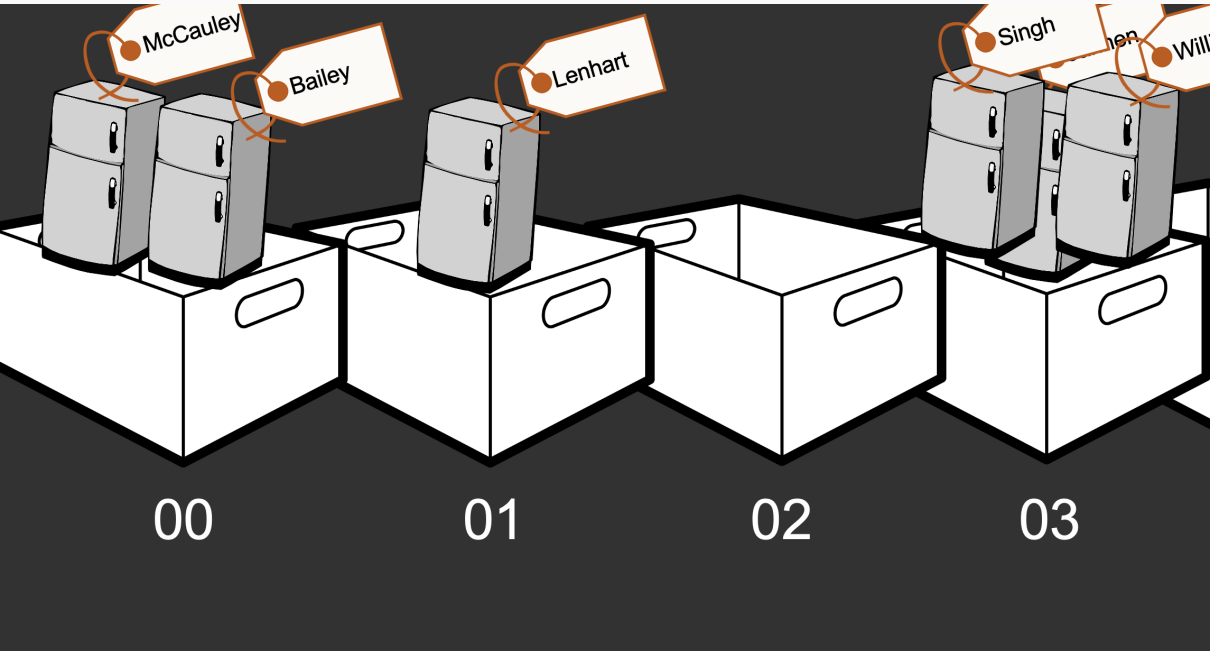
Admin



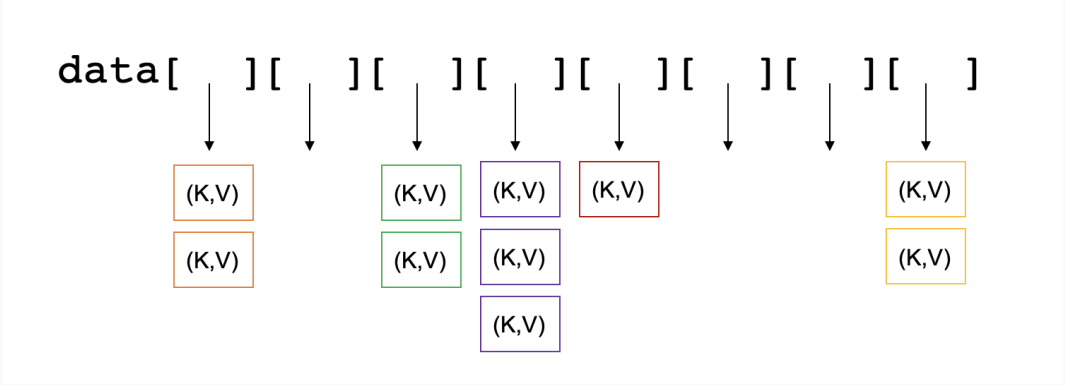
- Remember: decide if you want to take the final during reading period, let me know *today*!
- Today's lab is in-person; on implementing hash tables
- Any questions?

External Chaining

Resolving Collisions in Practice?



External Chaining



- Idea: instead of keeping individual items in each bin, we store a *list* in each bin
- `get()`, `put()`, and `remove()` proceed in two steps: identify the bin using the hash code; then perform a linked list operation

Hash Table with External Chaining Practice

- Let's do the following operations for a hash table with chaining of length 6. Let's assume we are storing a `Map` from the ID of each student to the corresponding `Student` object. We'll assume the hash code of each ID is the ID itself.

- `put(27, new Student("Joyce"));`
- `put(38, new Student("Cora"));`
- `put(9, new Student("Kristie"));`
- `put(14, new Student("Jose"));`
- `put(7, new Student("Evan"));`
- `getValue(14);`

Downsides to External Chaining?

- Each slot in our array stores a list, even if the slot is empty
  - Consumes some extra space (but not much)
- Potentially poor locality
  - Not something we've talked about in the course, but a general rule of thumb:
  - *It is faster to access things that are near to each other than it is to access things that are far away*
  - While array elements are contiguous (near), list elements may be scattered throughout memory (far)

Negative Hash Codes

Negative Hash Codes

- We said that we want to store key k in slot `k.hashCode() % arr.length`
- What happens if `k.hashCode()` is negative?
  - Answer: mod of a negative value is negative in Java. So: we get an *exception*
- How do we fix this? Let's come up with some ideas on the board.

Fixing Negative Hash Codes

- Option 1: use an if statement
  - Works well! Not often used (slightly slow)
- Option 2: Use `Math.abs()` to get the absolute value of the hash code
  - Does *not* work!
  - In Java, the absolute value of the smallest negative integer is negative
- Option 3: Use bit tricks to ensure the first bit of the integer is positive
  - `int slot = (s.hashCode() & 0x7FFFFFFF) % arraySize;`
  - We'll give this to you in the starter code for the lab

Linear Probing

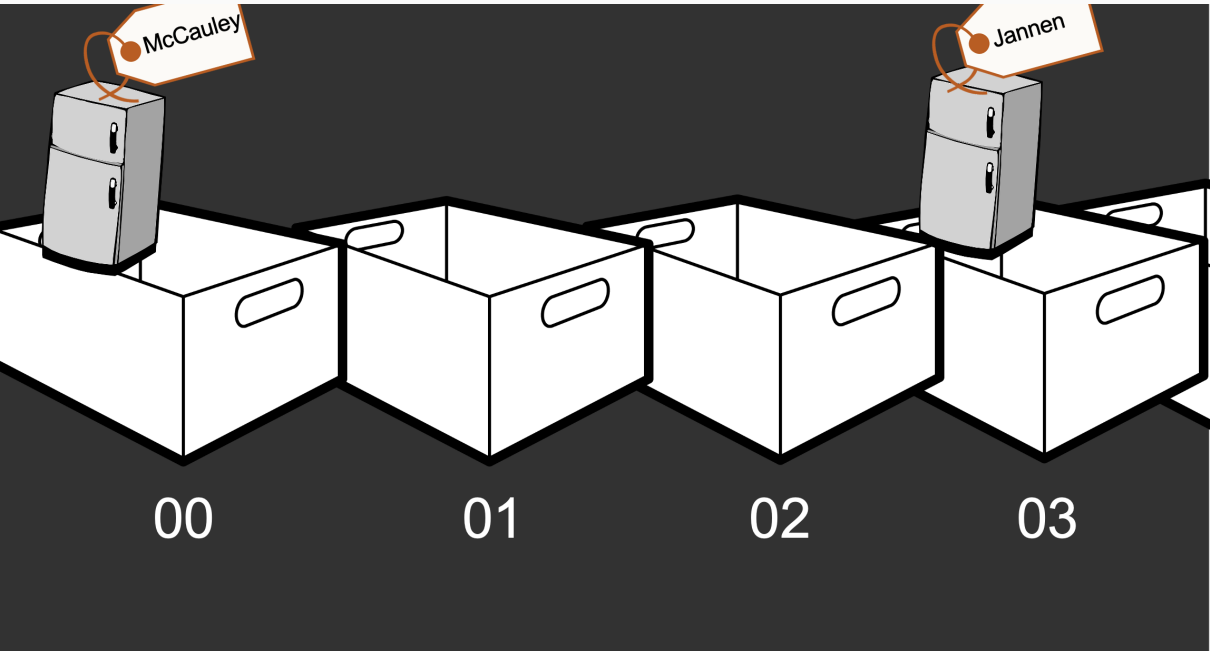
Rethinking Collisions

- Let's define an item's *canonical slot* as the place where the item begins (ignoring collisions)
- Something like the absolute value of the hash code modulo the table size
- If no two items have the same canonical slot, we're done!
- If multiple items do map to the same canonical slot, we need to figure out:
  - Which one goes in the canonical slot?
  - Among items that cannot be stored in their canonical slot, where should they go? How can we find them in the future?

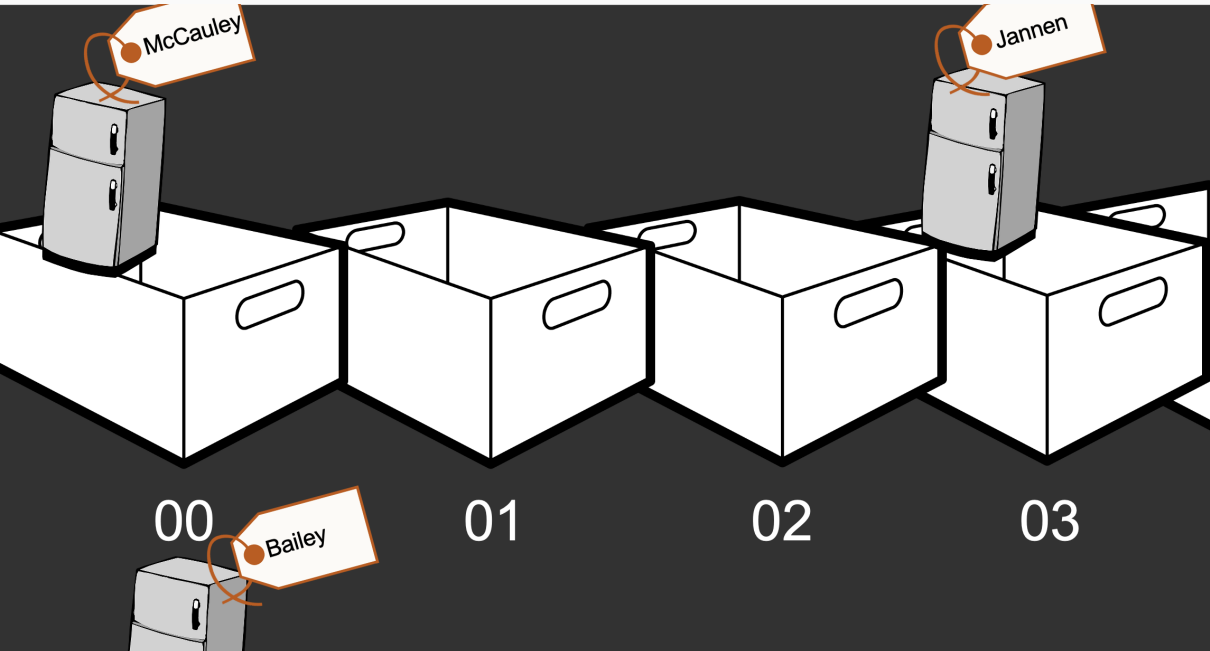
Linear Probing

- General idea: store each key-value pair in the first open slot on or after its canonical slot
- Insertion: if a collision occurs at the bin, just scan forward (linearly) until an empty slot is available; store the item there
  - We “wrap around” at the end of the array
  - Let’s call a contiguous region of full bins a *run*
- Lookup: to find a key-value pair, calculate the bin. Then, scan linearly until the item is found or you reach an empty slot

Linear Probing Example



Linear Probing Example



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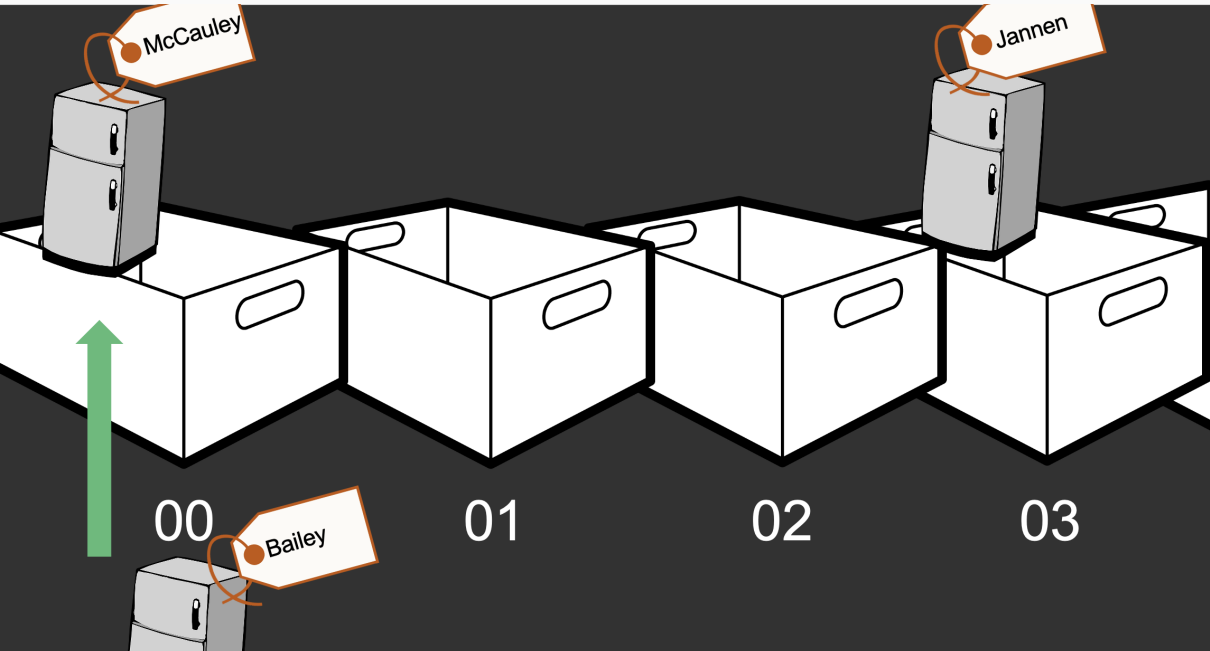

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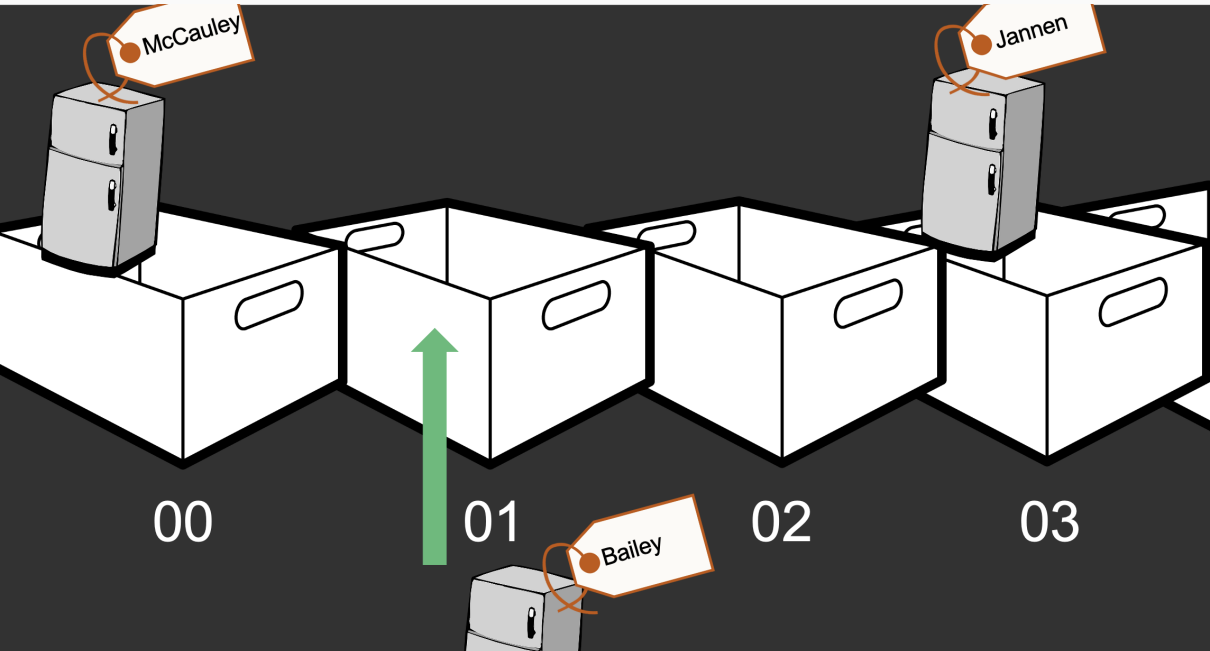

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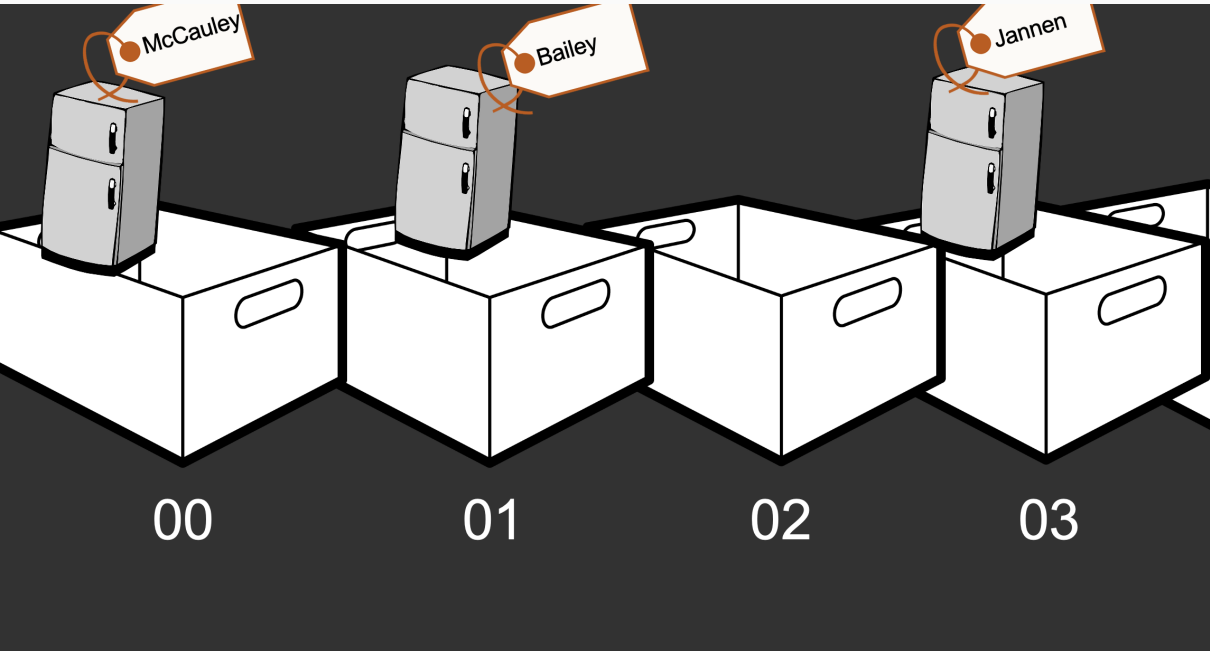

Hash Table with Linear Probing Practice

- Let's do the following operations for a hash table with linear probing of length 6. Let's assume we are storing a `Map` from the ID of each student to the corresponding `Student` object. We'll assume the hash code of each ID is the ID itself.

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- `getValue(7);`

Tricky Part: Deletes

- How do we delete items using linear probing?
- Let's try it on the board: `remove(9)`
- Problem: the "run" is broken up
- In pairs: how can we fix this?

Linear Probing Deletes

- Solution for today: insert a “placeholder”
  - If we see the placeholder during a lookup, we treat it as a collision, and keep scanning until we find a true hole
  - If we see the placeholder during an insertion, we treat it as an open slot
    - Must still scan the whole run to make sure the key isn’t present later on!
- It *is* possible to move elements back to fix the run
  - Uses the hash table more efficiently
  - Finicky: need to ensure you don’t move any items before their canonical slot (may need to scan ahead and take future items)
  - Placeholders are fairly common in practice: save compute time and programmer effort. We’ll stick to placeholders in this course.

Linear Probing Observations

- Downsides of linear probing?
- What if the array is almost full?
  - *Very* long runs
- Does external chaining avoid this problem?
  - Short answer: yes
  - Only scan through collisions, not the entire run
  - Never scans more items than linear probing!
  - But: worse cache behavior (locality)

Hashtable Size

Maintaining Hashtable Size

- Like ArrayList, we need to grow when we run out of space
- What do we mean by running out of space?
- We need to make a trade-off between *space* and performance:
  - We want our table size to be large to minimize collisions (and run/chain lengths): leads to good performance, *bad space*
  - We want our table size to be small to minimize wasted space (empty slots): leads to *good space*, bad performance
- Some flexibility (like with `ArrayLists`): we don't know the size up front

Load Factor

- Suppose a hash table with m slots stores n elements
- *Load factor* is a measure of how full the hash table is

$$\text{load factor} = \frac{\text{\# elements}}{\text{\# slots}} = \frac{n}{m}$$

- A smaller load factor means the hashtable is less full, which likely gives better performance

Using the Load Factor

- We can keep a running count of the table's elements so that we always know the load factor
- Given a hashtable's load factor, what should we do?
  - If the load factor is high (say $> .5$), we grow our table
- How to grow?
- `ArrayList`: `ensureCapacity()` allocates a new `Object` array, then copies elements over
- Does this work for hashtables?

Making Hashtables Larger

- Cannot just copy values! (why?)
- The canonical slot might change
- Example: suppose `key.hashCode() == 11`
- Then `11 % 8 == 3` but `11 % 16 == 11`
- How can we handle this?
- To grow our hashtable, we must recompute the canonical slot for each item, then reinsert the item into the new array

When to grow?

- Choose some load factor
- .50 and .66 are very popular; depends a bit on the use case
- Tradeoff between size and performance
- In the lab we'll use very full hash tables: .8. (It will still be quite fast.) Feel free to play around with this value once you're done with the lab.

Array Sizes

- Some people like using hash tables whose size is a prime
- Reason: remember that we use `% array.length` to calculate the canonical slot
- A prime size can help “spread out” the items
- Downside: need to find a prime size when doubling
- We won’t worry about this in this class; just a heads up. You’ll often see a hash table of size 997 or something—this is why.

Choosing Hash Functions

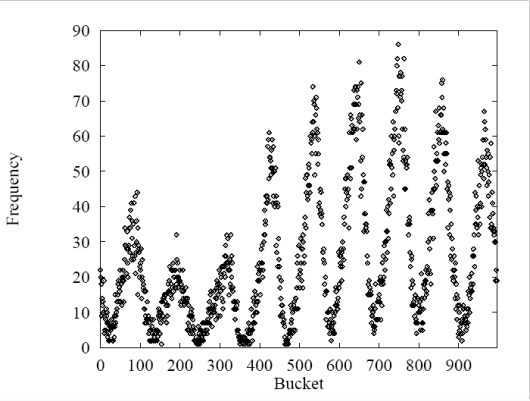
Good Hash Functions

- Good hash functions:
  - Are fast to compute
  - Uniformly distribute keys across the range
- Rules of thumb to make good hash functions?
  - Not really. We almost always have to test “goodness” empirically

Hashing Strings

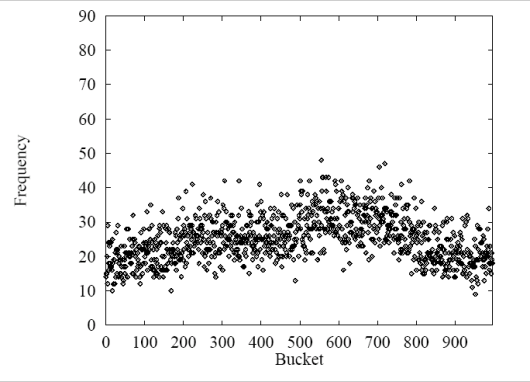
- What are some reasonable hash functions for `Strings`
- One idea: use the first character's Unicode value? (Every character is stores as a number in Java). Problems with this?
  - Can only return `0`–255
  - Not uniform (some letters far more common)
- Sum of the Unicode values of all characters?
  - Still not uniform! (We'll see in a second)
  - Doesn't work well for large hashtables
  - Not good at avoiding collisions: smile, limes, miles, and slime are all the same

Sum of Unicode Values



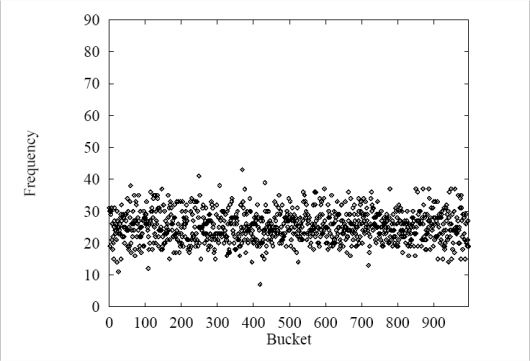
- This is the hash of all words in the UNIX spellchecking dictionary
  - *x*-axis is bucket; *y*-axis is number of words that hash to the bucket
- Uses 997 buckets
- Hash of a string *s*: $\sum_{i=0}^{s.length}$ `s.charAt(i)`

Sum of Unicode Values



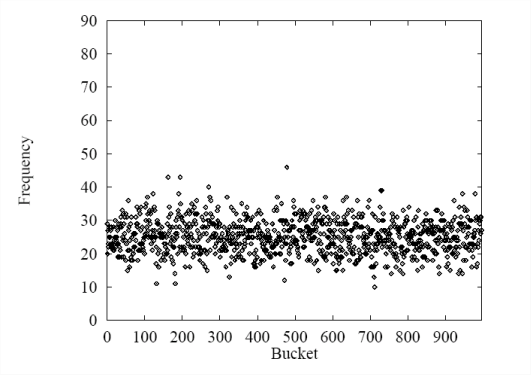
- Hash of a string s: $\sum_{i=0}^{s.length} 2^i \cdot$ `s.charAt(i)`
- Better! But still not great.

Sum of Unicode Values



- Hash of a string s: $\sum_{i=0}^{s.length} 256^i \cdot$ `s.charAt(i)`
- Really good! But do we need numbers as big as 256^i?

Sum of Unicode Values



- Hash of a string *s*: $\sum_{i=0}^{s.length} 31^i \cdot$ `s.charAt(i)`
- This is (essentially) what Java uses to hash strings!

Other Objects?

- Integers?
  - In Java: `i.hashCode()` is `i`
  - Could be terrible depending on your data
  - Might want to use another `hashCode()` method in that case
  - One popular one (has theoretical performance guarantees!):

$$h(x) = (ax + b) \% p$$

- What about other classes?
- Write your own (probably similar) `hashCode()` methods. Test empirically to make sure elements are spread out

Hashtable Performance

Hashtable Performance

- Given the hash code of an object `o`, how long does `get(o)` take?
- *O*(run length) for linear probing; *O*(chain length) for external chaining
- How long does calculating a hash code take?
  - Can be long for, say, a long string.
  - *O*(1) in terms of the number of items in the hash table
  - Another example of being careful about how we're stating our running time. Usually: in terms of number of strings in the table. But do we care about the length of our strings?

Impact on Performance

- Let's say we have constant load factor
- Assume we have a good hash function
  - Spreads objects out "like random"
- Then an *average* bucket has constant chain length
- An *average* bucket is in a run of constant length
- (With overwhelming probability, never gets worse than $O(\log n)$ for any bucket)
- Usually we say we have $O(1)$ performance. True on average; the actual worst case might be a bit worse

Summary of Map Performance

	`put`	`get`	space
Unsorted ArrayList	$O(n)$	$O(n)$	$O(n)$
Unsorted LinkedList	$O(n)$	$O(n)$	$O(n)$
Sorted ArrayList	$O(n)$	$O(\log n)$	$O(n)$
Balanced BST (AVL Tree)	$O(\log n)$	$O(\log n)$	$O(n)$
Hashtable (average)	$O(1)$	$O(1)$	$O(n)$